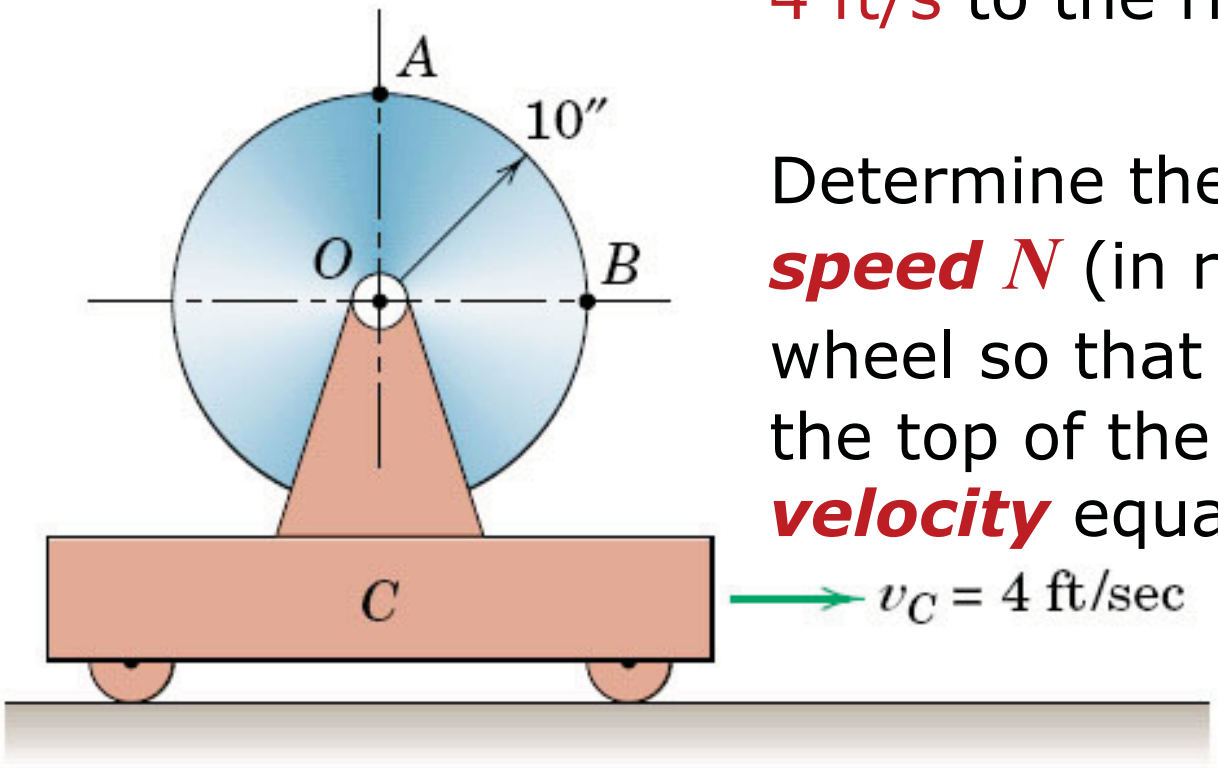
Relative Velocity

Lecture 11

ME 231: Dynamics

Question of the Day



The ***velocity*** of the cart is 4 ft/s to the right.

Determine the ***angular speed*** N (in rpm's) of the wheel so that ***point*** A on the top of the rim has a ***velocity*** equal to ***zero***.

Outline for Today

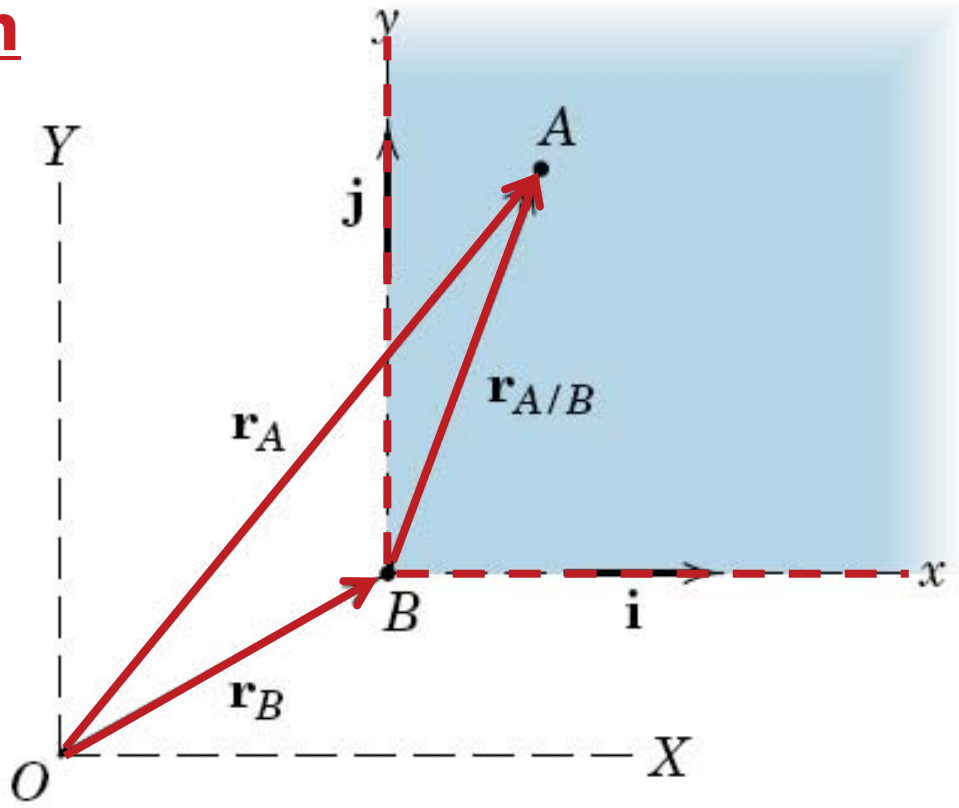
- Question of the day
- Relative velocity due to rotation
- Interpretation of $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$
- Solution of relative-velocity equation
- Answer your questions!

Recall: Relative Motion

$$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$$

$$\boxed{\mathbf{v}_A = \dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \dot{\mathbf{r}}_{A/B}}$$

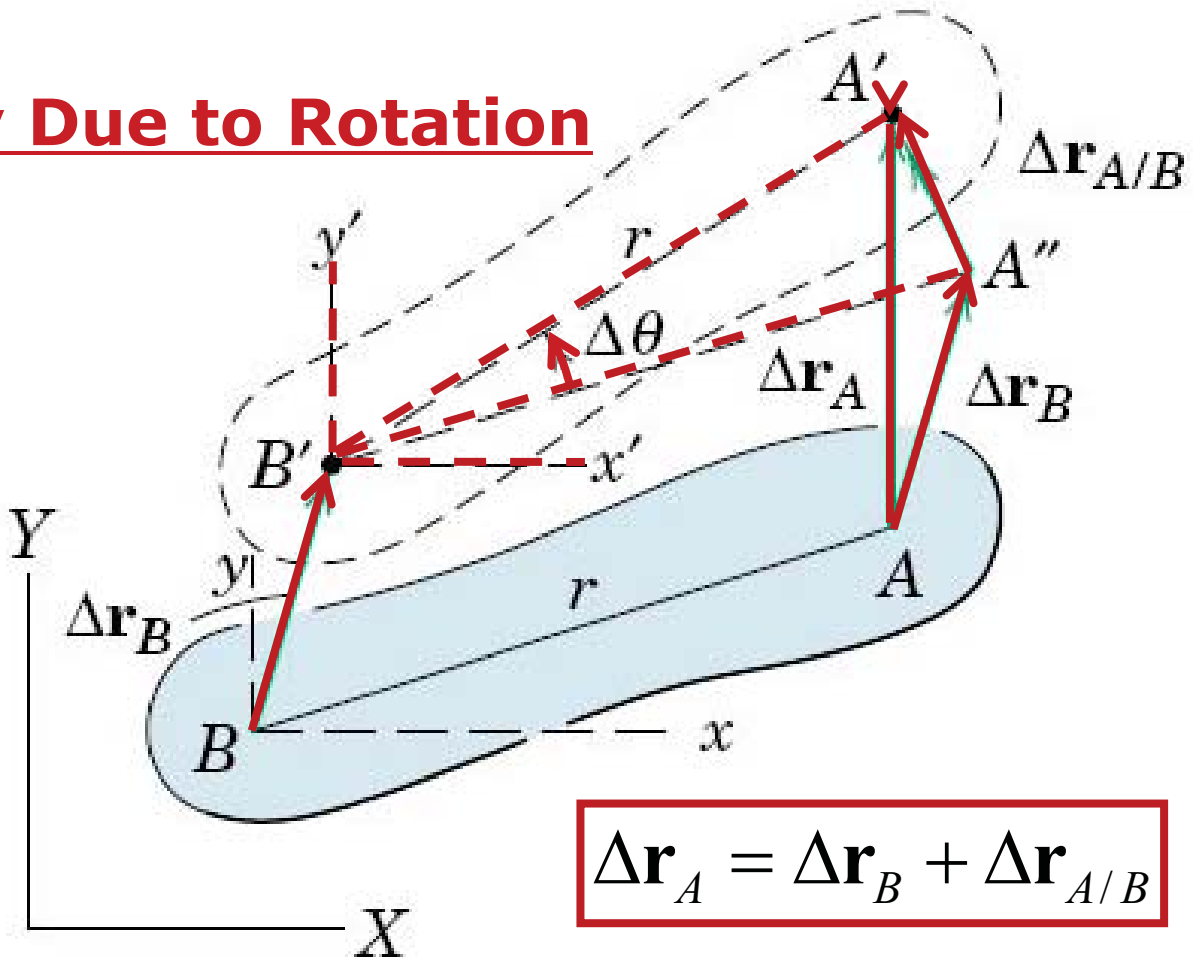
$$\mathbf{a}_A = \dot{\mathbf{v}}_A = \ddot{\mathbf{r}}_A = \ddot{\mathbf{r}}_B + \ddot{\mathbf{r}}_{A/B}$$



- Absolute position of ***B*** is defined in an inertial coordinate system ***X*-*Y***
- Attach a set of translating (*non-rotating*) axes ***x*-*y*** to particle ***B*** and define the position of ***A***
- Define position of "***A relative to B***" ("***A/B***") in ***x*-*y***

Relative Velocity Due to Rotation

- Movement in ***two parts***
- First, body ***translates*** $\Delta \mathbf{r}_B$ to parallel position
- Second, body ***rotates*** about ***B'*** through angle $\Delta\theta$



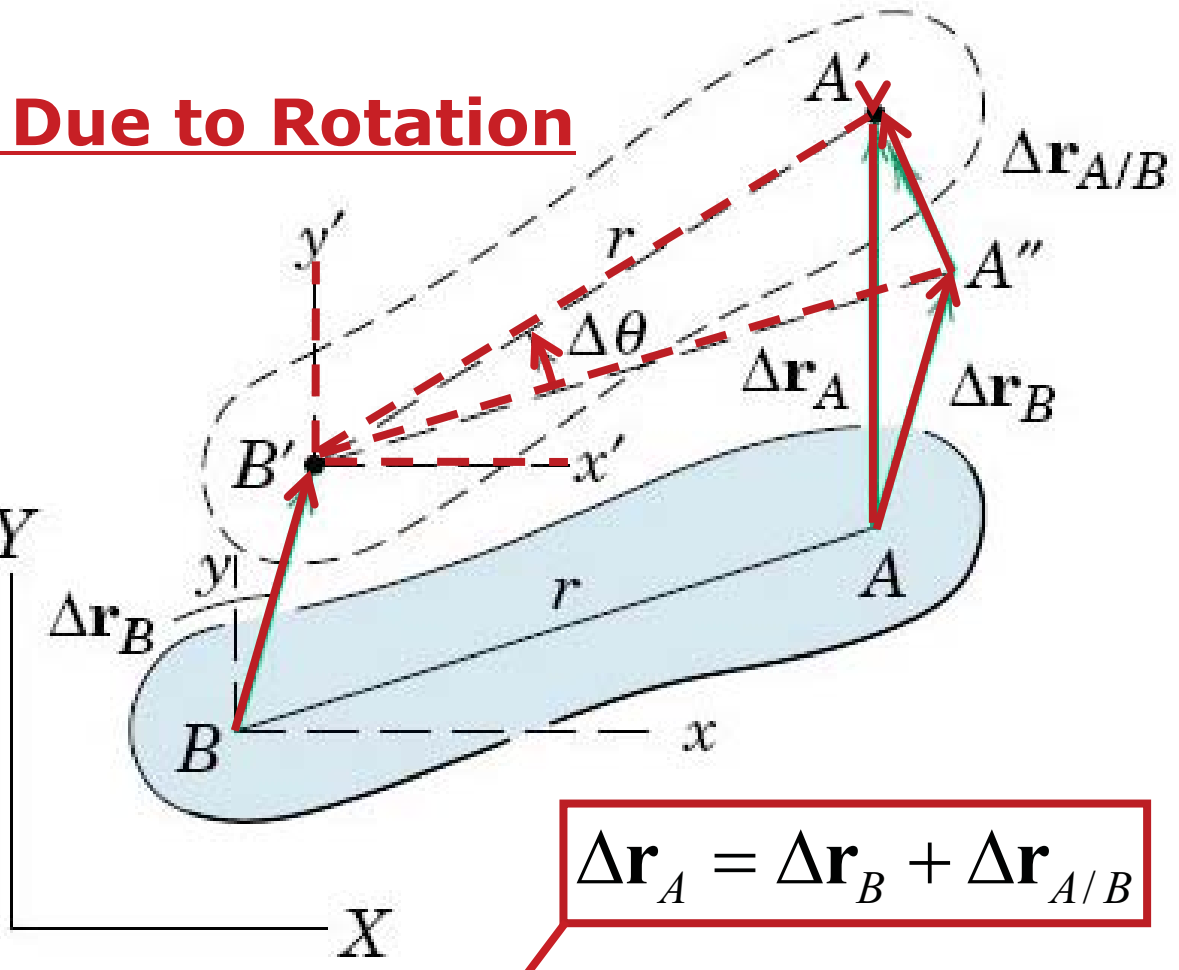
$$\Delta \mathbf{r}_A = \Delta \mathbf{r}_B + \Delta \mathbf{r}_{A/B}$$

From ***translating*** (*non-rotating*) axes ***x'-y'*** attached to point ***B'***, the remaining motion is a simple ***rotation*** about ***B'*** giving $\Delta \mathbf{r}_{A/B}$

Relative Velocity Due to Rotation

Relative velocity as a ***scalar***

$$v_{A/B} = \lim_{\Delta t \to 0} \frac{|\Delta \mathbf{r}_{A/B}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{r \Delta \theta}{\Delta t} = r\omega$$



$$\Delta \mathbf{r}_A = \Delta \mathbf{r}_B + \Delta \mathbf{r}_{A/B}$$

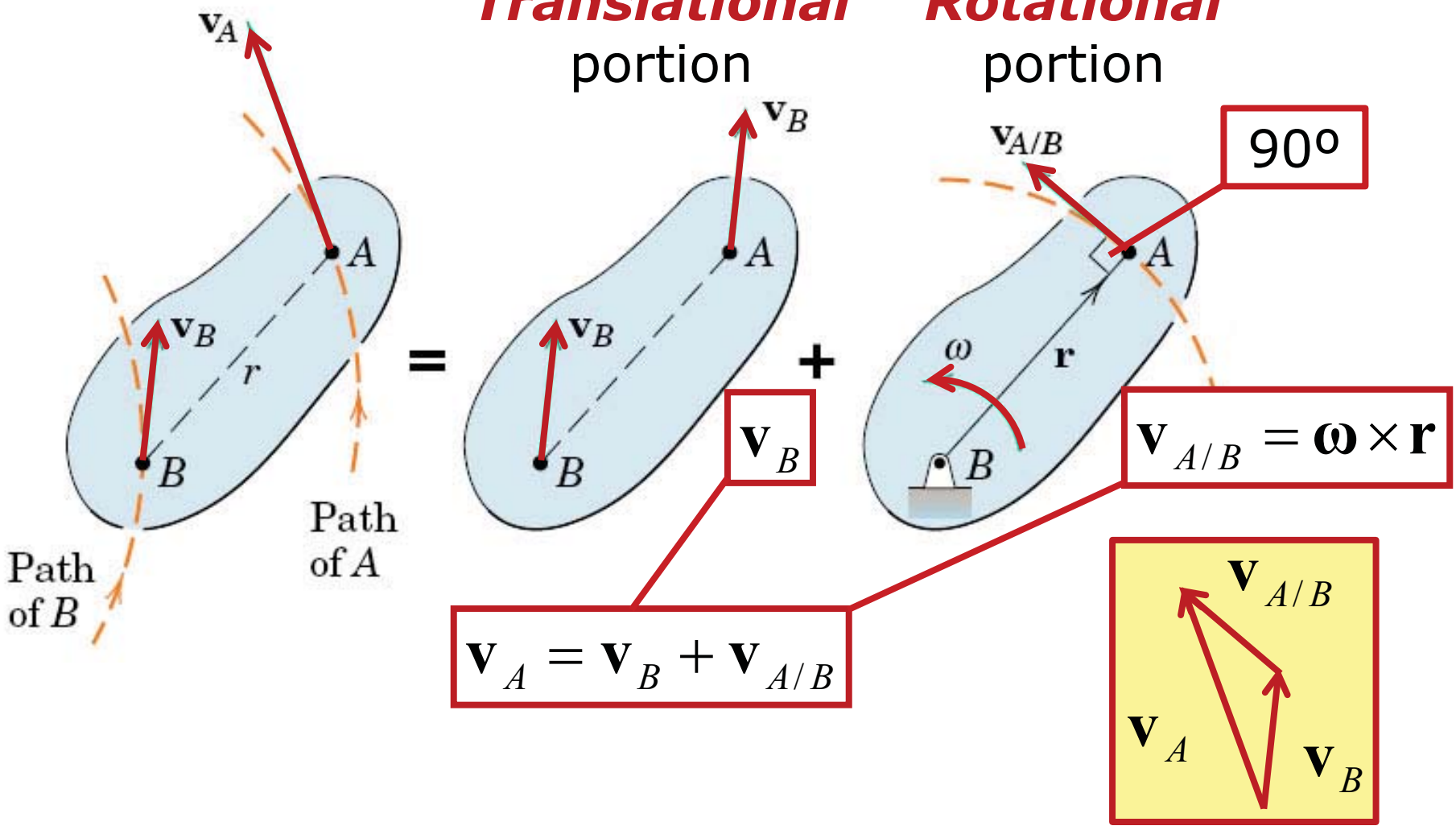
…as a ***vector***

$$\mathbf{v}_{A/B} = \boldsymbol{\omega} \times \mathbf{r}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

Dividing by Δt

Interpretation of Relative-Velocity Equation



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Solution of Relative-Velocity Equation: Case #1

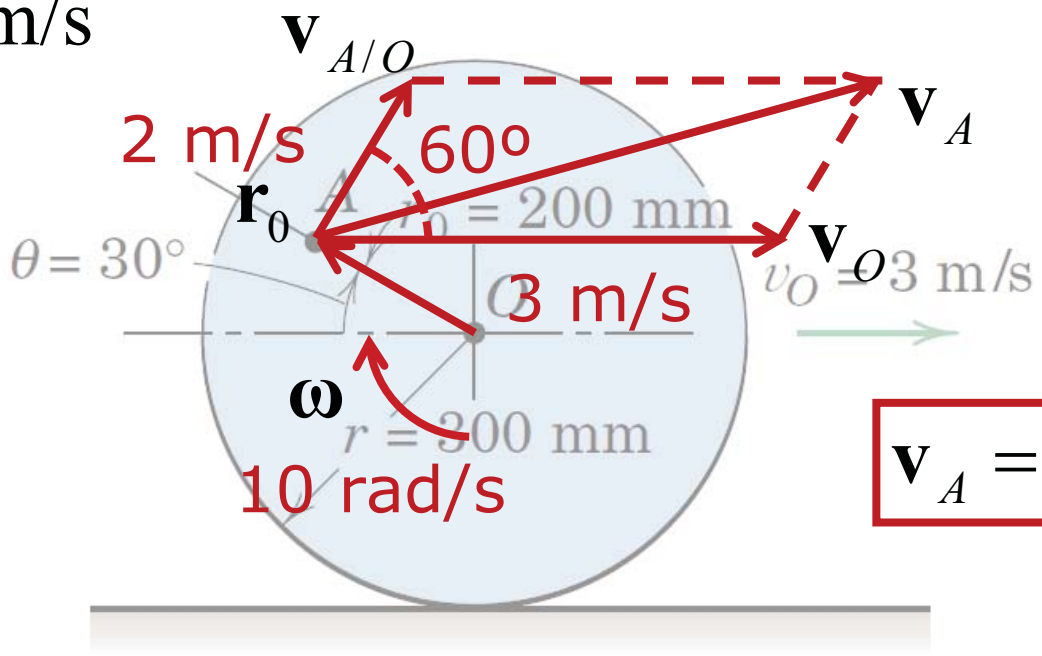
Scalar-Geometric

$$v_{A/O} = r_O \omega = r_O \left(\frac{v_O}{r} \right) = 2 \text{ m/s}$$

$$v_A^2 = v_{A/O}^2 + v_O^2 - 2(v_{A/O})(v_O)\cos 120^\circ$$

$$v_A^2 = 2^2 + 3^2 - 2(2)(3)\cos 120^\circ$$

$$v_A = 4.36 \text{ m/s}$$



$$\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A/O}$$

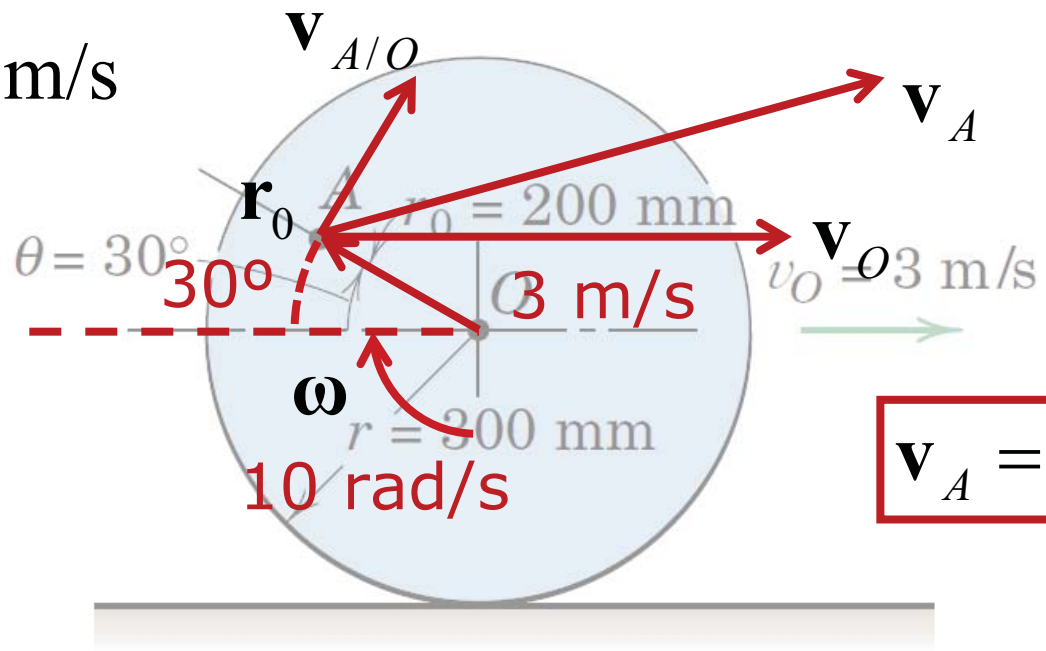
Solution of Relative-Velocity Equation: Case #2

Vector

$$\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A/O} = \mathbf{v}_O + \boldsymbol{\omega} \times \mathbf{r}_0$$

$$\mathbf{v}_A = 3\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -10 \\ -0.17 & 0.1 & 0 \end{vmatrix} = 3\mathbf{i} + 1.7\mathbf{j} + \mathbf{i} = 4\mathbf{i} + 1.7\mathbf{j}\ \text{m/s}$$

$$|\mathbf{v}_A| = 4.36\ \text{m/s}$$

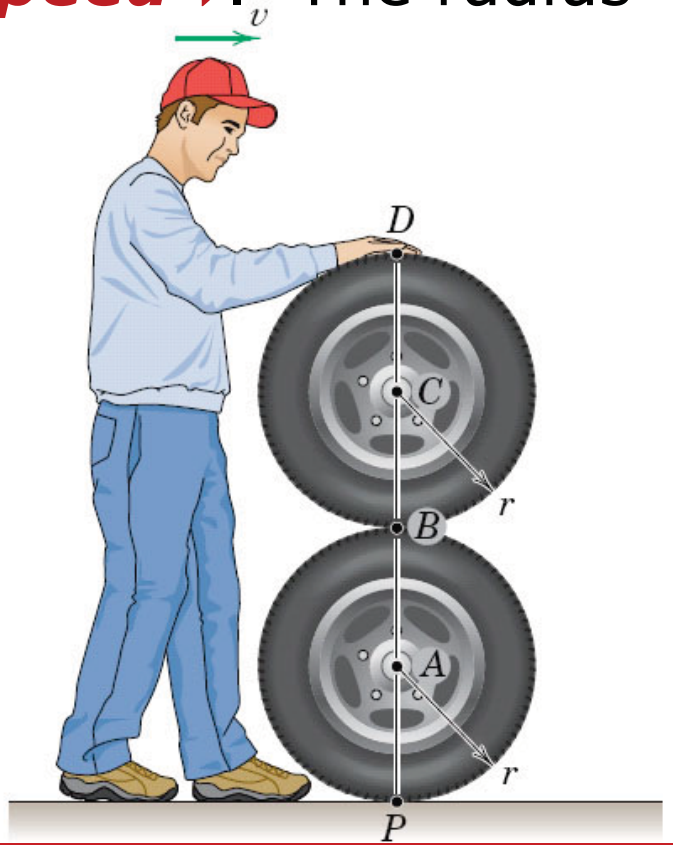


$$\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A/O}$$

Relative Velocity: Exercise

A mechanic "walks" a two-tire unit that rolls without slipping at constant ***speed v***. The radius of both tires is ***r***.

Determine the ***velocities*** of points ***A***, ***B***, ***C***, and ***D***.



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For Next Time…

- Complete Homework #4 due on Thursday (9/20)
- Read Chapter 6, Section 6.2