

Rectangular coordinates (x, y, z): Exercise

A particle moving in three-dimensions has a **position** vector (\mathbf{r}) as a function of **time** (t) with coordinates given by

$$x(t) = 30 \cos(2t), \quad y(t) = 40 \sin(2t), \quad z(t) = 20t + 3t^2$$

where \mathbf{r} is measured in millimeters and t is in seconds.

Determine the magnitude of the **velocity** (\mathbf{v}) and the **acceleration** (\mathbf{a}) at time $t = 2$ s.

ME 231: Dynamics

See Notes Page view for solution.

$$\begin{array}{l} 2/171 \\ \left\{ \begin{array}{l} x = .30 \cos 2t ; \quad y = 40 \sin 2t ; \quad z = 20t + 3t^2 \\ \dot{x} = -60 \sin 2t ; \quad \dot{y} = 80 \cos 2t ; \quad \dot{z} = 20 + 6t \\ \ddot{x} = -120 \cos 2t ; \quad \ddot{y} = -160 \sin 2t ; \quad \ddot{z} = 6 \end{array} \right. \end{array}$$

At $t = 2$ s :

$$\begin{array}{l} \left\{ \begin{array}{l} x = -19.61 \text{ mm} ; \quad y = -30.3 \text{ mm} ; \quad z = 52 \text{ mm} \\ \dot{x} = 45.4 \text{ mm/s} ; \quad \dot{y} = -52.3 \text{ mm/s} ; \quad \dot{z} = 32 \text{ mm/s} \\ \ddot{x} = 78.4 \text{ mm/s}^2 ; \quad \ddot{y} = 121.1 \text{ mm/s}^2 ; \quad \ddot{z} = 6 \text{ mm/s}^2 \end{array} \right. \\ r = (x^2 + y^2 + z^2)^{1/2} = 63.3 \text{ mm} \quad \parallel \quad a = (\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2)^{1/2} \\ v = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2} = 76.3 \text{ mm/s} \quad \parallel \quad = 144.4 \text{ mm/s}^2 \end{array}$$