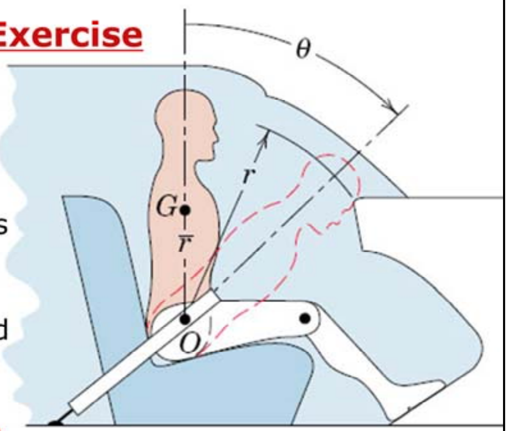
General Plane Motion: Exercise



In a study of head injury during a crash where lap belts are used, the segmented human model is analyzed. The ***hip joint O*** remains fixed relative to the car, and the ***torso*** is treated as a rigid body of ***mass m*** pivoted at ***O***.

$m = 50\text{ kg}$ $\quad \bar{r} = 450\text{ mm}$ $\quad r = 800\text{ mm}$

$k_O = 550\text{ mm}$ $\quad \theta = 45°$ $\quad a = 10g$

Determine the ***velocity v*** relative to the car with which the model's head strikes the instrument panel when the car is stopped with a constant ***deceleration a***.

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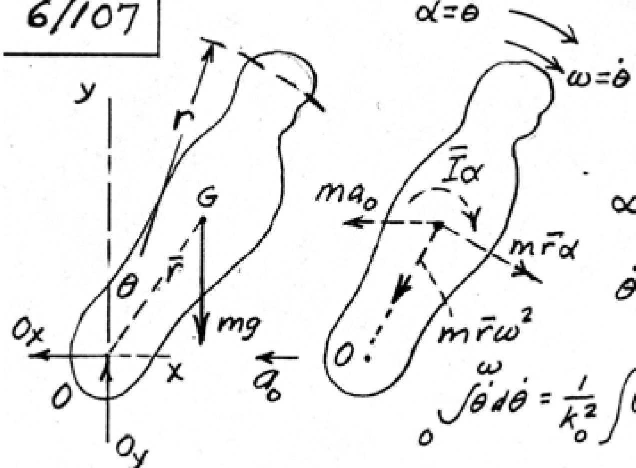

$\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d$

$mg\bar{r}\sin\theta = m(k_O^2 - \bar{r}^2)\alpha + m\bar{r}\alpha(\bar{r}) - ma_O\bar{r}\cos\theta$

$\alpha = \frac{1}{k_O^2}(g\bar{r}\sin\theta + a_O\bar{r}\cos\theta)$

$\dot{\theta}\,d\dot{\theta} = \ddot{\theta}\,d\theta$

$\int_0^\omega \dot{\theta}\,d\dot{\theta} = \frac{1}{k_O^2}\int (g\bar{r}\sin\theta + a_O\bar{r}\cos\theta)\,d\theta$

$\frac{\omega^2}{2} = \frac{1}{k_O^2}\left[g\bar{r}(1-\cos\theta) + a_O\bar{r}\sin\theta\right]$ where $a_O = a$

$v = r\omega = \frac{r\sqrt{2}}{k_O}\sqrt{g\bar{r}(1-\cos\theta) + a\bar{r}\sin\theta}$

For $\bar{r} = 0.45\text{ m}$, $r = 0.8\text{ m}$, $k_O = 0.55\text{ m}$, $\theta = 45°$, $a = 10g$,

$v = \frac{0.80\sqrt{2}}{0.55}\sqrt{9.81(0.45)(1 - 1/\sqrt{2}) + 10(9.81)(0.45)\,1/\sqrt{2}} = \underline{11.73\text{ m/s}}$

(Alternatively apply Eq. 6/3 with moment center at O)