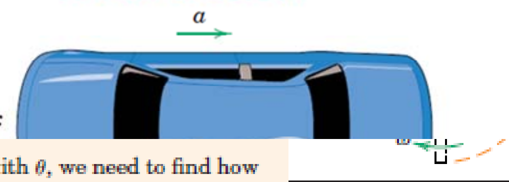
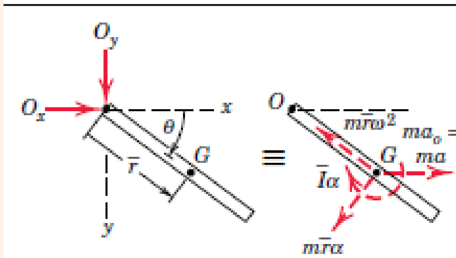
Question of the Day

The door of a moving car is inadvertently left slightly open. The brakes are applied to give the car a constant rearward ***acceleration***.

Determine expressions for the ***angular velocity*** of



Solution. Because the angular velocity ω increases with θ, we need to find how the angular acceleration α varies with θ so that we may integrate it over the interval to obtain ω. We obtain α from a moment equation about O. First, we draw the free-body diagram of the door in the horizontal plane for a general position θ. The only forces in this plane are the components of the hinge reaction shown here in the x- and y-directions. On the kinetic diagram, in addition to the resultant couple $\bar{I}\alpha$ shown in the sense of α, we represent the resultant force $m\bar{\mathbf{a}}$ in terms of its components by using an equation of relative acceleration with respect to O. This equation becomes the kinematic equation of constraint and is ①



$$\bar{\mathbf{a}} = \mathbf{a}_G = \mathbf{a}_O + (\mathbf{a}_{G/O})_n + (\mathbf{a}_{G/O})_t$$

The magnitudes of the $m\bar{\mathbf{a}}$ components are then

② $$ma_O = ma \qquad m(a_{G/O})_n = m\bar{r}\omega^2 \qquad m(a_{G/O})_t = m\bar{r}\alpha$$

where $\omega = \dot{\theta}$ and $\alpha = \ddot{\theta}$.

For a given angle θ, the three unknowns are α, O_x, and O_y. We can eliminate O_x and O_y by a moment equation about O, which gives

③ $[\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d]$ $\qquad 0 = m(k_O^2 - \bar{r}^2)\alpha + m\bar{r}\alpha(\bar{r}) - ma(\bar{r}\sin\theta)$

④ Solving for α gives $\qquad \alpha = \dfrac{a\bar{r}}{k_O^2}\sin\theta$

Now we integrate α first to a general position and get

$[\omega\, d\omega = \alpha\, d\theta]$ $$\int_0^\omega \omega\, d\omega = \int_0^\theta \frac{a\bar{r}}{k_O^2}\sin\theta\, d\theta$$

$$\omega^2 = \frac{2a\bar{r}}{k_O^2}(1 - \cos\theta)$$

For $\theta = \pi/2$, $$\omega = \frac{1}{k_O}\sqrt{2a\bar{r}} \qquad \textit{Ans.}$$

⑤ To find O_x and O_y for any given value of θ, the force equations give

$[\Sigma F_x = m\bar{a}_x]$ $$O_x = ma - m\bar{r}\omega^2\cos\theta - m\bar{r}\alpha\sin\theta$$
$$= m\left[a - \frac{2a\bar{r}^2}{k_O^2}(1 - \cos\theta)\cos\theta - \frac{a\bar{r}^2}{k_O^2}\sin^2\theta\right]$$
$$= ma\left[1 - \frac{\bar{r}^2}{k_O^2}(1 + 2\cos\theta - 3\cos^2\theta)\right] \qquad \textit{Ans.}$$

$[\Sigma F_y = m\bar{a}_y]$ $$O_y = m\bar{r}\alpha\cos\theta - m\bar{r}\omega^2\sin\theta$$
$$= m\bar{r}\frac{a\bar{r}}{k_O^2}\sin\theta\cos\theta - m\bar{r}\frac{2a\bar{r}}{k_O^2}(1 - \cos\theta)\sin\theta$$
$$= \frac{ma\bar{r}^2}{k_O^2}(3\cos\theta - 2)\sin\theta \qquad \textit{Ans.}$$

Helpful Hints

① Point O is chosen because it is the only point on the door whose acceleration is known.

② Be careful to place $m\bar{r}\alpha$ in the sense of positive α with respect to rotation about O.

③ The free-body diagram shows that there is zero moment about O. We use the transfer-of-axis theorem here and substitute $k_O^2 = \bar{k}^2 + \bar{r}^2$. If this relation is not totally familiar, review Art. B/1 in Appendix B.

④ We may also use Eq. 6/3 with O as a moment center

$$\Sigma \mathbf{M}_O = I_O\boldsymbol{\alpha} + \bar{\boldsymbol{\rho}} \times m\mathbf{a}_O$$

where the scalar values of the terms are $I_O\alpha = mk_O^2\alpha$ and $\bar{\boldsymbol{\rho}} \times m\mathbf{a}_O$ becomes $-\bar{r}ma\sin\theta$.

⑤ The kinetic diagram shows clearly the terms which make up $m\bar{a}_x$ and $m\bar{a}_y$.