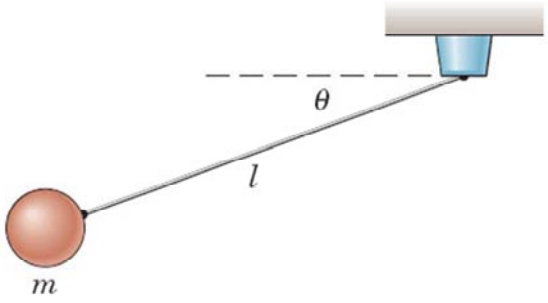
Angular Impulse and Momentum: Exercise 1



Using only the angular impulse-momentum principle, determine the expression for $\ddot{\theta}$ in terms of θ and the ***velocity v*** of the pendulum at $\theta = 90^o$.

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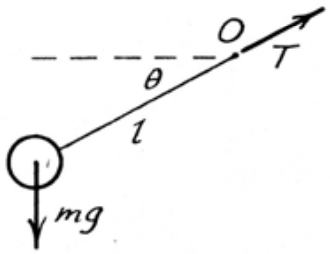

$\Sigma M_o = \dot{H}_o$: $mgl\cos\theta = \frac{d}{dt}(ml^2\dot{\theta}) = ml^2\ddot{\theta}$

$$\ddot{\theta} = \frac{g}{l}\cos\theta$$

From $\int \dot{\theta}\,d\dot{\theta} = \int \ddot{\theta}\,d\theta$, $\left.\frac{\dot{\theta}^2}{2}\right|_0^{\dot{\theta}} = \int_0^{\theta} \frac{g}{l}\cos\theta\,d\theta$,

$$\dot{\theta}^2 = \frac{2g}{l}\sin\theta, \quad \dot{\theta}_{\theta=90^\circ} = \sqrt{\frac{2g}{l}}$$

so at $\theta = 90^\circ$, $v = l\dot{\theta} = \sqrt{2gl}$

By work-energy $U = \Delta T$, $mgl = \frac{1}{2}mv^2$, $v = \sqrt{2gl}$