HOMEWORK #1

BME 473 ~ Applied Biomechanics

Tuned Track and Locomotion for Midterm 1 - Not Due

1. Use equation 8.7 and figure 8.30 from McMahon.

   a. Derive an expression for the track stiffness k_t^*, below which running would be impossible because the runner's hips would descend below the track surface.

   b. *Briefly* discuss your results.

   c. Describe what assumptions were made in deriving equation 8.7 and how do you think they affect your answer?

2. We used a spring-mass-dashpot system in the track design problem (i.e., figure 8.25 from McMahon). For that mechanical system, please answer the following questions.

   a. Derive an expression for the system's natural frequency, ω_n, with no damping (b=0).

   b. What is the physical significance of the natural frequency calculated in (a)?

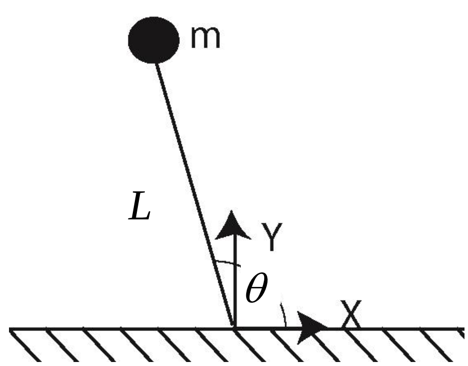
   c. Derive an expression for the system's damped natural frequency, ω_d, and damping ratio, ζ, when $k_t \rightarrow \infty$ (rigid track)

   d. What is the physical significance of the damped natural frequency calculated in (c) and how is it related to the damping ratio?

   Hint: you may want to look in a differential equations, dynamics, or control systems book under natural frequency, damping, or vibrations.

3. McMahon states that "the model of Fig. 8.10 is the least complicated mechanical configuration mechanical configuration one out to have in mind when thinking about the dynamics of walking. A compound pendulum alone or an inverted pendulum alone is not enough."

   Explore this statement by analyzing a simple inverted pendulum of mass m, length L, and angular position with respect to the ground given by θ.



   a. Based on the figure to the right, derive expressions for the x and y positions of the mass, m, as a function of the length of the limb, L, and rotation angle, θ.

b. Derive expressions for the vertical and horizontal ground reaction forces as a function of the length of the limb, L, and rotation angle, θ. (*Hint: assume the presence of gravity that is directed in the $-y$ direction. Use this term if you need it*)

c. Given the following:

L = 1 m $\quad\quad$ m = 1 kg

$\dot{\theta}$ = 1 °/sec $\quad\quad$ $\ddot{\theta}$ = 0 °/s^2

Plot the vertical position of the mass as θ varies over a range from 120° to 60°. Briefly discuss how this plot relates to the motion paths shown in the determinants of gait discussion from class.

d. For the same constants given in part (c) above, plot the vertical ground reaction force as θ varies over a range from 120° to 60°. Also, plot the vertical ground reaction force when $\ddot{\theta}$ = 1 °/s^2 (θ over the same range as before). Briefly discuss how your plot compares to the plot of vertical ground reaction force that was shown in class.

e. What are the assumptions made in modeling gait as an inverted pendulum?

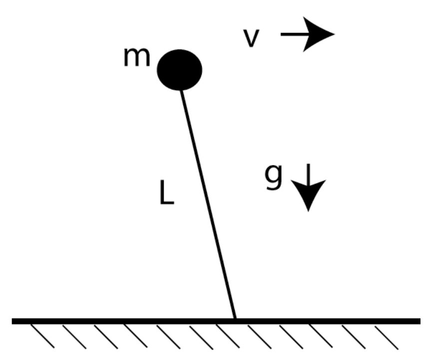
f. We took a very simple approach for the model in this problem (i.e., constant velocities and angular accelerations). Can you think of modifications of an inverted pendulum model so that it would more accurately represent aspects of normal locomotion? (*If you're really adventurous, try out some ideas and plot your results*)

4. The Froude number (sounds like "food" with an "r") is a useful parameter to study how locomotion changes with body size. It is a dimensionless variable that relates a scalar dimension of the body (typically leg length) to the forward velocity of the body. Theoretically, animals of different sizes that rely on pendulum-like mechanics for gait will use the same form of locomotion at a given Froude number (e.g., a T-Rex and a chicken traveling at the same Froude number would look the same). The Froude number is defined as:

$$Fr = \frac{v^2}{(g * L)}$$

where v = the speed of movement (in m/s)
g = the acceleration due to gravity (in m/s^2)
L = the effective length of the limb (in m)

a. Using the simple inverted pendulum model from the previous question, explain why the walk-run transition theoretically occurs at Fr = 1. (*Note:*



the model is redrawn to the right with the linear velocity and gravitational terms shown)

b. Dr. Evil (a somewhat normal adult male with a leg length of 0.9 m) and Mini-Me (an exact 1/8 replica of Dr. Evil) need to go to the other side of their secret lair in Nevada to feed their sharks and repair some laser beams. Dr. Evil demands that Mini-Me hold his hand so they can remain side-by-side as they travel to the shark tank. Dr. Evil walks to the shark tank at optimum walking speed (Fr = 0.25). Once arriving at the destination, Mini-Me says that he is too tired from running to keep up with Dr. Evil to feed the sharks. How can you explain this?

c. Dr. Evil is tired of having a miniature version of himself and orders the biomechanical engineers on his lunar base ($g_{Moon}=g_{Earth}/6$) to build a robot that looks, walks, and behaves exactly how he does on Earth. Why is the lunar engineering team doomed to failure?

d. Walking speed is a good indicator of mobility and is often used in research of gait disorders. It has been suggested that walking speed should be reported in terms of the Froude number to remove the effects of individual subject height in experiments. Suppose a patient with cerebral palsy walks with excessive knee flexion (this is called crouch gait). Knowing that she needs a leg length to compute Fr, the clinician measures the patient's leg length while he lies on the exam table. She also measures his walking speed. She is pleased when she computes Fr = 0.25, which suggests a reasonable walking speed for the patient’s stature and thus good energy efficiency. However, after measuring the patient's oxygen consumption, she finds that his gait is not energetically efficient at all. Explain this discrepancy in her results using the Froude number. Based on this information, do you think that Fr is a good clinical indicator of walking speed?