HOMEWORK #2

BME 473 ~ Applied Biomechanics

Muscle Architecture, Force, and Locomotion for Midterm 1
Muscle-tendon, Moment Arms, and Adaptation for Midterm 2
Not Due

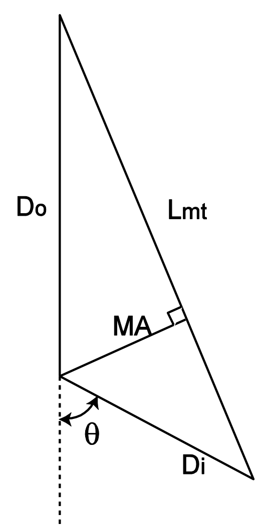
1. One way to estimate moment arm of a muscle with respect to a particular degree of freedom is to measure the length of the muscle-tendon complex over a range of joint motion and then use the equation:

$$ma = {\delta l^{MT}}/{\delta\theta} \tag{1}$$

   where ma is the moment arm, l^{MT} is the muscle-tendon length, and θ is the joint angle. Prove that equation (1) is true. You may prove this with geometry or the principal of virtual work.

2. Moment arms determine the moment-generating potential as well as the amount a muscle changes length with joint rotation. Moment arms vary with joint angle. They are determined by the geometry of the joint (e.g., center of rotation and the relative locations of the attachment points of the muscle with respect to the joint). This question allows you to explore this relationship:

   a. Two important parameters about moment arm verses joint angle relationship are: 1) the peak moment arm and 2) the angle at which the moment arm peaks. In order to get a larger peak moment arm, would you want your muscle attachments to be closer or further to the joint?



   b. Based on the triangle model to the right, determine the value of the peak moment arm (MA) and the angle (θ) at which the moment arm peaks, in terms of distances from the attachment sites to the joint center, Do and Di. Assume that Di < Do.

   c. What are the limitations to assuming a model such as this one?

   d. Why are peak moment arm and the angle at which the moment arms peak important?

3. Muscles Hans and Franz have both entered the prestigious Muscle Olympics (leaving their pesky tendons at home). They will each compete in three events in an attempt to bring home the coveted Golden Sarcomere. Being the poor

student that you are, you're hoping to make a little extra cash by placing some bets on the competition. The Olympic program with the competitor's vital stats has just arrived in the mail and your bookie is on the phone – what bets do you want to place?

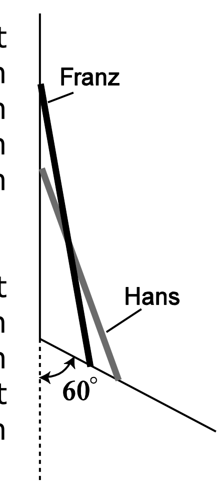
For each event below, predict whether Hans or Franz will win. Defend your choice using muscle mechanics principles and equations we've discussed in class. (*Note: You get credit for your reasons, not for just correctly guessing the winner*). If there is a muscle property that you think is important that isn't listed, assume that it is the same for both Hans and Franz. Assume both muscles have linear force-length properties and moment arms that are constant with joint angle. For all events, the muscles are loaded into a revolute joint with the given moment arm.

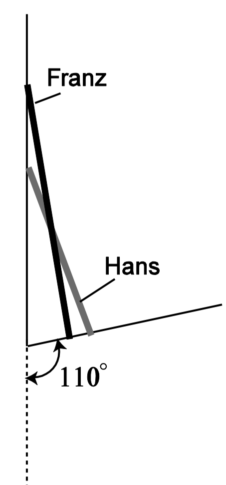
Muscle Property	**Hans**	**Franz**
Max isometric force F_{max} (N)	150	150
Optimal fiber length L_O (cm)	6	6
Moment arm ma (cm)	2	1.75

a. "Clean and Jerk": The joint moves through a range of motion at a constant angular velocity of 100°/sec. The winning muscle is that which produces the most force at the instant that it reaches optimal fiber length.

b. "60° Torque-off": Each joint is fixed at 60°. The winning muscle is that which produces the largest torque during an isometric contraction. Assume that each muscle is at optimal fiber length when the joint is at 60°.

c. "110° Torque-off": Each joint is fixed at 110°. The winning muscle is that which produces the largest torque during an isometric contraction. Assume again that each muscle is at optimal fiber length when the joint is at 60°.





d. Hans and Franz were not satisfied with their performances in the Torque-off competitions – they each thought they should be able to win both the 60° and 110° categories. They convince the Olympic officials to let them have a rematch. Over the weekend, both muscles worked hard to try to improve their odds. Franz went to the local Gold's Gym and bulked up. Hans went to the local yoga studio and stayed in downward-facing dog all weekend. After this intense conditioning, their vital stats had changed (see below) – get your bookie back on the phone and predict who will win the two Torque-off competitions now.

Muscle Property	**Hans**	**Franz**

Max isometric force F_{max} (N)	150	200
Optimal fiber length L_O (cm)	9	6
Moment arm ma (cm)	2	1.75

e. Using what you learned in this problem, comment on why we may have redundant muscles in our bodies.

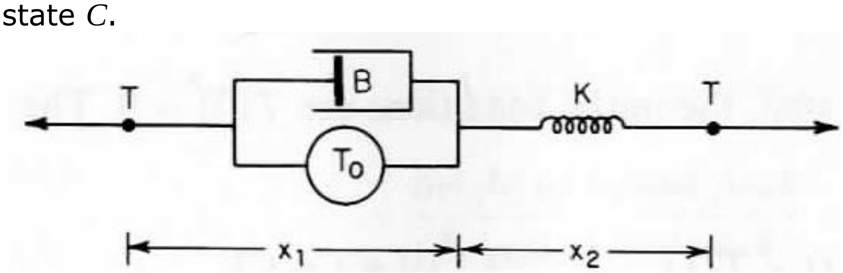
Disclaimer: Don't take Hans's and Franz's experiences literally. Only a chronic stretch and a sustained weight training program could actually change the properties of your muscles – but you already knew that.

4. In our dimensionless model of muscle and tendon, we have scaled the cross sectional area of tendon by maximum muscle force. This essentially assumes that the ratio of muscle cross-sectional area to tendon cross sectional area is constant among various muscles.

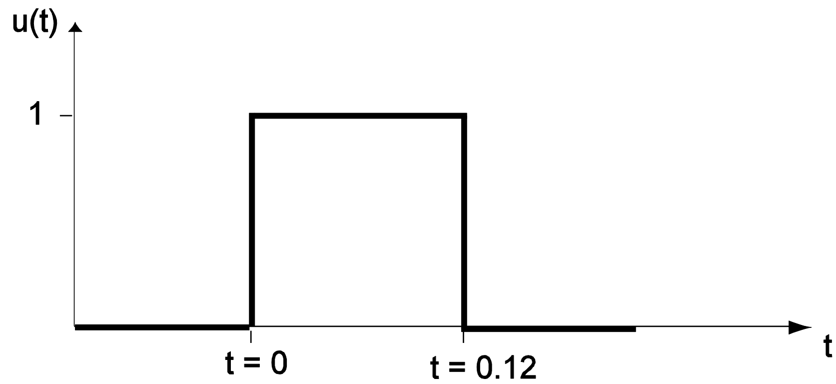
    a. Find the ratio of muscle cross-sectional area to tendon cross sectional area.

    b. Do you think this ratio being constant is a reasonable assumption? State why or why not.

5. The ratio of the peak tension developed in a single, isolated twitch T_C to tetanic tension T_0 is called the twitch/tetanus ratio. Experimental studies have shown that it is higher in fast muscles than in slow ones, and it declines in fatigued muscles. Use the linear three-element model below to calculate the twitch/tetanus ratio in terms of the parameters of the model and the duration of the active state C.



6. Given the neural excitation $u(t)$ shown below, plot the muscle activation $a(t)$ as a function of time. Assume that the time constant for activation is 0.12 seconds and the time constant for deactivation is 0.24 seconds.



7. A young biomechanist, Ted, is attempting to estimate a muscle's active force-length curve. However, he cannot isolate the muscle of interest, but instead can only perform experiments on the muscle-tendon actuator as whole. He performs a series of trials where the musculotendon actuator is held isometric and tendon force is measured in steady state. First, he plots the passive (i.e., $a(t) = 0$) force curve. Later, he measures the fully-activated (i.e., $a(t) = 1$) force curve, which includes passive muscle force (we have called this the total force curve). By subtracting the passive curve from the fully activated curve, he hopes to accurately estimate of the muscle's active force length curve.

   Assume the muscle and tendon properties are linear. **Find and plot** the passive force-length curve and fully activated force-length curve you would get in the experiment Ted did. From these curves estimate the active force-length curve by subtracting the former from the latter. Compare the estimated active force-length curves you get this way with the active force-length curve of muscle alone when:

   a. Tendon is stiff (i.e., when tendon slack length is 0.1 times optimal fiber length)
   b. Tendon is compliant (i.e., when tendon slack length is 10 times optimal fiber length).
   c. Discuss your results.

   Note: Please use axes similar to those shown below.