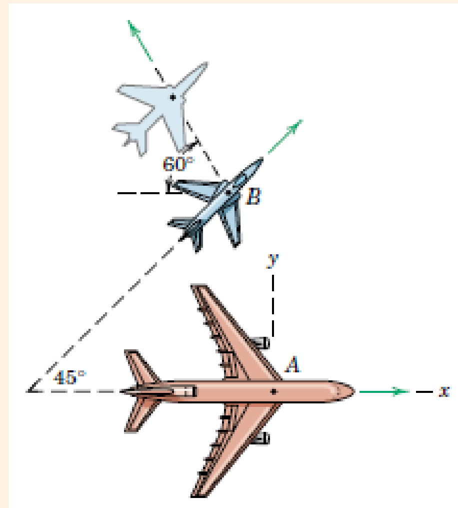
Question of the Day

Sample Problem 2/13

Passengers in the jet transport A flying east at a speed of 800 km/h observe a second jet plane B that passes under the transport in horizontal flight. Although the nose of B is pointed in the 45° northeast direction, plane B appears to the passengers in A to be moving away from the transport at the 60° angle as shown. Determine the true velocity of B.



Solution. The moving reference axes x-y are attached to A, from which the relative observations are made. We write, therefore,

① $$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

Next we identify the knowns and unknowns. The velocity $\mathbf{v}_A$ is given in both magnitude and direction. The 60° direction of $\mathbf{v}_{B/A}$, the velocity which B appears to
② have to the moving observers in A, is known, and the true velocity of B is in the 45° direction in which it is heading. The two remaining unknowns are the magni-
③ tudes of $\mathbf{v}_B$ and $\mathbf{v}_{B/A}$. We may solve the vector equation in any one of three ways.

(I) Graphical. We start the vector sum at some point P by drawing $\mathbf{v}_A$ to a convenient scale and then construct a line through the tip of $\mathbf{v}_A$ with the known direction of $\mathbf{v}_{B/A}$. The known direction of $\mathbf{v}_B$ is then drawn through P, and the intersection C yields the unique solution enabling us to complete the vector triangle and scale off the unknown magnitudes, which are found to be

$$v_{B/A} = 586 \text{ km/h} \qquad \text{and} \qquad v_B = 717 \text{ km/h} \qquad \textit{Ans.}$$

(II) Trigonometric. A sketch of the vector triangle is made to reveal the trigonometry, which gives

④ $$\frac{v_B}{\sin 60^\circ} = \frac{v_A}{\sin 75^\circ} \qquad v_B = 800\frac{\sin 60^\circ}{\sin 75^\circ} = 717 \text{ km/h} \qquad \textit{Ans.}$$

(III) Vector Algebra. Using unit vectors $\mathbf{i}$ and $\mathbf{j}$, we express the velocities in vector form as

$$\mathbf{v}_A = 800\mathbf{i} \text{ km/h} \qquad \mathbf{v}_B = (v_B \cos 45^\circ)\mathbf{i} + (v_B \sin 45^\circ)\mathbf{j}$$

$$\mathbf{v}_{B/A} = (v_{B/A} \cos 60^\circ)(-\mathbf{i}) + (v_{B/A} \sin 60^\circ)\mathbf{j}$$

Substituting these relations into the relative-velocity equation and solving separately for the $\mathbf{i}$ and $\mathbf{j}$ terms give

$$(\mathbf{i}\text{-terms}) \qquad v_B \cos 45^\circ = 800 - v_{B/A} \cos 60^\circ$$

$$(\mathbf{j}\text{-terms}) \qquad v_B \sin 45^\circ = v_{B/A} \sin 60^\circ$$

⑤ Solving simultaneously yields the unknown velocity magnitudes

$$v_{B/A} = 586 \text{ km/h} \qquad \text{and} \qquad v_B = 717 \text{ km/h} \qquad \textit{Ans.}$$

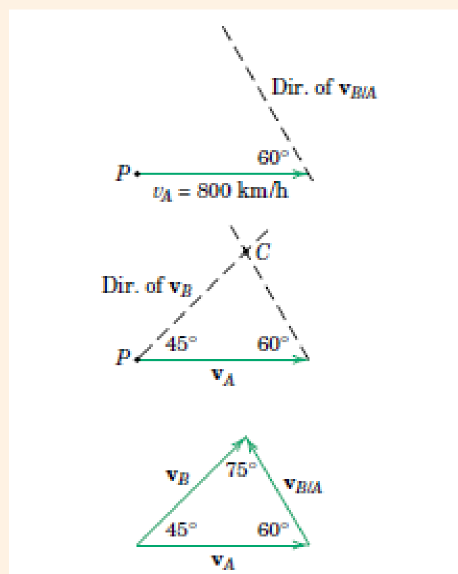
It is worth noting the solution of this problem from the viewpoint of an observer in B. With reference axes attached to B, we would write $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$. The apparent velocity of A as observed by B is then $\mathbf{v}_{A/B}$, which is the negative of $\mathbf{v}_{B/A}$.

Helpful Hints

① We treat each airplane as a particle.

② We assume no side slip due to cross wind.

③ Students should become familiar with all three solutions.



④ We must be prepared to recognize the appropriate trigonometric relation, which here is the law of sines.

⑤ We can see that the graphical or trigonometric solution is shorter than the vector algebra solution in this particular problem.