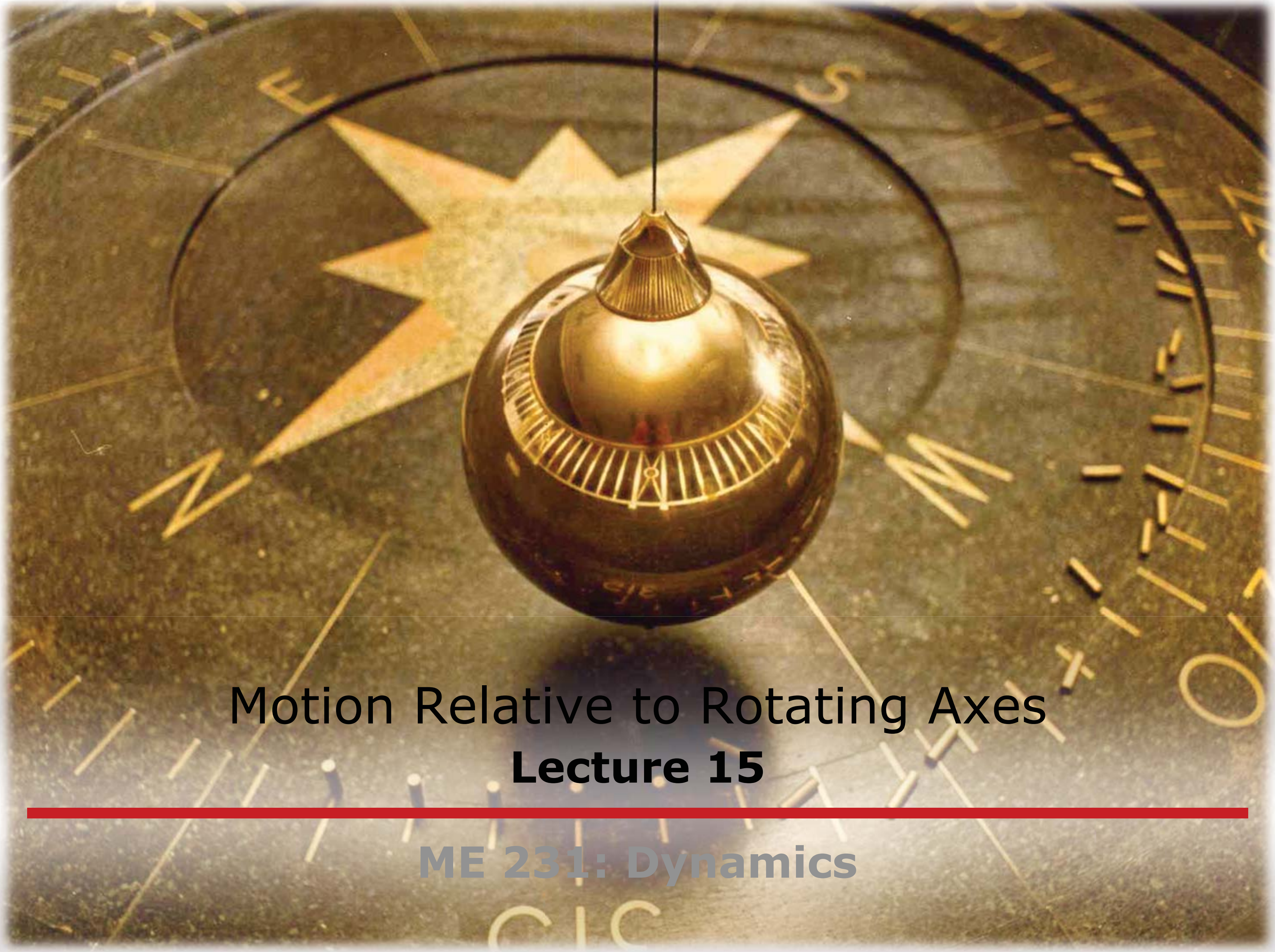


Motion Relative to Rotating Axes

Lecture 15

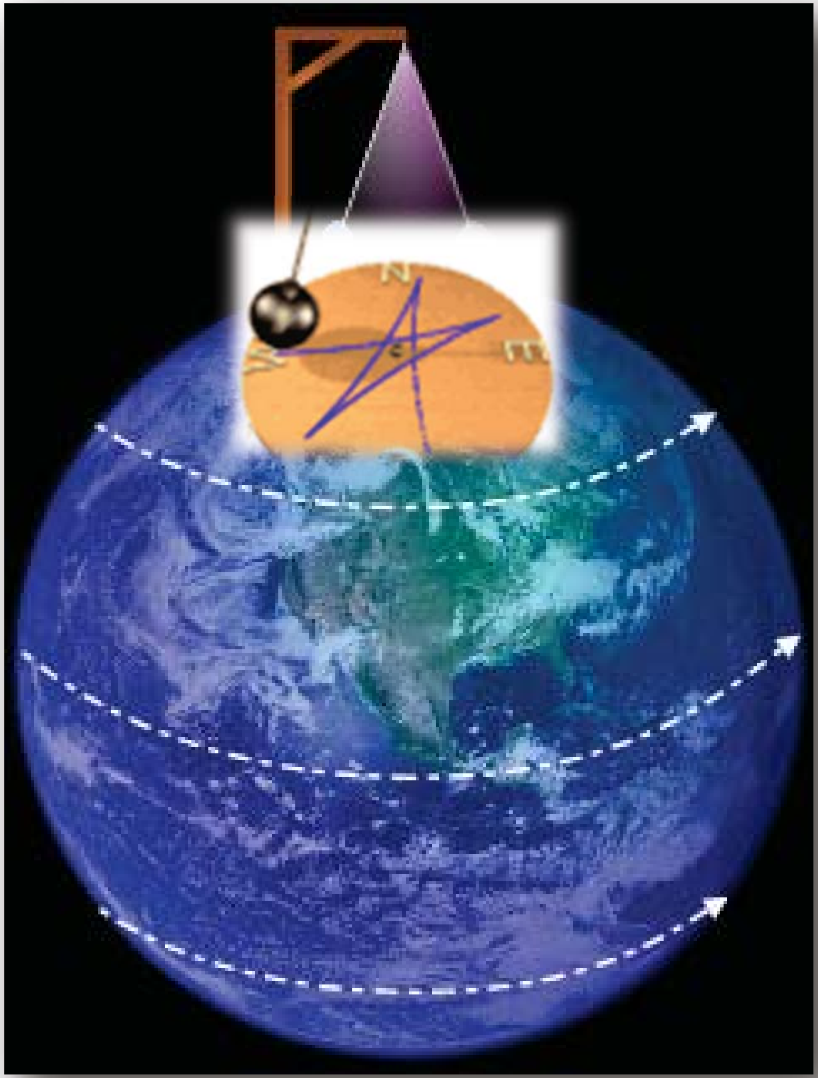
ME 231: Dynamics

Question of the Day

A tall pendulum free to oscillate in any vertical plane is set in motion at the north pole.

What direction would the pendulum swing in space?

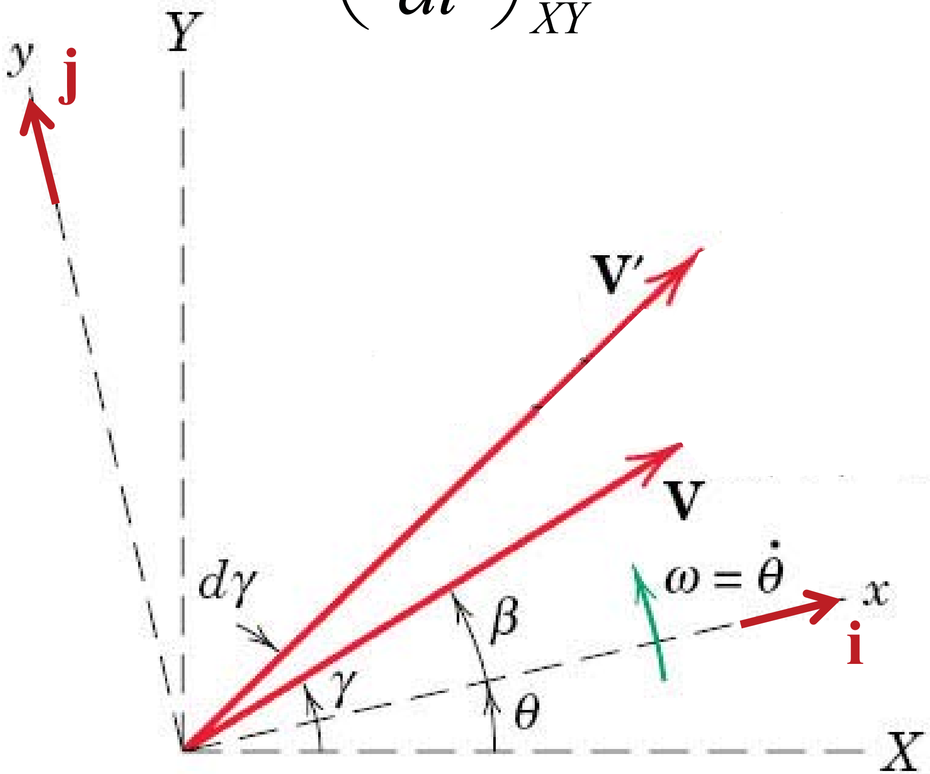
What direction on Earth?



Question of the Day (concept from last time)

Transformation of Time Derivative
One of the most important concepts in dynamics!

$$\left(\frac{d\mathbf{V}}{dt}\right)_{XY} = \left(\dot{V}_x\mathbf{i} + \dot{V}_y\mathbf{j}\right) + \left(V_x\dot{\mathbf{i}} + V_y\dot{\mathbf{j}}\right)$$



$$\left(\frac{d\mathbf{V}}{dt}\right)_{XY} = \left(\frac{d\mathbf{V}}{dt}\right)_{xy} + \boldsymbol{\omega}\times\mathbf{V}$$

$$\dot{\mathbf{i}} = \boldsymbol{\omega}\times\mathbf{i} \qquad \dot{\mathbf{j}} = \boldsymbol{\omega}\times\mathbf{j}$$

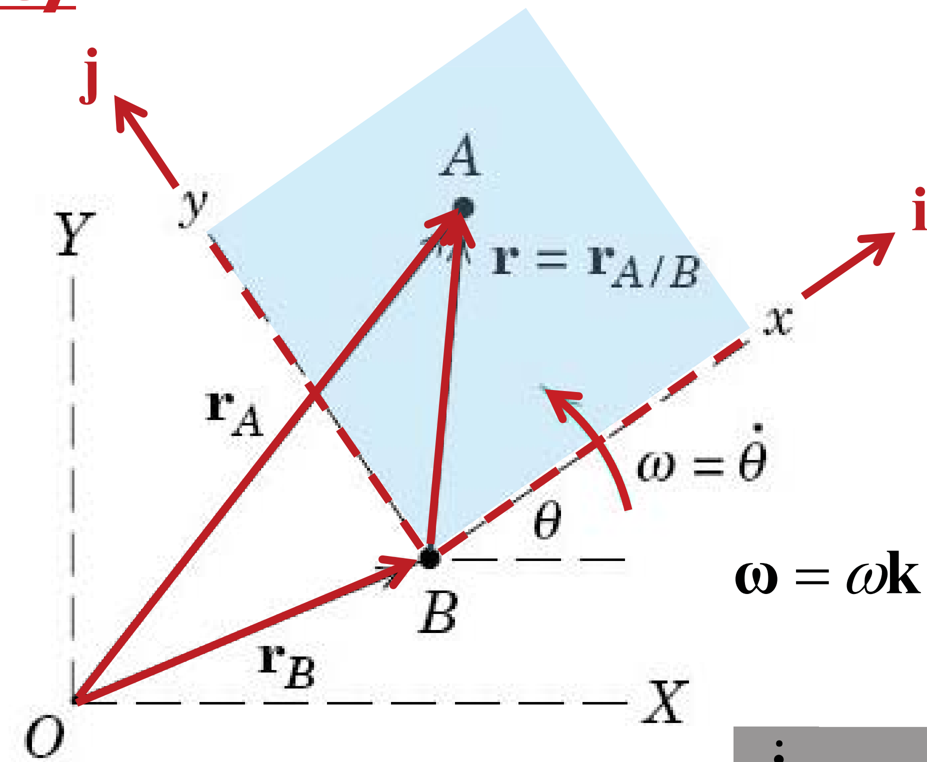
Outline for Today

- Question of the day
- Relative acceleration
- Coriolis acceleration: $2\boldsymbol{\omega} \times \mathbf{v}_{\text{rel}}$
- Rotating versus nonrotating systems
- Answer your questions!

Recall: Relative Velocity

$$\mathbf{r}_A = \mathbf{r}_B + (x\mathbf{i} + y\mathbf{j})$$

$$\boxed{\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}}$$

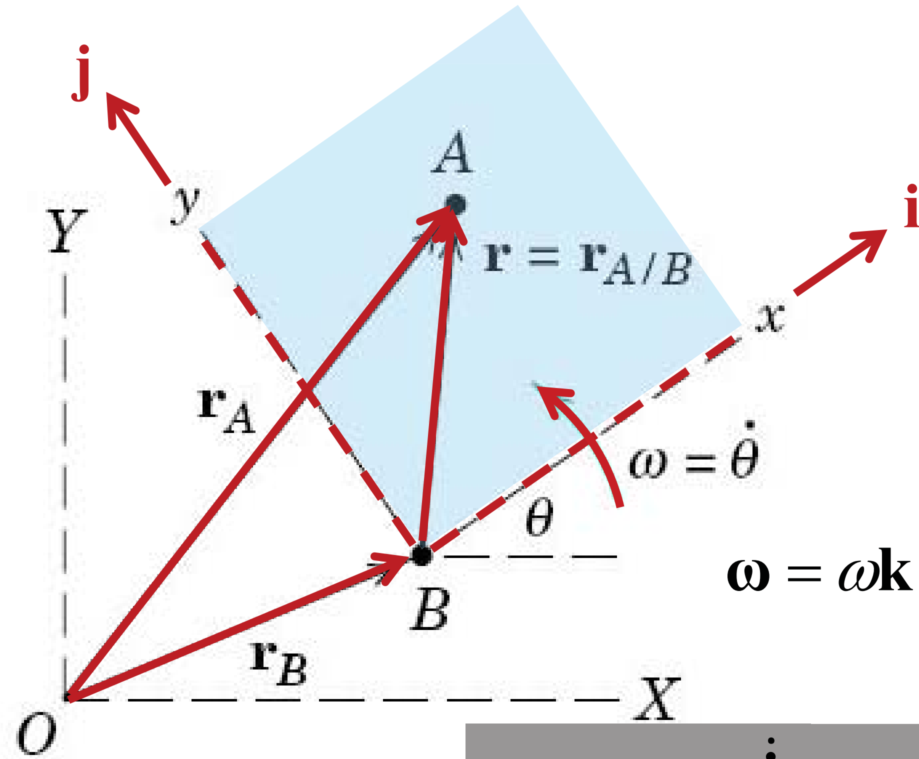


$$\boldsymbol{\omega} = \omega\mathbf{k}$$

$$\dot{\mathbf{i}} = \boldsymbol{\omega} \times \mathbf{i}$$
$$\dot{\mathbf{j}} = \boldsymbol{\omega} \times \mathbf{j}$$

- Absolute position of ***B*** is defined in an inertial coordinate system ***X***-***Y***
- Moving reference frame ***x***-***y*** has its origin at ***B*** and rotates with angular velocity ***ω***
- Define "***A relative to B***" using unit vectors in ***x***-***y***

Relative Acceleration



$$\mathbf{r}_A = \mathbf{r}_B + (x\mathbf{i} + y\mathbf{j})$$

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}$$

$$\boldsymbol{\omega} = \omega\mathbf{k}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times \dot{\mathbf{r}} + \dot{\mathbf{v}}_{rel}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}) + \dot{\mathbf{v}}_{rel}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \boldsymbol{\omega} \times \mathbf{v}_{rel} + \dot{\mathbf{v}}_{rel}$$

$$\boxed{\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}}$$

$$\dot{\mathbf{i}} = \boldsymbol{\omega} \times \mathbf{i}$$

$$\dot{\mathbf{j}} = \boldsymbol{\omega} \times \mathbf{j}$$

$$\dot{\mathbf{r}} = \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}$$

$$\dot{\mathbf{v}}_{rel} = \boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$

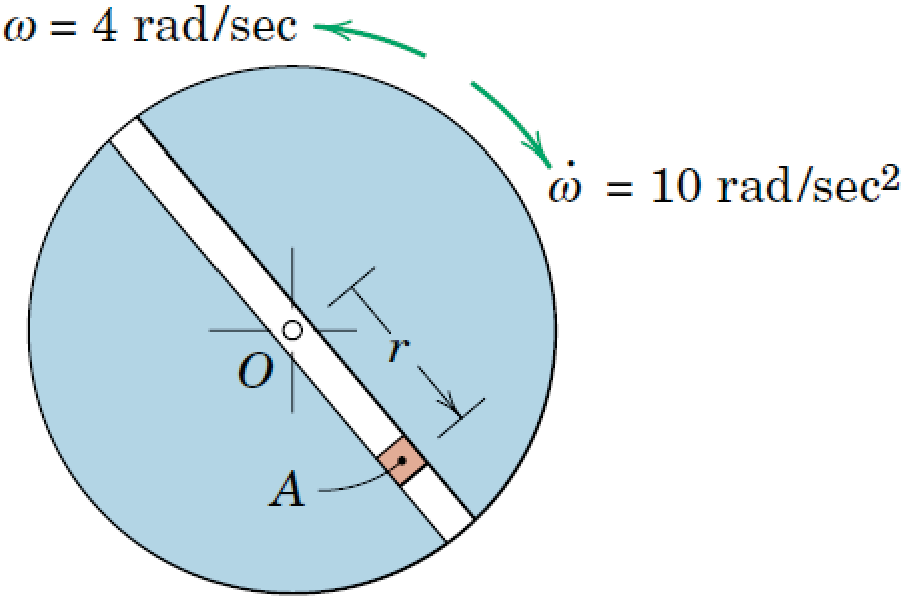
Relative Acceleration: Exercise

A ***disk*** with the radial slot is ***rotating*** about ***O*** with $\omega = 4$ rad/s and decreasing by 10 rad/s^2. The ***slider A*** motion is

$r = 6 \text{ in}, \dot{r} = 5 \text{ in/s}, \text{and}$

$\ddot{r} = 81 \text{ in/s}^2$

Determine the ***absolute acceleration*** of *A* for this position.



$$\mathbf{a}_A = \mathbf{a}_O + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$

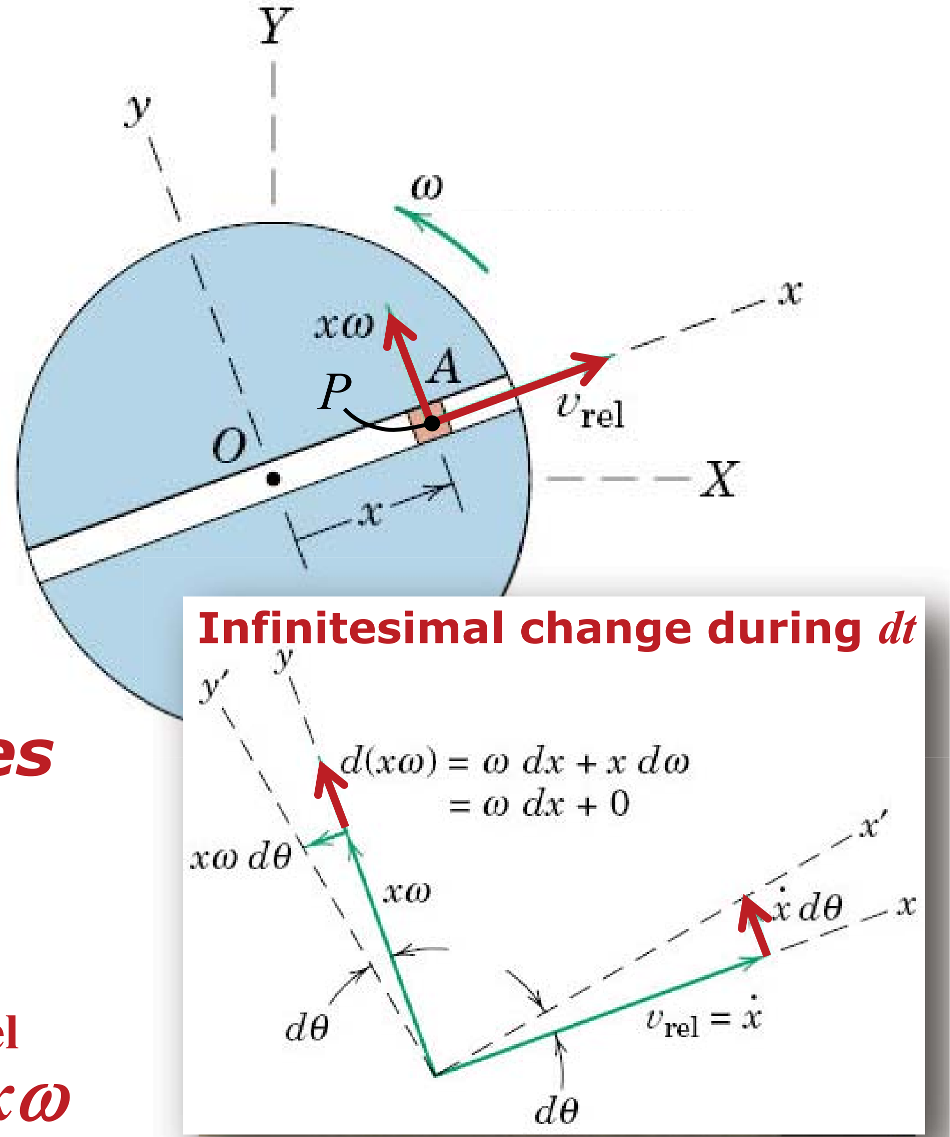
Outline for Today

- Question of the day
- Relative acceleration
- Coriolis acceleration: $2\boldsymbol{\omega} \times \mathbf{v}_{\text{rel}}$
- Rotating versus nonrotating systems
- Answer your questions!

Coriolis Acceleration:

$2\boldsymbol{\omega} \times \mathbf{v}_{rel}$

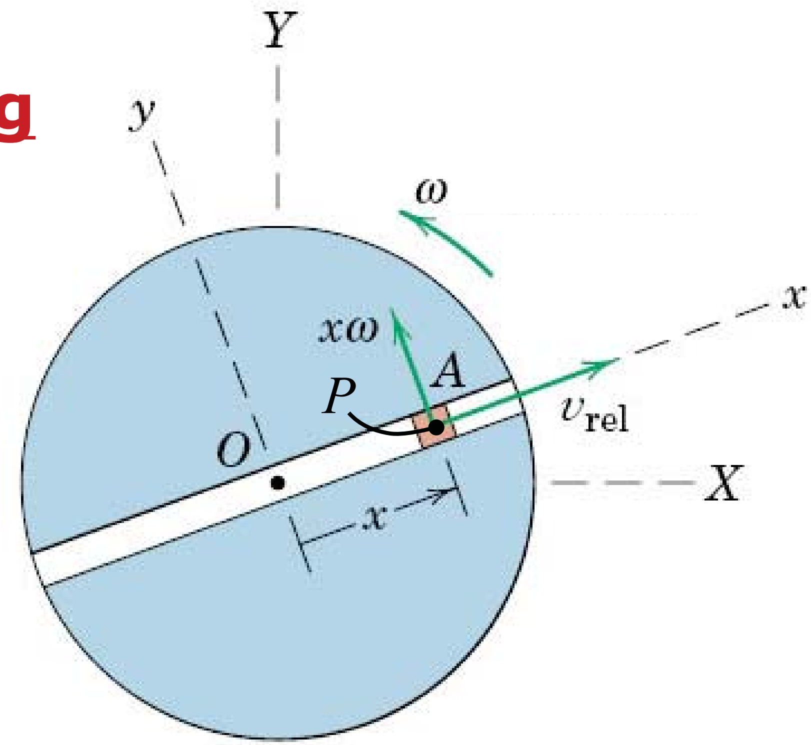
- Represents the ***difference*** between ***acceleration*** measured from ***nonrotating*** axes and ***rotating axes***
- Composed of ***two physical effects***
  1. Change in ***direction*** of v_{rel}
  2. Change in ***magnitude*** of $x\omega$



$$\left|2\boldsymbol{\omega} \times \mathbf{v}_{rel}\right| = \omega\dot{x} + \dot{x}\omega$$

$$\mathbf{a}_A = \mathbf{a}_O + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$

Rotating vs. Nonrotating Systems



$$\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}_{A/O}$$ **Relative acceleration for *A/O***

$$\mathbf{a}_A = \mathbf{a}_P + \mathbf{a}_{A/P}$$ **Relative acceleration for *A/P***

$$\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}_{P/O} + \mathbf{a}_{A/P}$$ **Tangent & normal for *P/O* & *A/P***

$$\mathbf{a}_A = \mathbf{a}_O + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$

Outline for Today

- Question of the day
- Relative acceleration
- Coriolis acceleration: $2\boldsymbol{\omega} \times \mathbf{v}_{rel}$
- Rotating versus nonrotating systems
- Answer your questions!

For Next Time…

- Begin Homework #6 due next ***Monday (10/1)***
- Review Chapters 2 & 6