KINEMATICS SUMMARY

ANGULAR VELOCITY (Section 6-1)

Definition

$$^{A}\boldsymbol{\omega}^{B} = \mathbf{b}_1 \frac{^{A}d\mathbf{b}_2}{dt} \cdot \mathbf{b}_3 + \mathbf{b}_2 \frac{^{A}d\mathbf{b}_3}{dt} \cdot \mathbf{b}_1 + \mathbf{b}_3 \frac{^{A}d\mathbf{b}_1}{dt} \cdot \mathbf{b}_2$$

where $^{A}d\mathbf{b}_i / dt$ (i = 1, 2, 3) can be determined by using $^{A}R^{B}$ to express the $\mathbf{b}_i$ unit vectors in terms of $\mathbf{a}_i$ unit vectors. This method of calculating $^{A}\boldsymbol{\omega}^{B}$ is rarely necessary in practice.

Differentiation in Two Reference Frames (or Million Dollar Formula)

$$\frac{^{A}d\mathbf{v}}{dt} = \frac{^{B}d\mathbf{v}}{dt} + {}^{A}\boldsymbol{\omega}^{B} \times \mathbf{v}$$

This equation is useful when you want to calculate $^{A}d\mathbf{v}/dt$ but you have $\mathbf{v}$ and $^{A}\boldsymbol{\omega}^{B}$ expressed in terms of $\mathbf{b}_i$ rather than $\mathbf{a}_i$ unit vectors. In essence, this relationship allows you to avoid re-expressing $\mathbf{v}$ in terms of $\mathbf{a}_i$ unit vectors in order to calculate the desired derivative in A.

Simple Angular Velocity

$$^{A}\boldsymbol{\omega}^{B} = \omega\, \mathbf{a}_i = \omega\, \mathbf{b}_i$$

When rigid bodies A and B share a common unit vector during some specified time period, then the $^{A}\boldsymbol{\omega}^{B}$ can be found immediately from the above relationship without resorting to the definition of angular velocity given above. This situation is commonly encountered when solving dynamics problems.

Addition Theorem for Angular Velocities

$$^{A}\boldsymbol{\omega}^{B} = {}^{A}\boldsymbol{\omega}^{A_1} + {}^{A_1}\boldsymbol{\omega}^{A_2} + \cdots + {}^{A_{n-1}}\boldsymbol{\omega}^{A_n} + {}^{A_n}\boldsymbol{\omega}^{B}$$

The individual angular velocities $^{A_i}\boldsymbol{\omega}^{A_j}$ are usually, but need not be, simple angular velocities. They are frequently associated with intermediate bodies (e.g., as in a gimbal joint), but they may also be associated with fictitious intermediate reference frames introduced by the analyst to simply the problem (e.g., the rolling coin in problem 20.10).

ANGULAR ACCELERATION (Section 6-2)

Definition

$$^{A}\boldsymbol{\alpha}^{B} = \frac{^{A}d\, {}^{A}\boldsymbol{\omega}^{B}}{dt} = \frac{^{B}d\, {}^{A}\boldsymbol{\omega}^{B}}{dt}$$

The nice thing about calculating $^{A}\boldsymbol{\alpha}^{B}$ is that the time derivative can be performed in either A or B (see book for proof involving differentiation in two reference frames). No addition theorem for angular accelerations exists similar to that for angular velocities.

VELOCITY AND ACCELERATION (Section 6-3)

Definition

$$ {}^{A}\mathbf{v}^{P} = \frac{{}^{A}d\mathbf{p}^{OP}}{dt} $$

$$ {}^{A}\mathbf{a}^{P} = \frac{{}^{A}d\mathbf{v}^{P}}{dt} $$

where P is the point whose velocity and acceleration are desired and O is *any* point fixed in reference frame A. Judicious selection of the point O can greatly simply the amount of work required to calculate ${}^{A}\mathbf{v}^{P}$.

Two Points Fixed on a Rigid Body (Section 6-5)

$$ {}^{A}\mathbf{v}^{P} = {}^{A}\mathbf{v}^{Q} + {}^{A}\boldsymbol{\omega}^{B} \times \mathbf{p}^{QP} $$

$$ {}^{A}\mathbf{a}^{P} = {}^{A}\mathbf{a}^{Q} + {}^{A}\boldsymbol{\omega}^{B} \times ({}^{A}\boldsymbol{\omega}^{B} \times \mathbf{p}^{QP}) + {}^{A}\boldsymbol{\alpha}^{B} \times \mathbf{p}^{QP} $$

where Q and P are points fixed on a rigid body B, P is the point whose velocity and acceleration are desired, B moves in a references frame A, and $\mathbf{p}^{QP}$, ${}^{A}\mathbf{v}^{Q}$, ${}^{A}\mathbf{a}^{Q}$, ${}^{A}\boldsymbol{\omega}^{B}$, and ${}^{A}\boldsymbol{\alpha}^{B}$ are known. These relationships are often easier to use than the definitions of velocity and acceleration given above, since it is not necessary to construct $\mathbf{p}^{OP}$ and then re-express it in terms of unit vectors fixed in reference frame A.

One Point Moving on a Rigid Body (Section 6-6)

$$ {}^{A}\mathbf{v}^{P} = {}^{A}\mathbf{v}^{Q} + {}^{B}\mathbf{v}^{P} $$

$$ {}^{A}\mathbf{a}^{P} = {}^{A}\mathbf{a}^{Q} + {}^{B}\mathbf{a}^{P} + 2\,{}^{A}\boldsymbol{\omega}^{B} \times {}^{B}\mathbf{v}^{P} $$

where P is the point whose velocity and acceleration are desired, P moves on a rigid body B, and at the instant under consideration, P is coincident with a point Q fixed on B. Frequently, the velocity and acceleration of Q in A can be found easily using two points fixed on a rigid body, while the velocity and acceleration of P in B can be found easily from the definition of velocity and acceleration. These are additional relationships which often allow one to avoid resorting to the definitions of velocity and acceleration to calculate ${}^{A}\mathbf{v}^{P}$ and ${}^{A}\mathbf{a}^{P}$, and hence, to avoid re-expressing vectors in terms of unit vectors fixed in reference frame A.

Rolling Contact (Section 6-9)

$$ {}^{N}\mathbf{v}^{P} = {}^{N}\mathbf{v}^{Q} $$

where P and Q points on the surfaces bounding bodies A and B, respectively, which are in rolling contact with each other at points P and Q at the instant under consideration, and N is *any* reference frame. This relationship is helpful in problems involving one body rolling on another, such as two gears in contact.