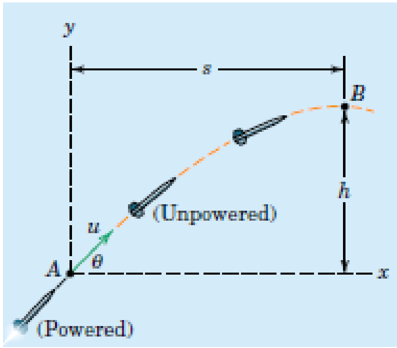
Projectile Motion: Exercise

Sample Problem 2/6

A rocket has expended all its fuel when it reaches position A, where it has a velocity of $\mathbf{u}$ at an angle θ with respect to the horizontal. It then begins unpowered flight and attains a maximum added height h at position B after traveling a horizontal distance s from A. Determine the expressions for h and s, the time t of flight from A to B, and the equation of the path. For the interval concerned, assume a flat earth with a constant gravitational acceleration g and neglect any atmospheric resistance.



Solution. Since all motion components are directly expressible in terms of horizontal and vertical coordinates, a rectangular set of axes *x-y* will be employed. With the neglect of atmospheric resistance, $a_x = 0$ and $a_y = -g$, and the resulting motion is a direct superposition of two rectilinear motions with constant acceleration. Thus, ①

$$[dx = v_x\,dt] \qquad x = \int_0^t u\cos\theta\,dt \qquad x = ut\cos\theta$$

$$[dv_y = a_y\,dt] \qquad \int_{u\sin\theta}^{v_y} dv_y = \int_0^t (-g)\,dt \qquad v_y = u\sin\theta - gt$$

$$[dy = v_y\,dt] \qquad y = \int_0^t (u\sin\theta - gt)\,dt \qquad y = ut\sin\theta - \tfrac{1}{2}gt^2$$

Position B is reached when $v_y = 0$, which occurs for $0 = u\sin\theta - gt$ or

$$t = (u\sin\theta)/g \qquad \textit{Ans.}$$

Substitution of this value for the time into the expression for y gives the maximum added altitude

$$h = u\left(\frac{u\sin\theta}{g}\right)\sin\theta - \frac{1}{2}g\left(\frac{u\sin\theta}{g}\right)^2 \qquad h = \frac{u^2\sin^2\theta}{2g} \qquad \textit{Ans.}$$

The horizontal distance is seen to be

② $$s = u\left(\frac{u\sin\theta}{g}\right)\cos\theta \qquad s = \frac{u^2\sin 2\theta}{2g} \qquad \textit{Ans.}$$

which is clearly a maximum when $\theta = 45^\circ$. The equation of the path is obtained by eliminating t from the expressions for x and y, which gives

$$y = x\tan\theta - \frac{gx^2}{2u^2}\sec^2\theta \qquad \textit{Ans.}$$

③ This equation describes a vertical parabola as indicated in the figure.

Helpful Hints

① Note that this problem is simply the description of projectile motion neglecting atmospheric resistance.

② We see that the total range and time of flight for a projectile fired above a horizontal plane would be twice the respective values of s and t given here.

③ If atmospheric resistance were to be accounted for, the dependency of the acceleration components on the velocity would have to be established before an integration of the equations could be carried out. This becomes a much more difficult problem.