X-Y Vector Representation: Exercise

A particle moving in two-dimensions has a position vector (**r**) as a function of time (***t***) with coordinates given by

$$x(t) = t^2 - 4t + 20 \quad , \quad y(t) = 3\sin(2t)$$

where **r** is measured in inches and ***t*** is in seconds.

Determine the magnitude of the ***velocity*** (**v**) and the ***acceleration*** (**a**) at time ***t*** =3 s.

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$x = t^2 - 4t + 20$ $\qquad$ $y = 3\sin 2t$

$\dot{x} = 2t - 4$ $\qquad$ $\dot{y} = 6\cos 2t$

$\ddot{x} = 2$ $\qquad$ $\ddot{y} = -12\sin 2t$

At time $t = 3$ sec:

$\dot{x} = 2$ in./sec $\qquad$ $\dot{y} = 5.76$ in./sec

$\ddot{x} = 2$ in./sec^2 $\qquad$ $\ddot{y} = 3.35$ in./sec^2

$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{2^2 + 5.76^2} = \underline{6.10 \text{ in./sec}}$

$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{2^2 + 3.35^2} = \underline{3.90 \text{ in./sec}^2}$

$\underline{v} = 2\underline{i} + 5.76\underline{j}$ in./sec, $\quad \underline{a} = 2\underline{i} + 3.35\underline{j}$ in./sec^2

$$\theta = \cos^{-1}\frac{\underline{v}\cdot\underline{a}}{va} = \cos^{-1}\left(\frac{2(2) + 5.76(3.35)}{(6.10)(3.90)}\right)$$

$= \underline{11.67^\circ}$