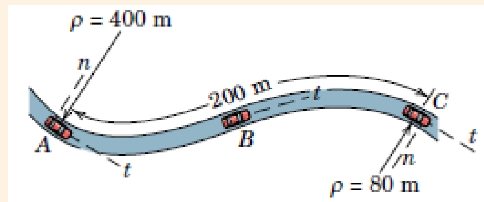
Normal and Tangential (n-t) Coordinates: Exercise

$$\mathbf{a} = \frac{v^2}{\rho}\mathbf{e_n} + \dot{v}\,\mathbf{e_t}$$

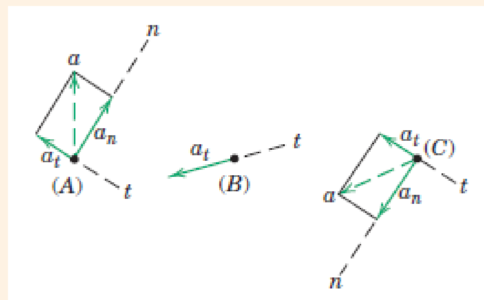
Sample Problem 3/8

A 1500-kg car enters a section of curved road in the horizontal plane and slows down at a uniform rate from a speed of 100 km/h at A to a speed of 50 km/h as it passes C. The radius of curvature ρ of the road at A is 400 m and at C is 80 m. Determine the total horizontal force exerted by the road on the tires at positions A, B, and C. Point B is the inflection point where the curvature changes direction.



Solution. The car will be treated as a particle so that the effect of all forces exerted by the road on the tires will be treated as a single force. Since the motion is described along the direction of the road, normal and tangential coordinates will be used to specify the acceleration of the car. We will then determine the forces from the accelerations.

The constant tangential acceleration is in the negative t-direction, and its magnitude is given by



① $[v_C{}^2 = v_A{}^2 + 2a_t\,\Delta s]$ $\qquad a_t = \left|\frac{(50/3.6)^2 - (100/3.6)^2}{2(200)}\right| = 1.447\text{ m/s}^2$

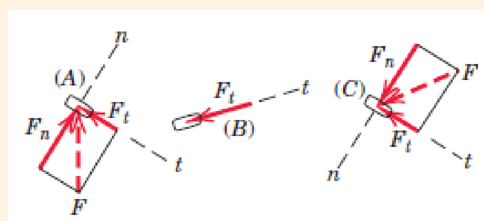
The normal components of acceleration at A, B, and C are

② $[a_n = v^2/\rho]$ $\qquad$ At A, $\quad a_n = \frac{(100/3.6)^2}{400} = 1.929\text{ m/s}^2$

At B, $\quad a_n = 0$

At C, $\quad a_n = \frac{(50/3.6)^2}{80} = 2.41\text{ m/s}^2$

Application of Newton's second law in both the n- and t-directions to the free-body diagrams of the car gives



$[\Sigma F_t = ma_t]$ $\qquad F_t = 1500(1.447) = 2170\text{ N}$

③ $[\Sigma F_n = ma_n]$ $\qquad$ At A, $\quad F_n = 1500(1.929) = 2890\text{ N}$

At B, $\quad F_n = 0$

At C, $\quad F_n = 1500(2.41) = 3620\text{ N}$

Thus, the total horizontal force acting on the tires becomes

At A, $\quad F = \sqrt{F_n{}^2 + F_t{}^2} = \sqrt{(2890)^2 + (2170)^2} = 3620\text{ N}$ $\qquad$ *Ans.*

At B, $\quad F = F_t = 2170\text{ N}$ $\qquad$ *Ans.*

④ At C, $\quad F = \sqrt{F_n{}^2 + F_t{}^2} = \sqrt{(3620)^2 + (2170)^2} = 4220\text{ N}$ $\qquad$ *Ans.*

Helpful Hints

① Recognize the numerical value of the conversion factor from km/h to m/s as 1000/3600 or 1/3.6.

② Note that a_n is always directed toward the center of curvature.

③ Note that the direction of F_n must agree with that of a_n.

④ The angle made by **a** and **F** with the direction of the path can be computed if desired.