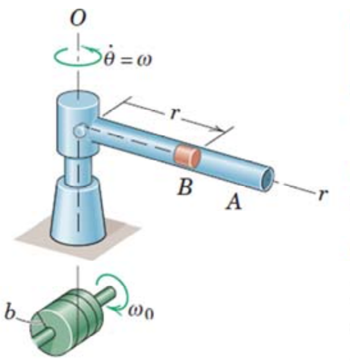
Polar (r-θ) Coordinates: Exercise



$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e_r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta$$

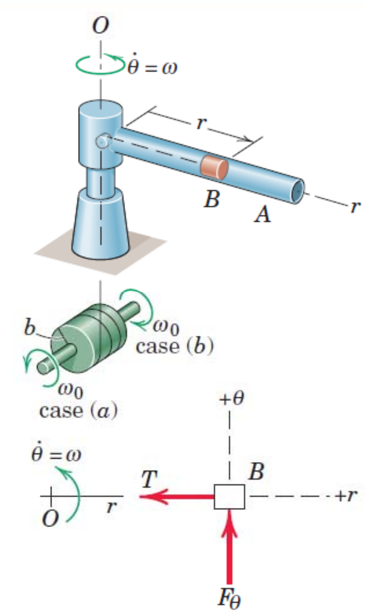
Tube A rotates about the vertical ***O-axis*** with constant ***angular velocity*** ω and contains a small ***cylinder*** B of ***mass*** m whose radial position is controlled by a cord passing through the tube and wound around a ***drum*** of ***radius*** b.

Determine the ***tension*** T in the cord and ***θ-component*** of ***force*** F_θ if the drum has a constant angular rate of rotation of ω_0 as shown.

ME 231: Dynamics

Sample Problem 3/10

Tube A rotates about the vertical O-axis with a constant angular rate $\dot{\theta} = \omega$ and contains a small cylindrical plug B of mass m whose radial position is controlled by the cord which passes freely through the tube and shaft and is wound around the drum of radius b. Determine the tension T in the cord and the horizontal component F_θ of force exerted by the tube on the plug if the constant angular rate of rotation of the drum is ω_0 first in the direction for case (a) and second in the direction for case (b). Neglect friction.



Solution. With r a variable, we use the polar-coordinate form of the equations of motion, Eqs. 3/8. The free-body diagram of B is shown in the horizontal plane and discloses only T and F_θ. The equations of motion are

$$[\Sigma F_r = ma_r] \qquad -T = m(\ddot{r} - r\dot{\theta}^2)$$

$$[\Sigma F_\theta = ma_\theta] \qquad F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

Case (a). With $\dot{r} = +b\omega_0$, $\ddot{r} = 0$, and $\ddot{\theta} = 0$, the forces become

$$T = mr\omega^2 \qquad F_\theta = 2mb\omega_0\omega \qquad \textit{Ans.}$$

① **Case (b).** With $\dot{r} = -b\omega_0$, $\ddot{r} = 0$, and $\ddot{\theta} = 0$, the forces become

$$T = mr\omega^2 \qquad F_\theta = -2mb\omega_0\omega \qquad \textit{Ans.}$$

Helpful Hint

① The minus sign shows that F_θ is in the direction opposite to that shown on the free-body diagram.