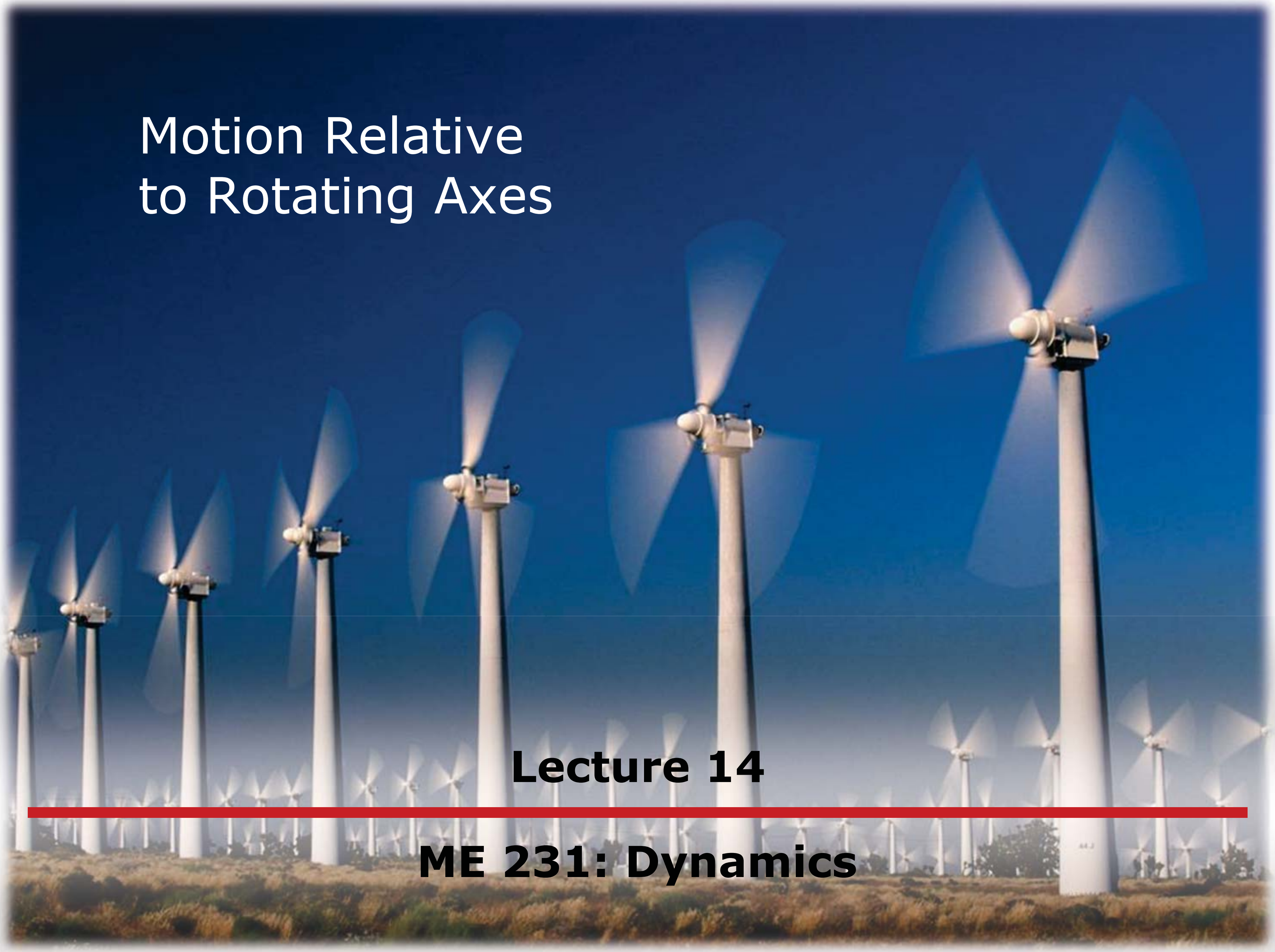


Motion Relative to Rotating Axes

Lecture 14

ME 231: Dynamics

Question of the Day

A wheel rotates 180º in the time it takes a ball dropped from the top of the wheel to reach the ground. What is the ***motion of the ball*** observed from a ***moving reference frame*** attached to the wheel?

Outline for Today

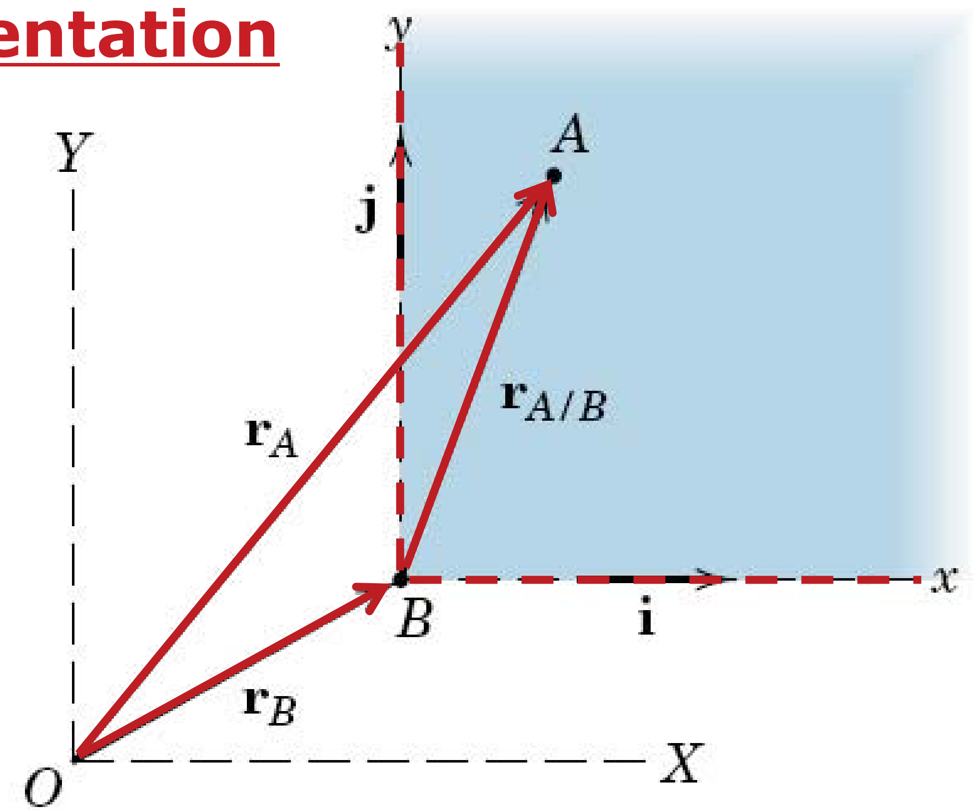
- Question of the day
- Vector representation: rotating axes
- Time derivatives of unit vectors
- Relative velocity
- Transformation of a time derivative
- Answer your questions!

Recall: Vector Representation

$$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$$

$$\mathbf{v}_A = \dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \dot{\mathbf{r}}_{A/B}$$

$$\mathbf{a}_A = \dot{\mathbf{v}}_A = \ddot{\mathbf{r}}_A = \ddot{\mathbf{r}}_B + \ddot{\mathbf{r}}_{A/B}$$



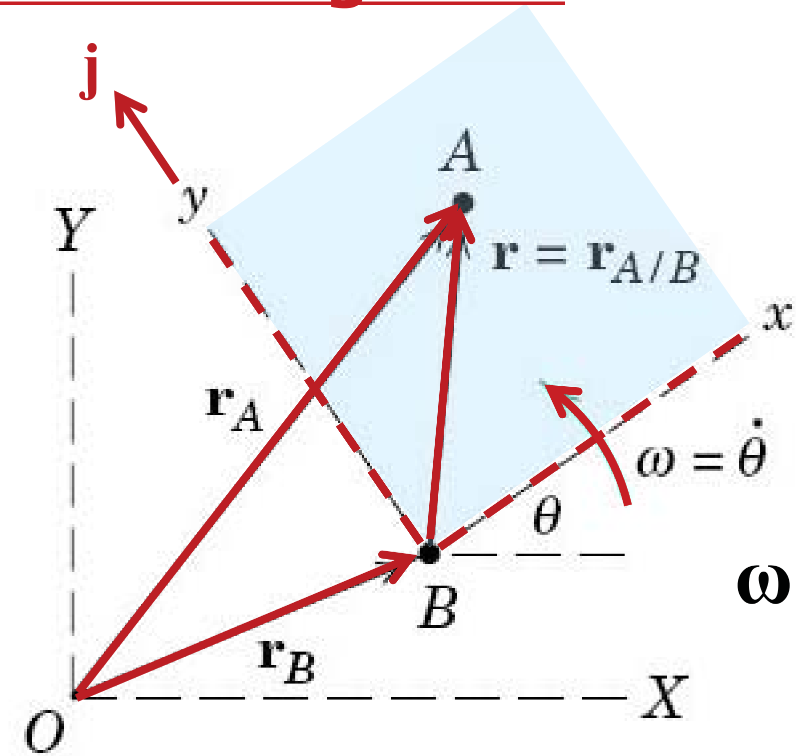
- Absolute position of ***B*** is defined in an inertial coordinate system ***X-Y***
- Attach a set of translating (*non-rotating*) axes ***x-y*** to particle ***B*** and define the position of ***A***
- Define position of "***A relative to B***" ("***A/B***") in ***x-y***

Vector Representation: Rotating Axes

$$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$$

$$\Downarrow$$

$$\boxed{\mathbf{r}_A = \mathbf{r}_B + (x\mathbf{i} + y\mathbf{j})}$$



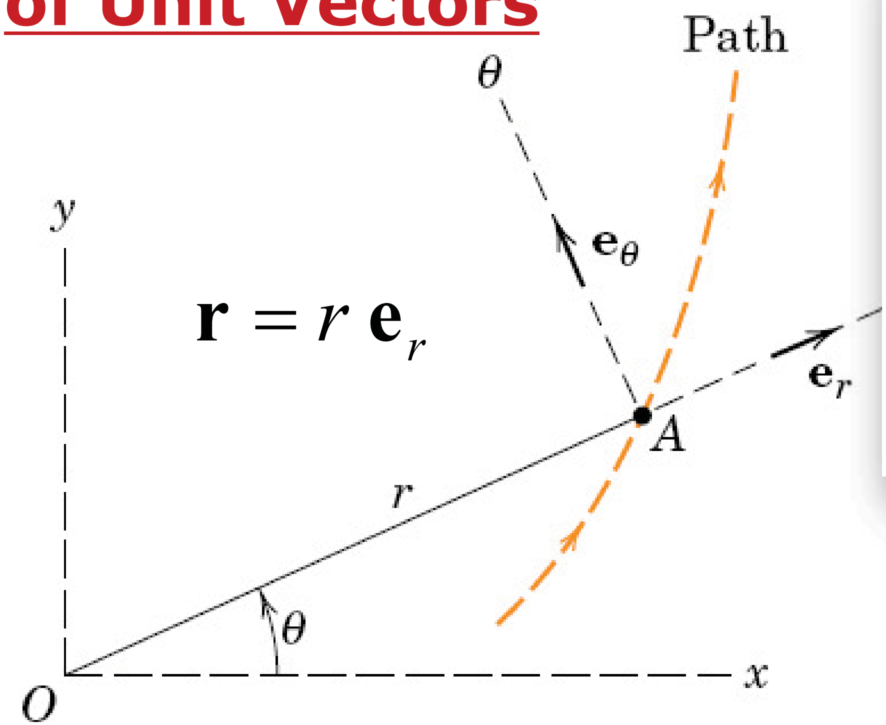
$$\boldsymbol{\omega} = \omega\mathbf{k}$$

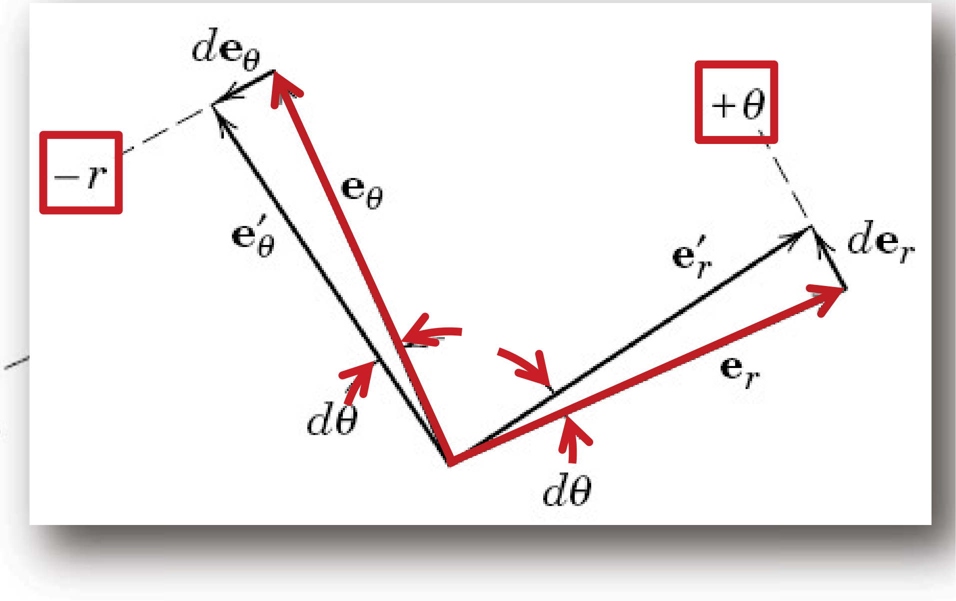
- Absolute position of B is defined in an inertial coordinate system X-Y
- Moving reference frame x-y has its origin at B and rotates with angular velocity ω
- Define "*A relative to B*" using unit vectors in x-y

Outline for Today

- Question of the day
- Vector representation: rotating axes
- Time derivatives of unit vectors
- Relative velocity
- Transformation of a time derivative
- Answer your questions!

Recall: Time Derivative of Unit Vectors



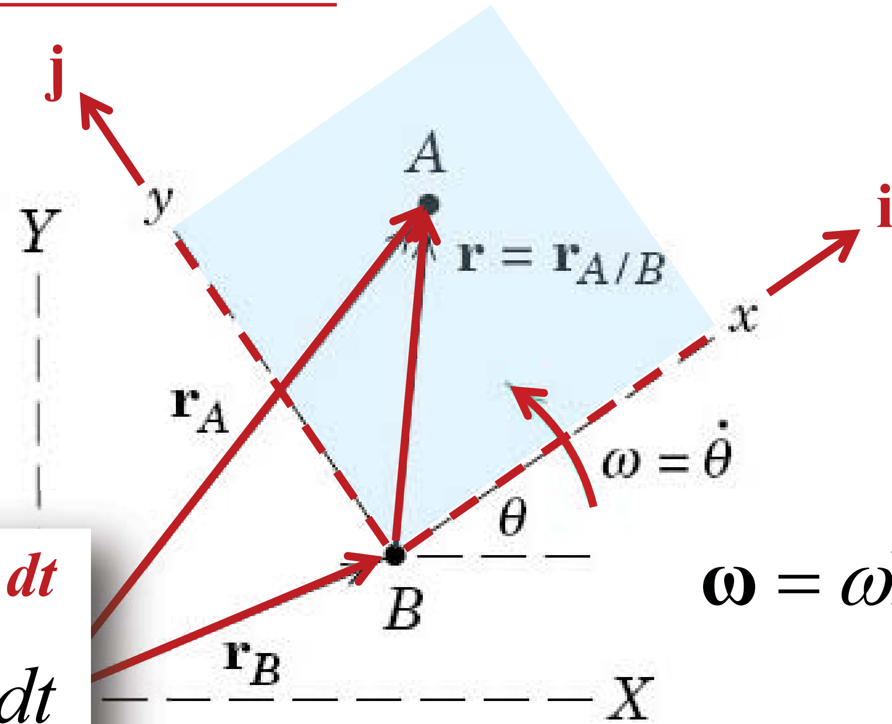


$$\dot{\mathbf{e}}_r = \frac{d\,\mathbf{e}_r}{dt} = \left(\frac{d\theta}{dt}\right)\mathbf{e}_\theta = \dot{\theta}\,\mathbf{e}_\theta$$

$$\dot{\mathbf{e}}_\theta = \frac{d\,\mathbf{e}_\theta}{dt} = -\left(\frac{d\theta}{dt}\right)\mathbf{e}_r = -\dot{\theta}\,\mathbf{e}_r$$

Time Derivatives of Unit Vectors

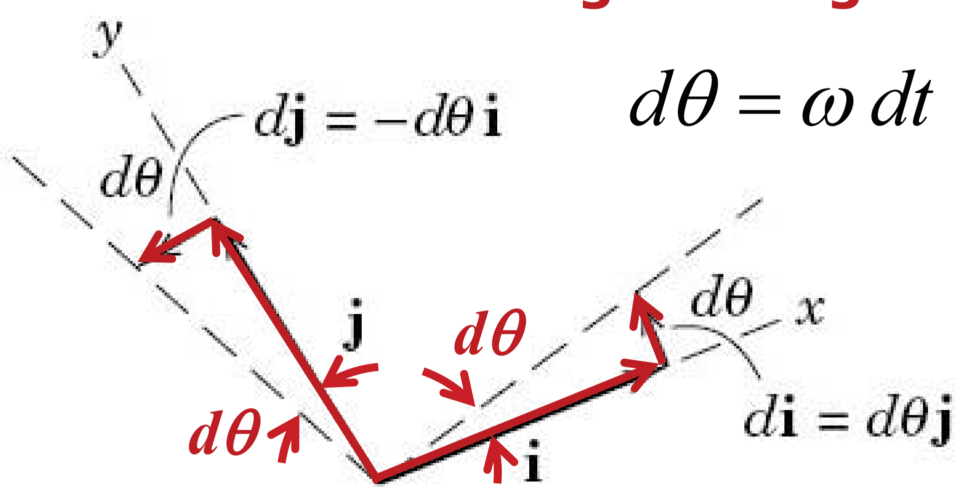
$$\mathbf{r}_A = \mathbf{r}_B + (x\mathbf{i} + y\mathbf{j})$$



$$\boldsymbol{\omega} = \omega\mathbf{k}$$

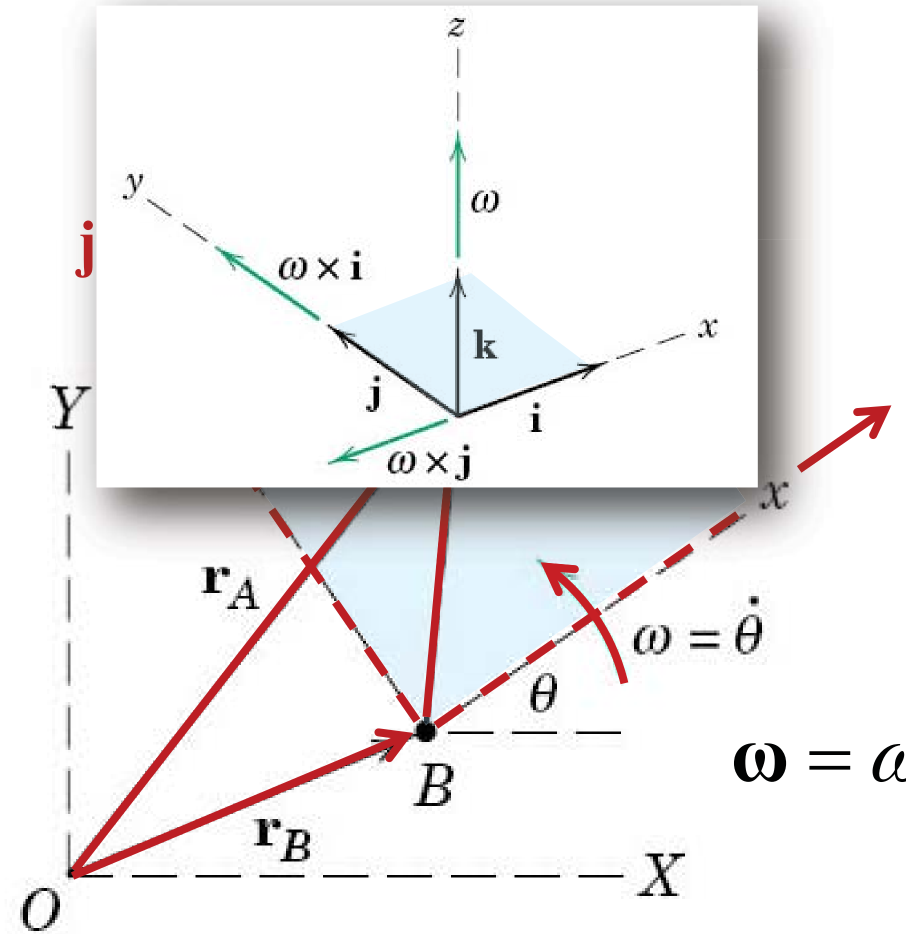
Infinitesimal change during *dt*

$$d\theta = \omega\, dt$$



$$\dot{\mathbf{i}} = \omega\mathbf{j} \qquad \dot{\mathbf{j}} = -\omega\mathbf{i}$$

Relative Velocity



$$\mathbf{r}_A = \mathbf{r}_B + \left(x\mathbf{i} + y\mathbf{j}\right)$$

$$\boxed{\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}}$$

$$\boldsymbol{\omega} = \omega\mathbf{k}$$

$$\dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \frac{d}{dt}\left(x\mathbf{i} + y\mathbf{j}\right)$$

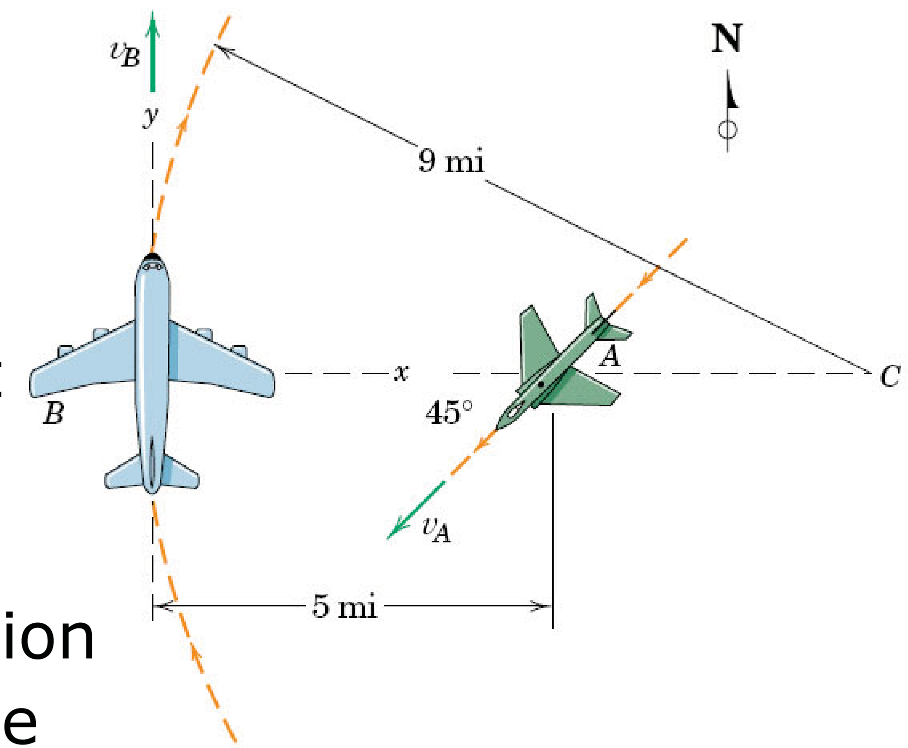
$$\dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \left(x\dot{\mathbf{i}} + y\dot{\mathbf{j}}\right) + \left(\dot{x}\mathbf{i} + \dot{y}\mathbf{j}\right) \qquad \dot{\mathbf{i}} = \omega\mathbf{j} \qquad \dot{\mathbf{j}} = -\omega\mathbf{i}$$

$$\dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \left(\boldsymbol{\omega} \times x\mathbf{i} + \boldsymbol{\omega} \times y\mathbf{j}\right) + \left(\dot{x}\mathbf{i} + \dot{y}\mathbf{j}\right) \qquad \dot{\mathbf{i}} = \boldsymbol{\omega} \times \mathbf{i} \qquad \dot{\mathbf{j}} = \boldsymbol{\omega} \times \mathbf{j}$$

$$\dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \left(\boldsymbol{\omega} \times \left(x\mathbf{i} + y\mathbf{j}\right)\right) + \left(\dot{x}\mathbf{i} + \dot{y}\mathbf{j}\right)$$

Relative Velocity: Exercise

Aircraft ***B*** has a constant ***speed*** of 480 mph along an arc with a ***radius*** of 9 miles. Aircraft ***A*** flies ***southwest*** at a constant ***speed*** of 360 mph.

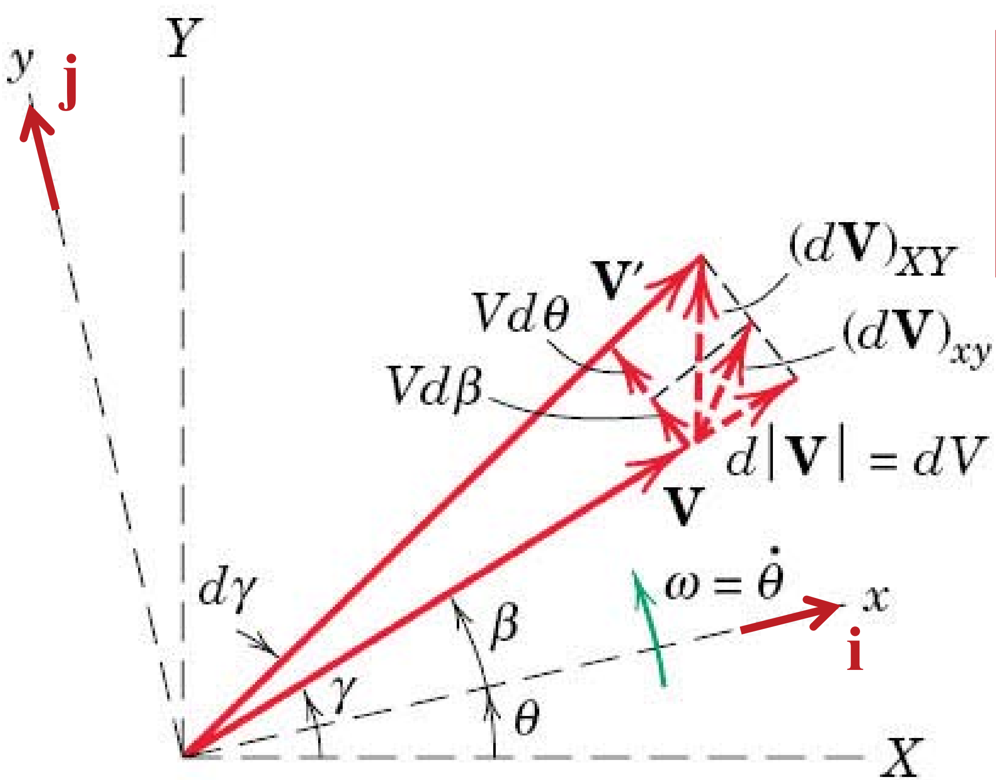


Write the ***vector*** expression (x-y attached to B) for the ***velocity*** of ***A relative to B***.

Transformation of a Time Derivative

One of the most important concepts in dynamics!

$$\left(\frac{d\mathbf{V}}{dt}\right)_{XY} = \left(\dot{V}_x\mathbf{i} + \dot{V}_y\mathbf{j}\right) + \left(V_x\dot{\mathbf{i}} + V_y\dot{\mathbf{j}}\right)$$

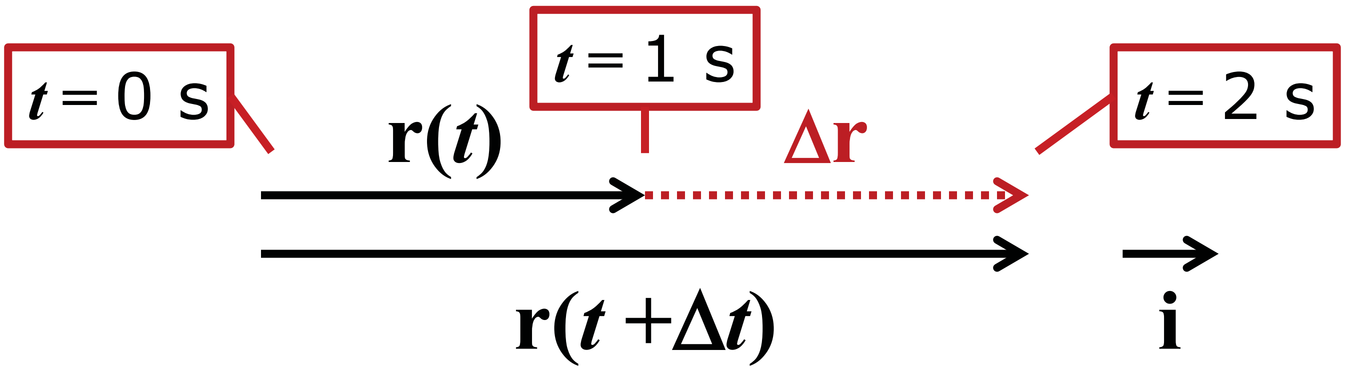


$$\boxed{\left(\frac{d\mathbf{V}}{dt}\right)_{XY} = \left(\frac{d\mathbf{V}}{dt}\right)_{xy} + \boldsymbol{\omega}\times\mathbf{V}}$$

$$\dot{\mathbf{i}} = \boldsymbol{\omega}\times\mathbf{i} \qquad \dot{\mathbf{j}} = \boldsymbol{\omega}\times\mathbf{j}$$

Recall: Time Derivative of a Vector

Magnitude changes, but direction constant



$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = 2\,\mathbf{i}$$

$$\mathbf{r}(t) = 2t\,\mathbf{i}$$

$$\Delta\mathbf{r} = (4-2)\,\mathbf{i} = 2\,\mathbf{i}$$

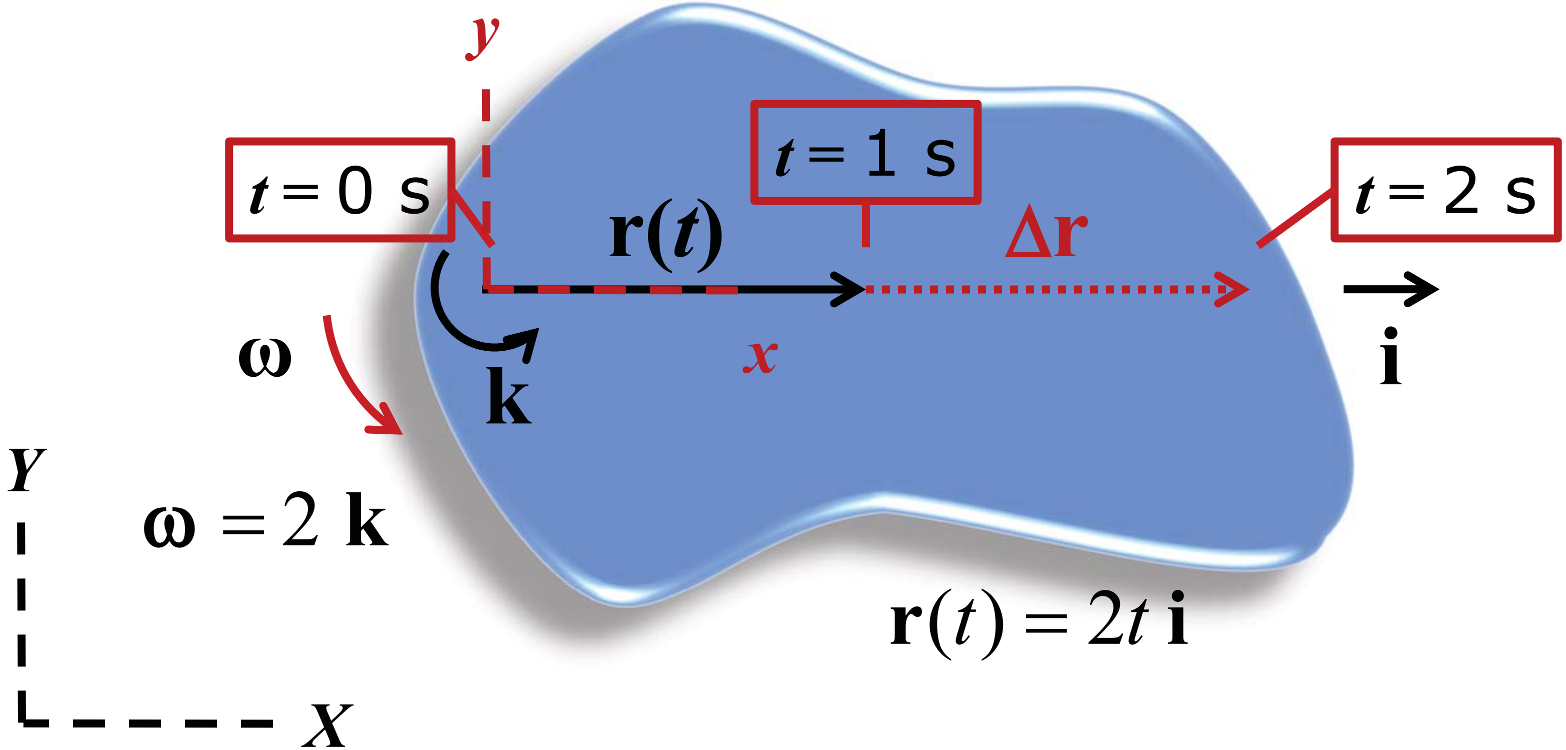
$\mathbf{v}$ has

direction of $\Delta\mathbf{r}$

and magnitude $|\Delta\mathbf{r}/\Delta t|$

$$\left|\frac{\Delta\mathbf{r}}{\Delta t}\right| = \left|\frac{(4-2)\,\mathbf{i}}{(2-1)}\right| = \left|\frac{2\,\mathbf{i}}{1}\right| = 2$$

Transformation of a Time Derivative: Exercise



$$\dot{\mathbf{r}}_{XY} = \dot{\mathbf{r}}_{xy} + \boldsymbol{\omega} \times \mathbf{r}$$

$$= 2\,\mathbf{i} + 2\,\mathbf{k} \times 2t\,\mathbf{i}$$

$$= 2\,\mathbf{i} + 4t\,\mathbf{j}$$

Outline for Today

- Question of the day
- Vector representation: rotating axes
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For Next Time…

- Complete Homework #5 due on Wednesday (9/26) at the ***beginning of class***
- Read Chapter 6, Sections 6.3 and 6.4