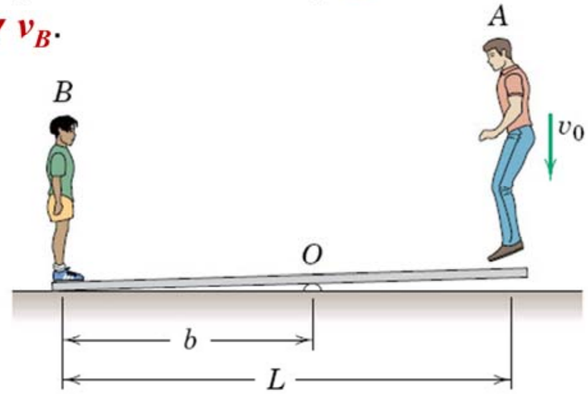
Impulse-Momentum for Rigid Bodies: Exercise 2

The man A of ***mass*** m_A drops with ***velocity*** v_0 onto the end of a light beam. The boy B is sent up with a ***velocity*** v_B.

Determine b in terms of L to maximize the v_B for a given ratio $n = m_B/m_A$.



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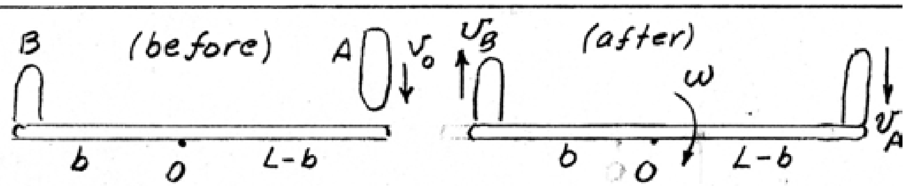

Before: $H_O = m_A v_0 (L-b)$

After: $H_O = m_A v_A (L-b) + m_B v_B b$

$\Delta H_O = 0$ along with $\omega = v_B/b = v_A/(L-b)$ give

$$m_A v_0 (L-b) = m_A \frac{L-b}{b} v_B (L-b) + m_B v_B b$$

$$v_B = v_0 \frac{1}{\frac{L-b}{b} + n\frac{b}{L-b}} \quad \text{where } n = m_B/m_A$$

$$\frac{dv_B}{db} = v_0 \frac{-\left(\frac{L}{b^2} + n\frac{L-b-b(-1)}{(L-b)^2}\right)}{\left(\frac{L-b}{b} + n\frac{b}{L-b}\right)^2} = v_0 \frac{L\left(\frac{1}{b^2} - \frac{n}{(L-b)^2}\right)}{\left(\frac{L-b}{b} + n\frac{b}{L-b}\right)^2} = 0 \text{ for } v_{B\,max}$$

So $\frac{1}{b^2} = \frac{n}{(L-b)^2}$, $b = \frac{L}{1 \pm \sqrt{n}}$ (+ sign gives positive v_B)

Thus $\underline{b = \frac{L}{1+\sqrt{n}}}$ which gives $v_B = \frac{v_0}{2\sqrt{n}}$