Review of: Dynamic Optimization Analysis for Equipment Setup Problems in Endurance Cycling

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Why Do We Care?

- Dynamic Optimization
  - Powerful tool for investigating human movement
  - Optimal control theory allows estimation of neuromuscular system controls
    - Net joint torques or muscle activations

- Example Applications of Optimal Control Theory
  - Gait, human jumping, pedaling at maximal speed, and other general movements

What About Cycling?

- Inverse Dynamics vs. Dynamic Optimization
  - Inverse dynamics needs pedal forces
    - Depend on inertial effects of leg motion
  - Inverse dynamics has problems with pedal force changes due to altered coordination strategy (control theory)
  - How can coordination be altered without worrying about pedal forces?
    - Dynamic Optimization

Specific Goals

- Develop a dynamic optimization framework for analysis of equipment setup problems in endurance cycling

- Illustrate the application of this framework by determining the optimal chainring shape

Mathematical Model

- Similar to Fregly and Zajac
  - 3 DOF
  - Seated
  - 2 legs
  - Cycling ergometer
- Equations of Motion

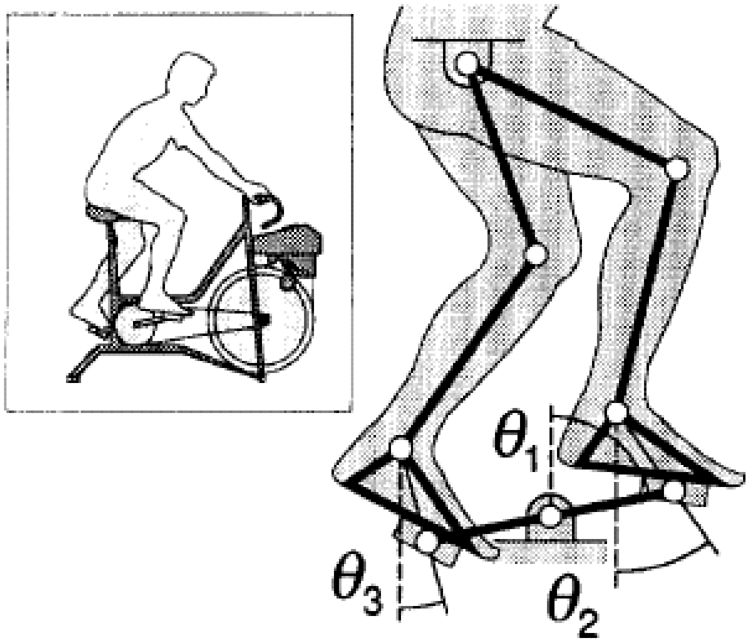
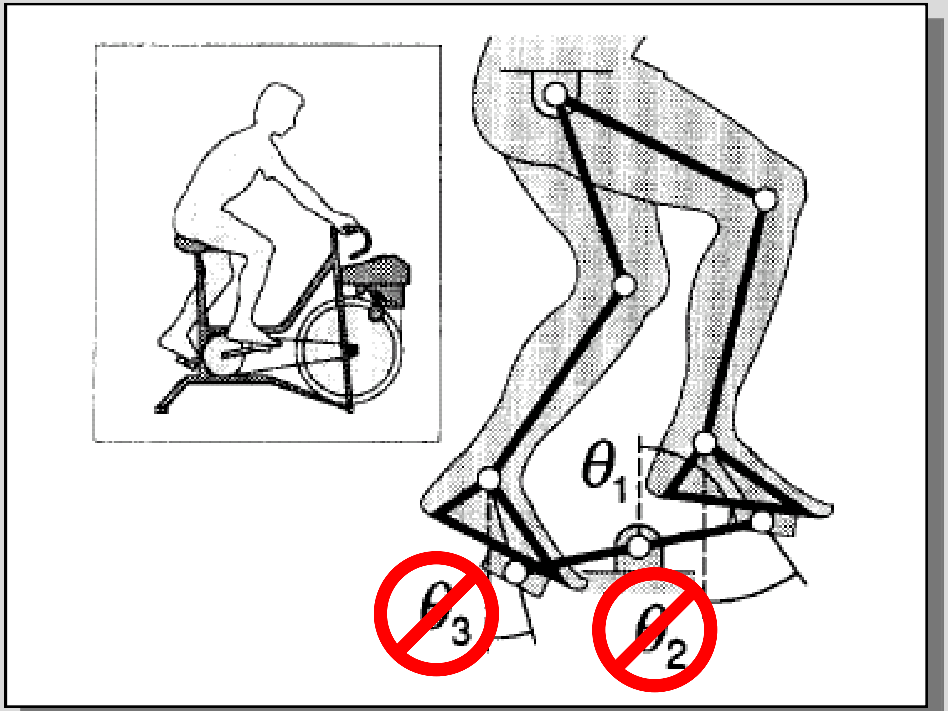


Fig. 1. Three degree-of-freedom dynamical model of seated, two-legged pedaling of a stationary bicycle ergometer. The crank angle θ_1 and the two foot segment angles θ_2 and θ_3 with respect to an inertial reference frame are the three generalized coordinates. All joints are assumed to be frictionless and revolute, and both hips are assumed to remain stationary. The total load that the cyclist experiences at the crank due to all ergometer components is modeled by an 'effective' inertia and an 'effective' friction.

Mathematical Model

- Simplification
  - 1 DOF
  - Pedal angles as functions of time



- Equation of Motion

$$\underbrace{\boxed{M(\theta,t)}}_{\text{Mass}}\ddot{\theta} = \underbrace{\boxed{T(\theta,t)}}_{\text{Torque}} + \underbrace{\boxed{V(\theta,\dot{\theta},t)}}_{\text{Velocity}} + \underbrace{\boxed{G(\theta,t)}}_{\text{Gravity}} + \underbrace{\boxed{F(\theta,t)}}_{\text{Friction}}$$

Mathematical Model

- Prescribed Pedal Angle
  - M_1 fulfills constraint

$$M_a = M_1$$
$$M_k = M_k{}^* + M_1$$
$$M_h = M_h{}^* + M_1$$

- Inputs
  - $M_h{}^*$ and $M_k{}^*$
- Outputs
  - M_h , M_k , M_a , $\ddot{\theta}$

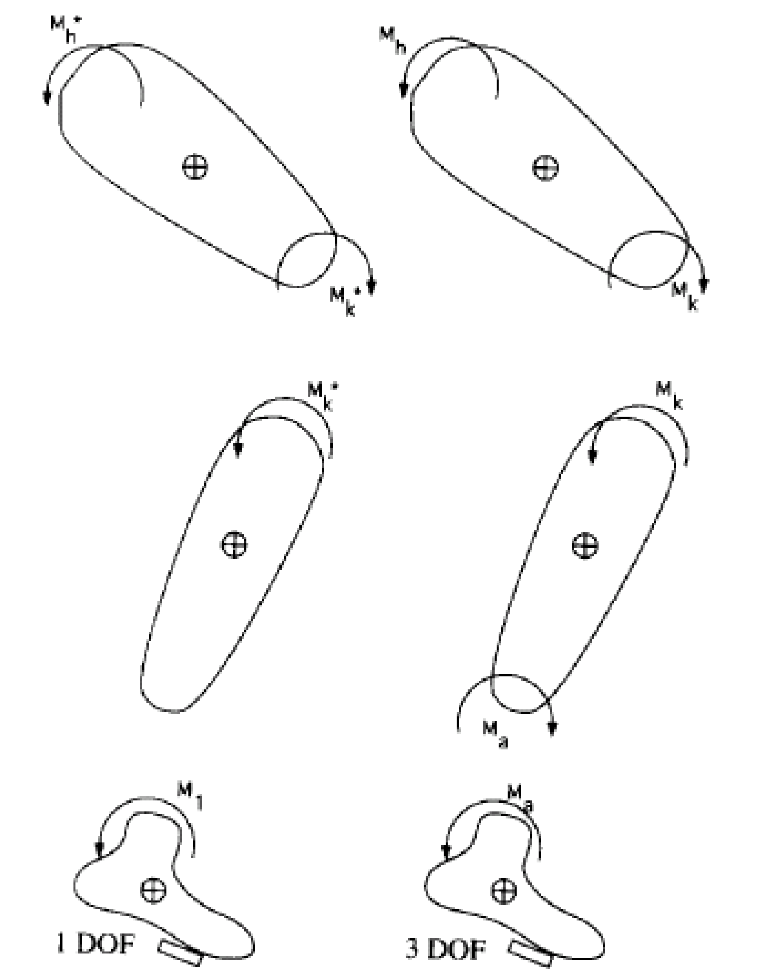


Fig. 1. Comparison of moments applied to each leg in 1 DOF vs. 3 DOF model.

Mathematical Model

- Dynamic response of ergometer equipped with a non-circular chainring

$$I_{eq} = I_c + \left(\frac{R_c}{R_s}\right)^2 I_f \qquad T_{res} = \frac{R_c}{R_s} F_{fric} R - \frac{R_c}{R_s^{\,2}} I_f \, \omega_c^{\,2} \frac{\mathrm{d}R_c}{\mathrm{d}\theta}$$

I_c	moment of inertia of the crankset
R_c	instantaneous radius of the chainring
R_s	radius of the flywheel sprocket
I_f	moment of inertia of the flywheel
R	radius of the flywheel
ω_c	angular velocity of the crank
F_{fric}	frictional force applied to flywheel

Mathematical Model

Equivalent Inertia

$$I_{eq} = I_c + \left(\frac{R_c}{R_s}\right)^2 I_f$$

Resistance About Crank

$$T_{res} = \frac{R_c}{R_s} F_{fric} R - \frac{R_c}{R_s^{\,2}} I_f \, \omega_c^{\,2} \frac{\mathrm{d}R_c}{\mathrm{d}\theta}$$

$$\boxed{M(\theta,t)}\,\ddot{\theta} = T(\theta,t) + V(\theta,\dot{\theta},t) + G(\theta,t) + \boxed{F(\theta,t)}$$

- Equation of motion as a function of chainring shape and control moments

Dynamic Optimization Framework

- Objective Function

$$\min \int_0^{t_f} L(x(t), u(t), t)\, dt = \min \int_0^{t_f} 2\,(M_h{}^2 + M_k{}^2 + M_a{}^2)\, dt$$

- Design Variables

$u_1 = M_h{}^*$	(hip moment of right leg)
$u_2 = M_k{}^*$	(knee moment of right leg)
$u_3 = M_h{}^*$	(hip moment of left leg)
$u_4 = M_k{}^*$	(knee moment of left leg)
$u_5 = dR_c/d\theta$	(chainring rate of change)

Dynamic Optimization Framework

- Constraint Equations

$t_f = 0.66$ s	(90 rpm cadence)
$x_1(0) = 0$	(initial crank angle)
$x_1(t_f) = 2\pi$	(1 full revolution of crank)
$x_2(t_f) = x_2(0)$	(cyclical angular velocity)
$x_4(t_f) = x_4(0)$	(cyclical chainring radius)

Computational Biomechanics Laboratory

Dynamic Optimization Framework

- Constraint Equations (continued)

$u_1(t + t_f) = u_1(t)$	(periodic right hip control torque)
$u_2(t + t_f) = u_2(t)$	(periodic right knee control torque)
$u_3(t) = u_1(t + t_f/2)$	(left hip ½ period out of phase)
$u_4(t) = u_2(t + t_f/2)$	(left knee ½ period out of phase)
$u_5(t + t_f/2) = u_5(t)$	(symmetric chainring about crank)

Dynamic Optimization Framework

- Non-linear Programming Algorithm
  - Readily available
  - Initial conditions and control nodes adjusted simultaneously
    - 22 nodes for right hip control
    - 22 nodes for right knee control
    - 11 nodes for chainring rate of change
    - 1 initial crank angular velocity
    - 1 initial chainring radius

Example Application

- Determine the optimal shape of the chainring to reduce the cost of pedaling (i.e., joint moments)
- Assess the decrease in cost along with corresponding joint moments and pedal forces
  - Non-circular chainring
  - Circular chainring
    - Radius is average of non-circular radius
- Compare with independent reference case

Optimal Chainring Shape

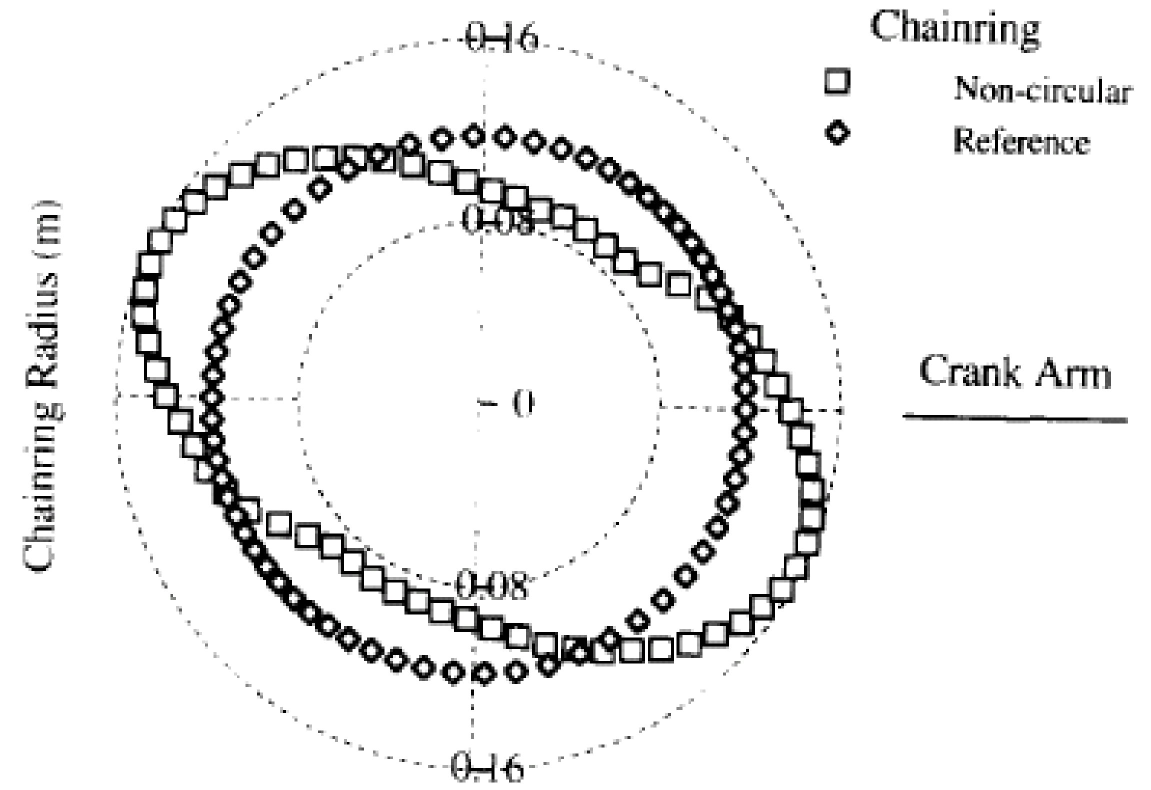


Fig. 2. Polar plot of chainring shape with cranks mounted horizontally (cranks rotate clockwise). Note the cusps in the non-circular chainring. The desired angular velocity profile could not implemented using conventional cycling equipment.

Equivalent Cadence

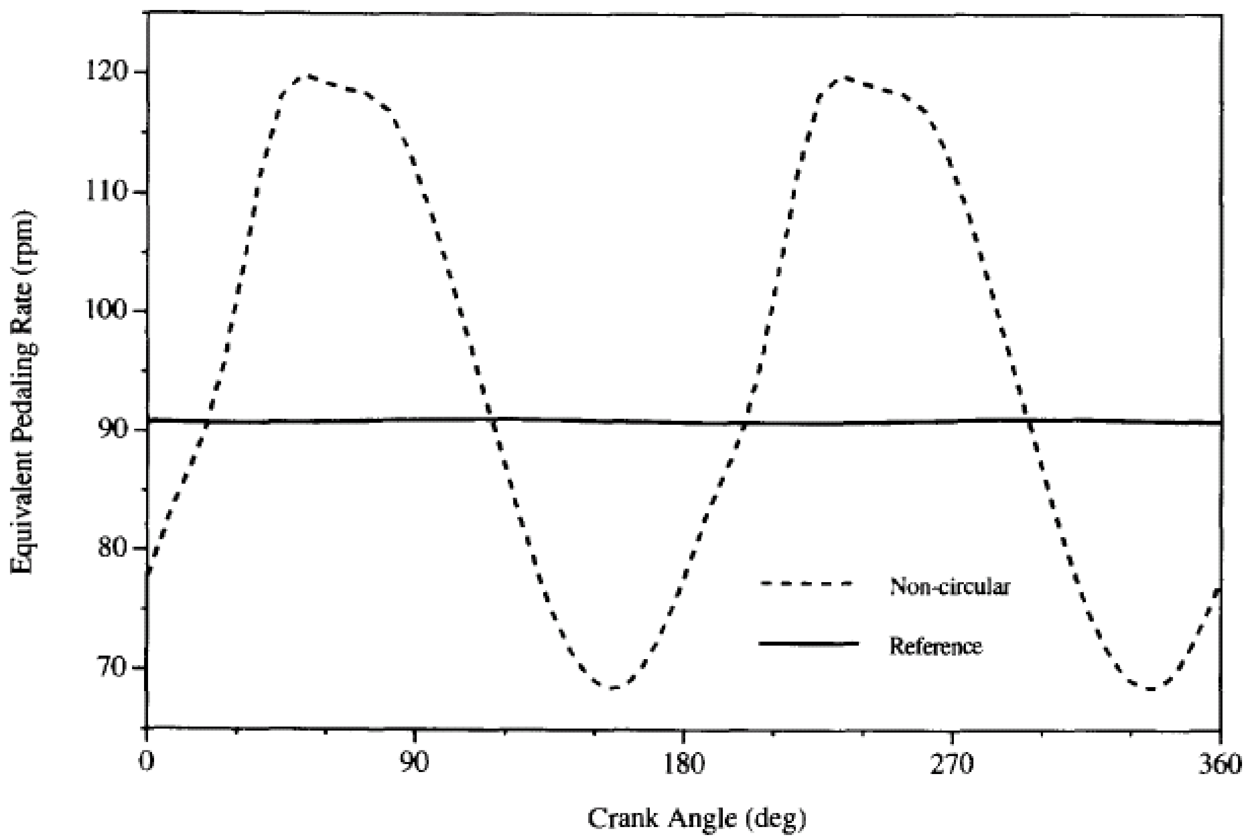
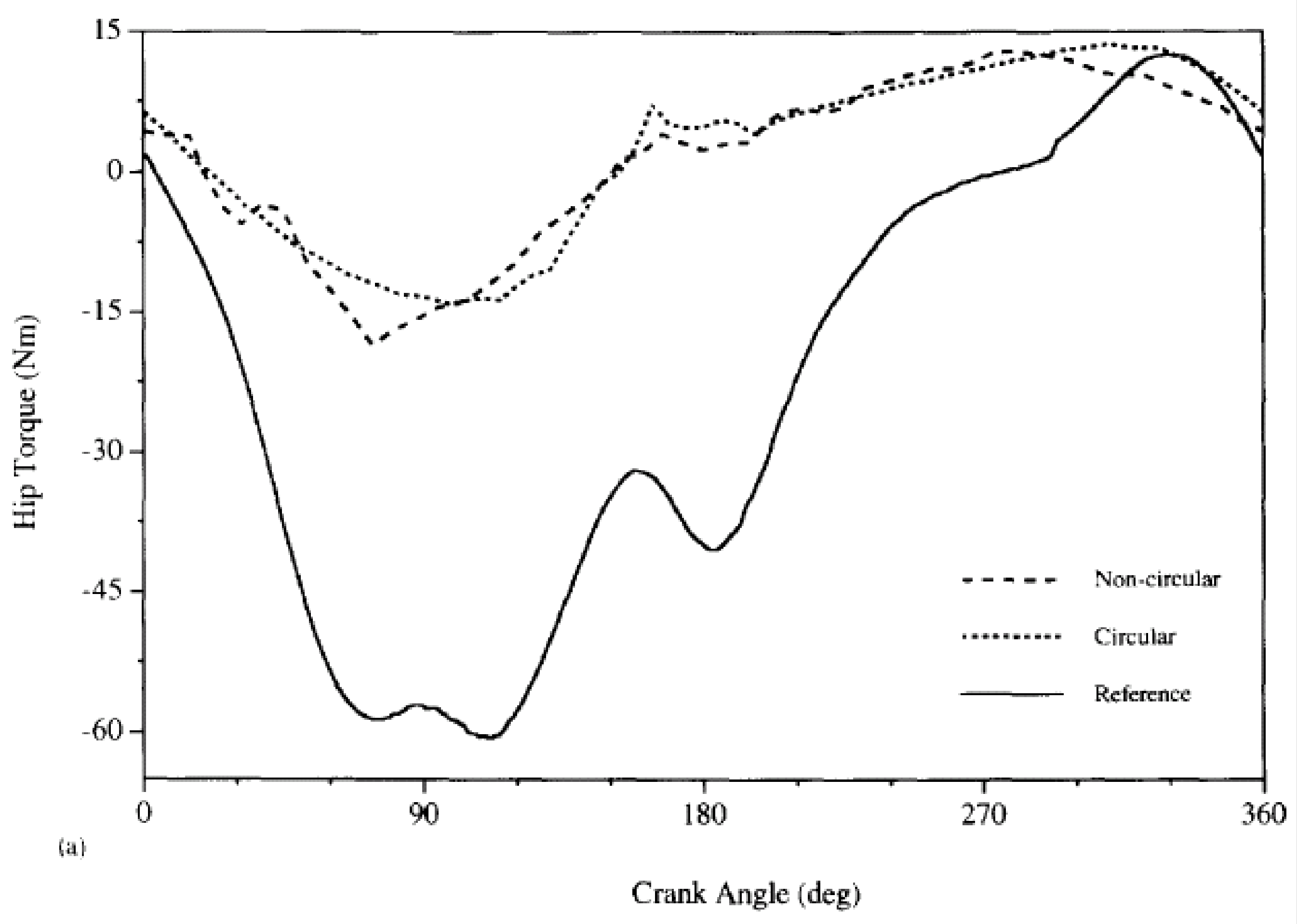
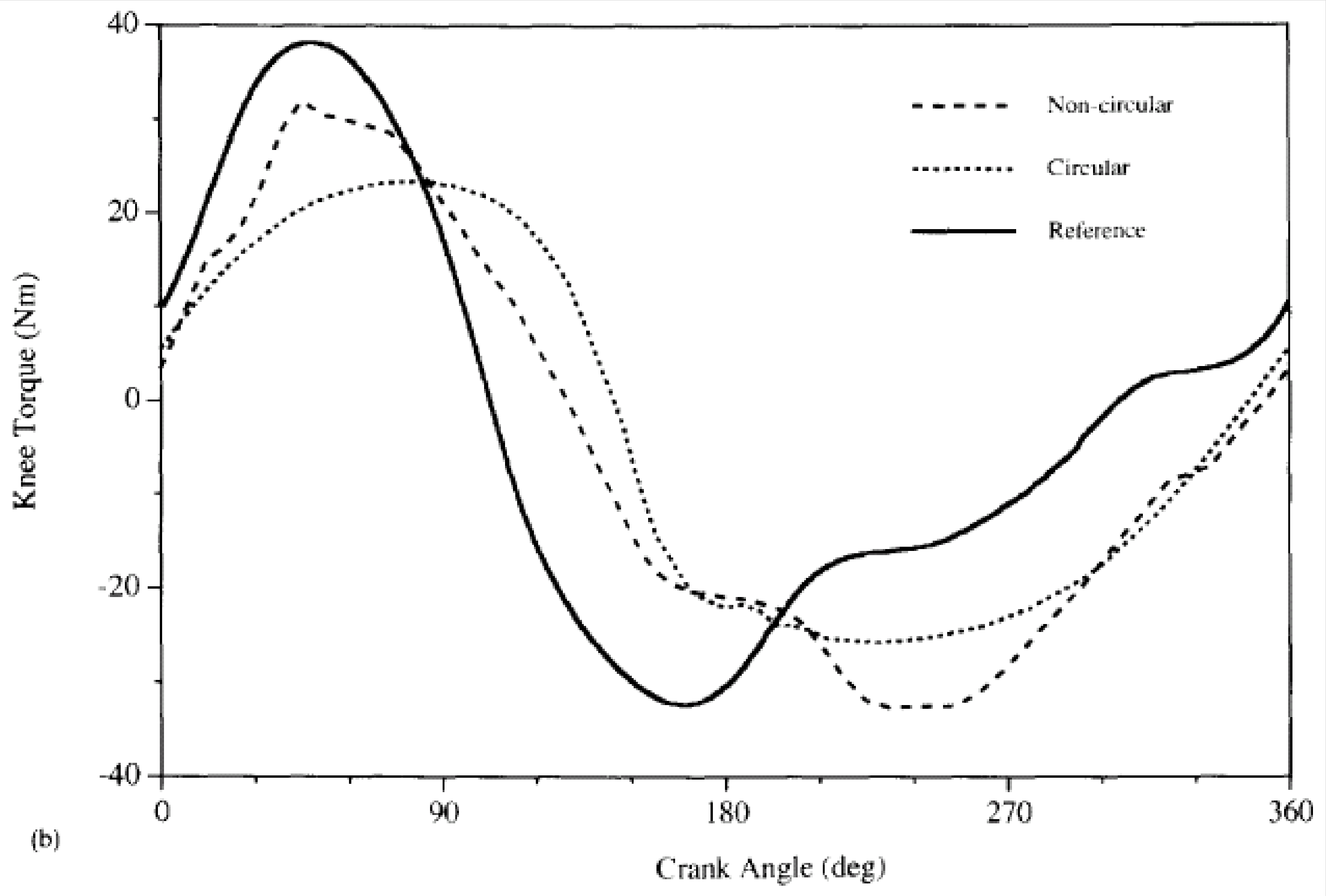


Fig. 3. (a) Vertical radius for the non-circular and equal perimeter circular chainring plotted as a function of crank angle. (b) Instantaneous angular velocity (expressed as an equivalent cadence) induced by the non-circular and circular chainring plotted as a function of crank angle.

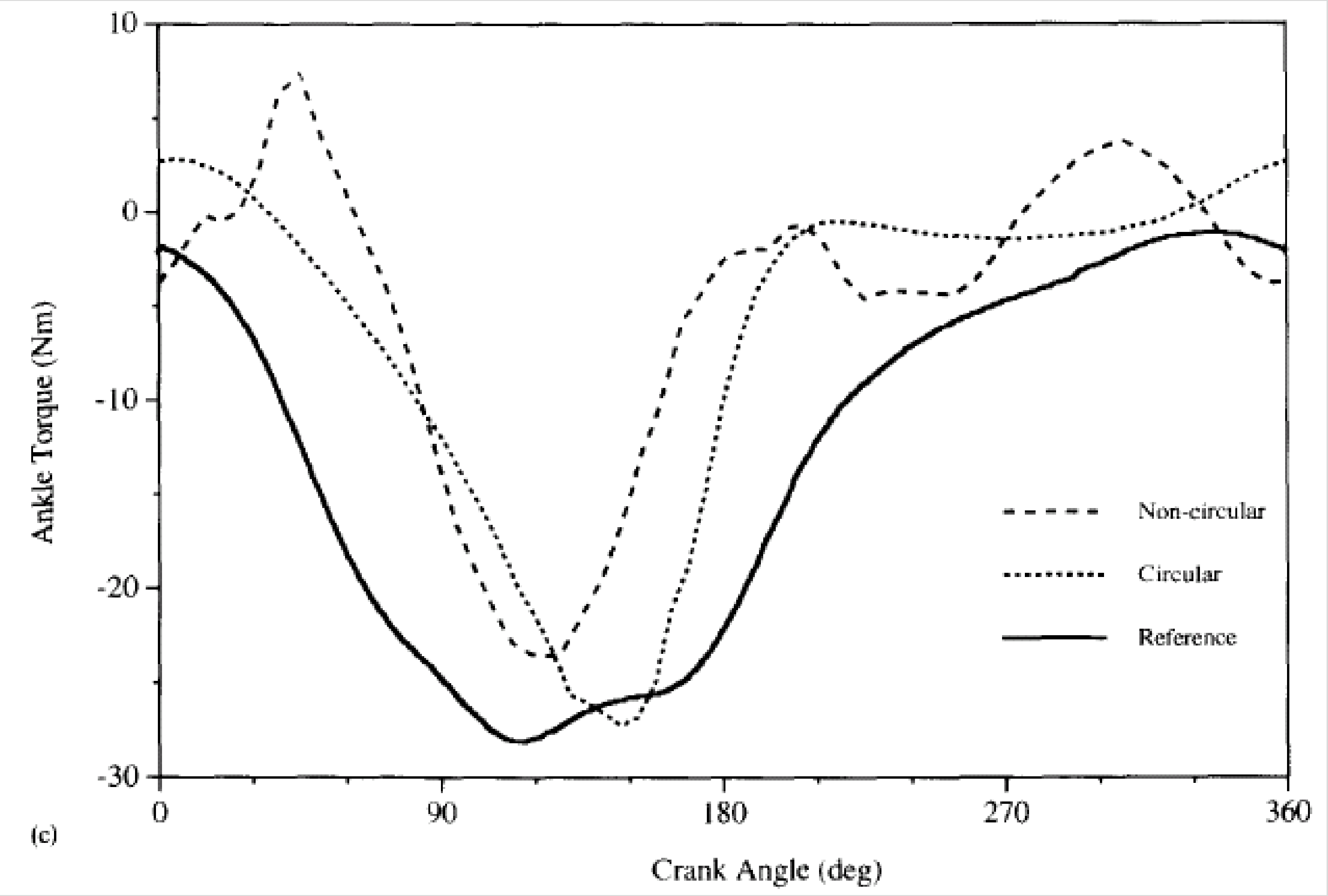
Hip Joint Torque



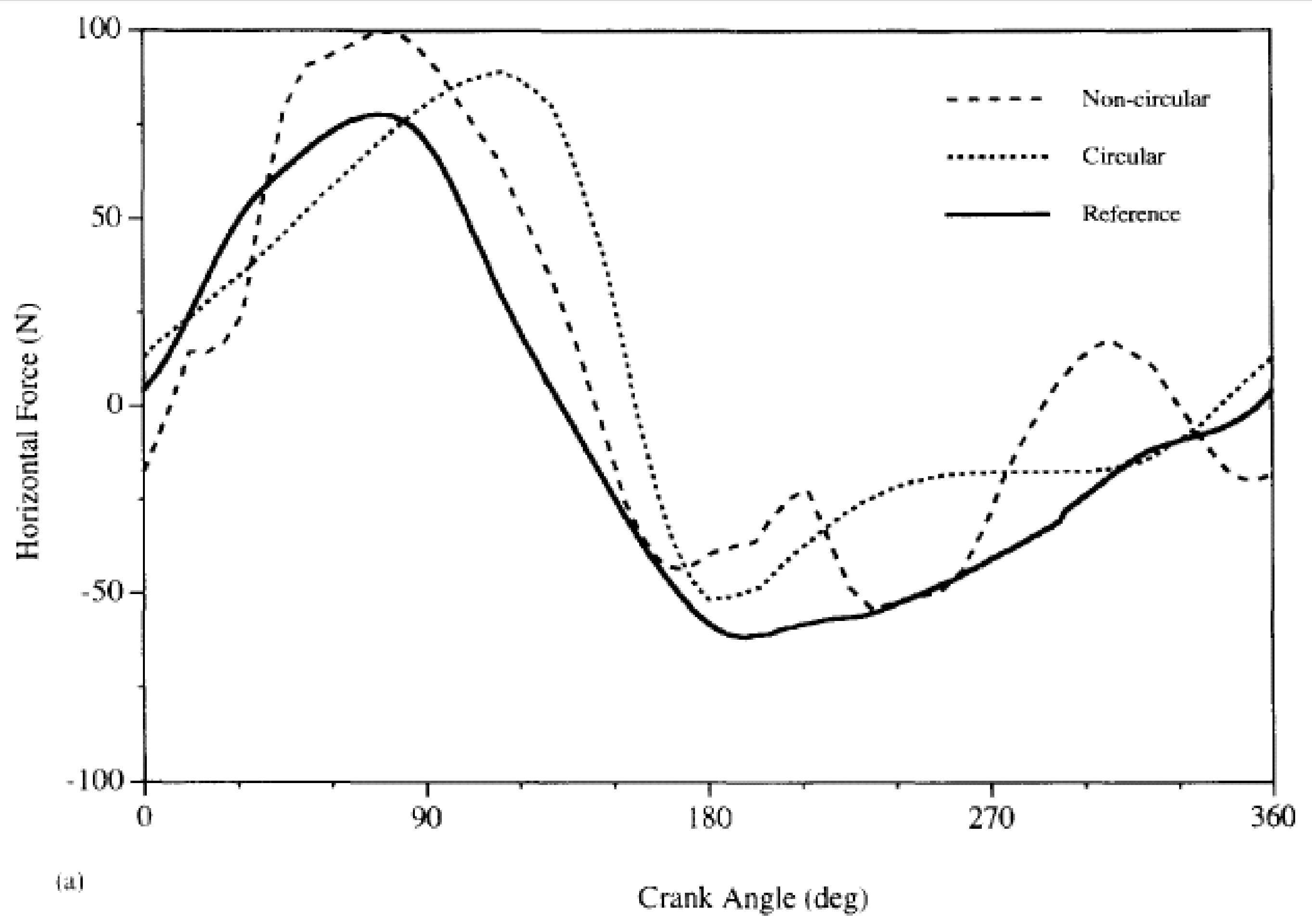
Knee Joint Torque



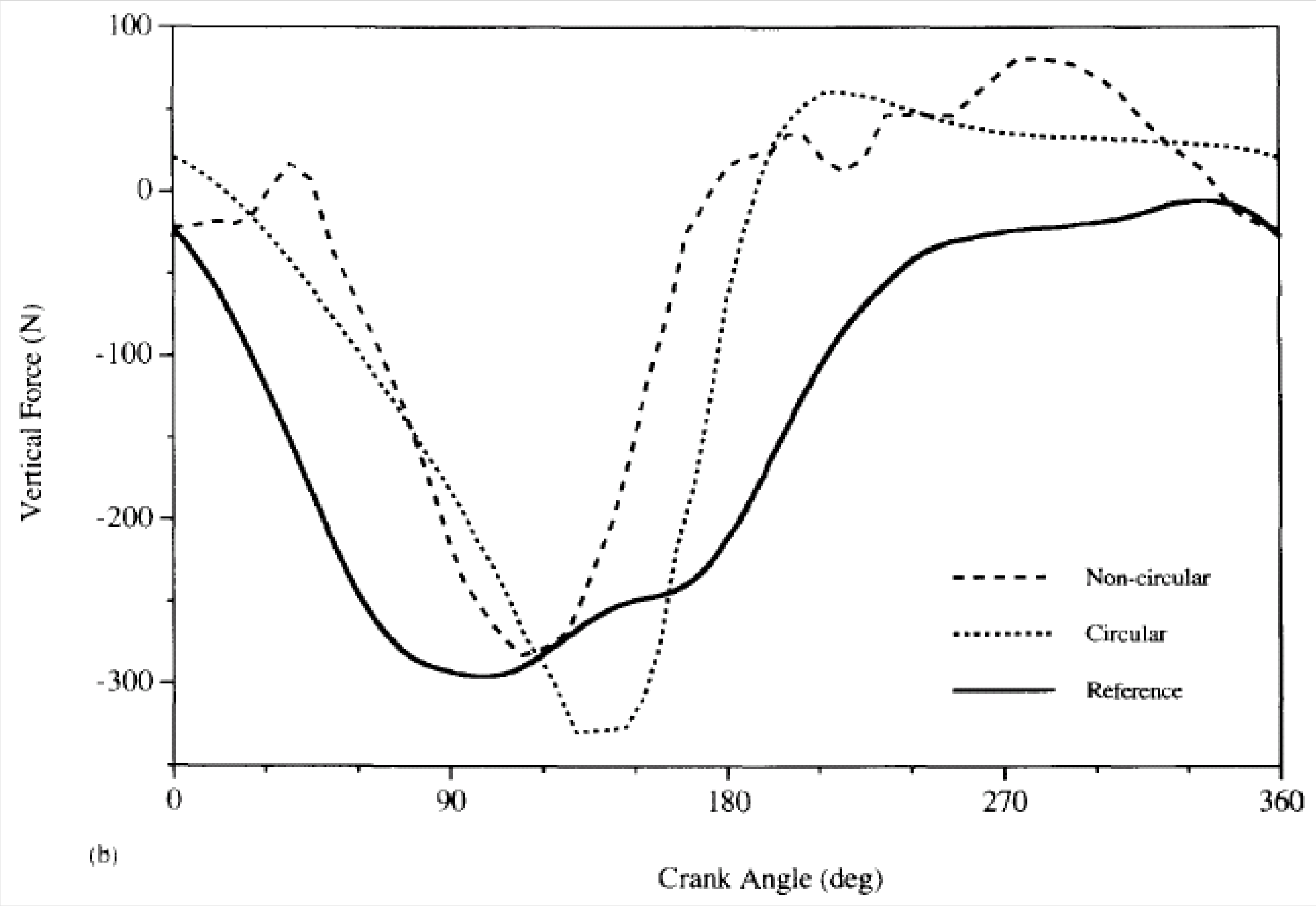
Ankle Joint Torque



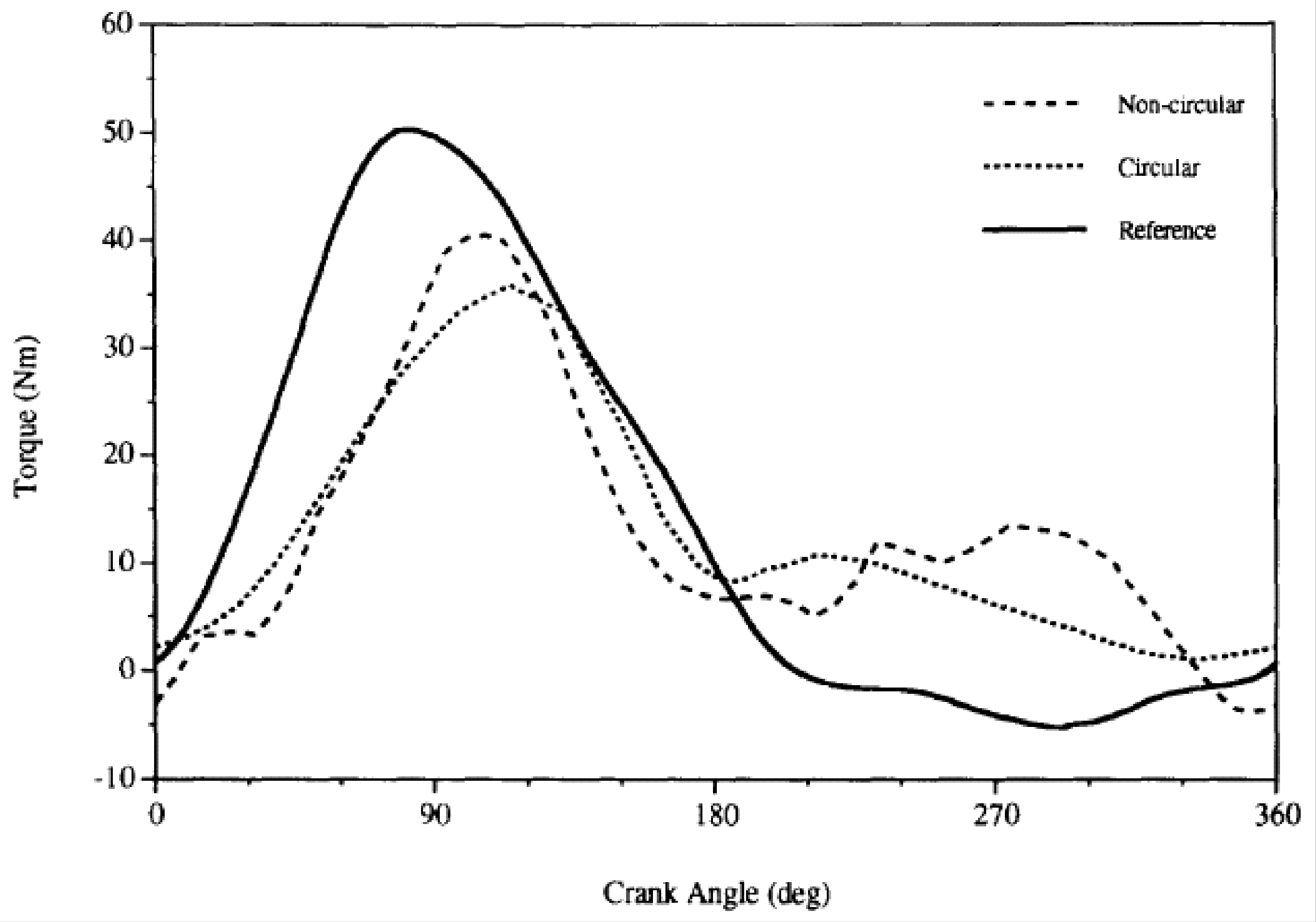
Horizontal Pedal Force



Vertical Pedal Force



Single-Leg Torque About Crank



Discussion

- Both goals were successfully achieved
  – Dynamic optimization framework
  – Optimal chainring shape
- Endurance cycling studies justified choice of a moment based objective function
- Quantitative difficulty with objective function
  – Ankle and knee were good, but hip was bad
- Qualitative difficulty with objective function
  – Large muscle activations required

Discussion

- Is the optimal chainring shape invalid?
  – Objective function predicted cadence well
- Should muscle mechanics be explicitly modeled?
  – Dependence of muscle force on velocity
- Is there a need to consider muscle coordination as opposed to net joint moments?
  – Objective function reflecting energetics

- Objective function with individual muscles necessary to fully evaluate impact of dynamic optimization on cycling performance