A close-up photograph of a Foucault pendulum bob, a polished metal sphere with a decorative ring of vertical lines near the top, suspended by a thin wire. The bob is positioned over a circular dial with a compass rose and degree markings. The dial is dark with gold-colored markings and letters. The lighting is warm, highlighting the metallic sheen of the bob and the intricate details of the dial.

Motion Relative to Rotating Axes  
**Lecture 15**

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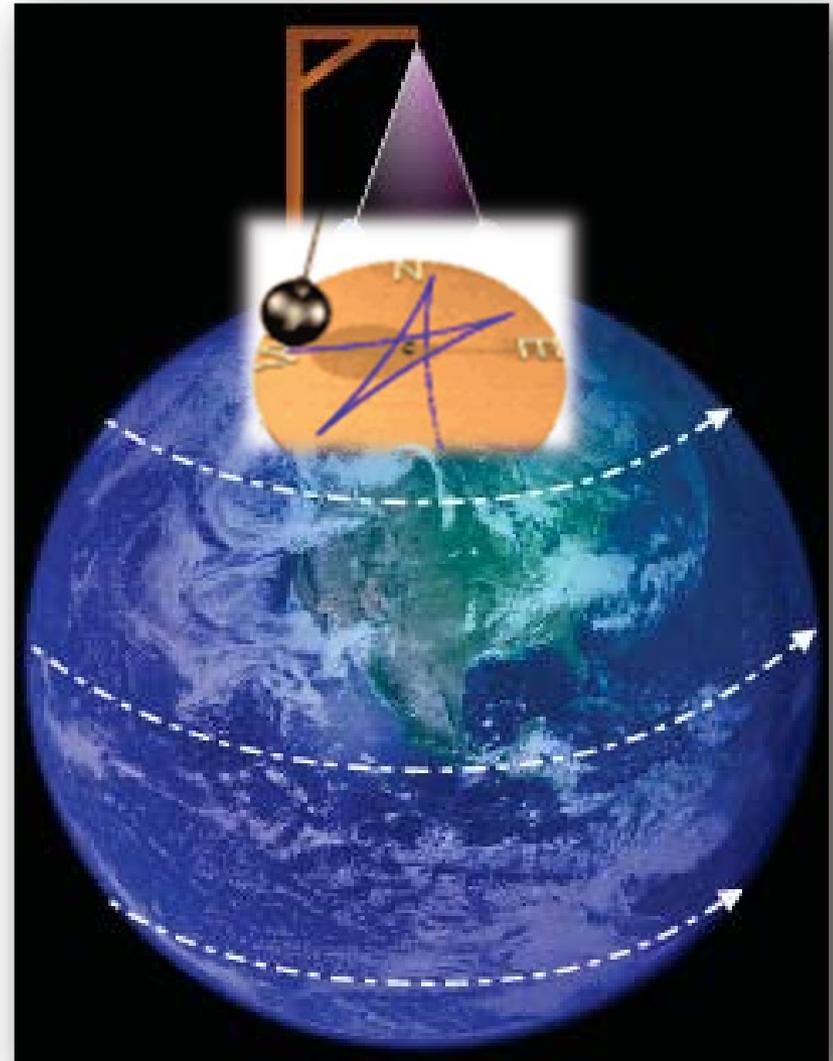
ME 231: Dynamics

## Question of the Day

A tall pendulum free to oscillate in any vertical plane is set in motion at the north pole.

What direction would the pendulum swing in space?

What direction on Earth?



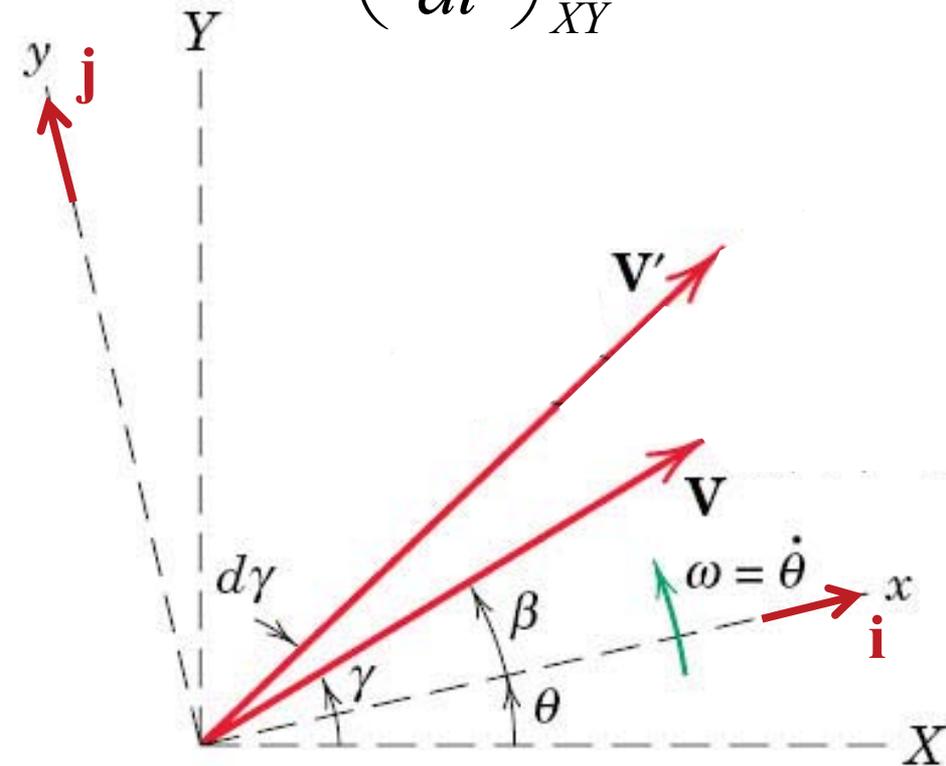
# Question of the Day (concept from last time)

## Transformation of Time Derivative

One of the most important concepts in dynamics!

$$\left(\frac{d\mathbf{V}}{dt}\right)_{XY} = (\dot{V}_x \mathbf{i} + \dot{V}_y \mathbf{j}) + (V_x \dot{\mathbf{i}} + V_y \dot{\mathbf{j}})$$

$$\left(\frac{d\mathbf{V}}{dt}\right)_{XY} = \left(\frac{d\mathbf{V}}{dt}\right)_{xy} + \boldsymbol{\omega} \times \mathbf{V}$$



$$\dot{\mathbf{i}} = \boldsymbol{\omega} \times \mathbf{i} \quad \dot{\mathbf{j}} = \boldsymbol{\omega} \times \mathbf{j}$$

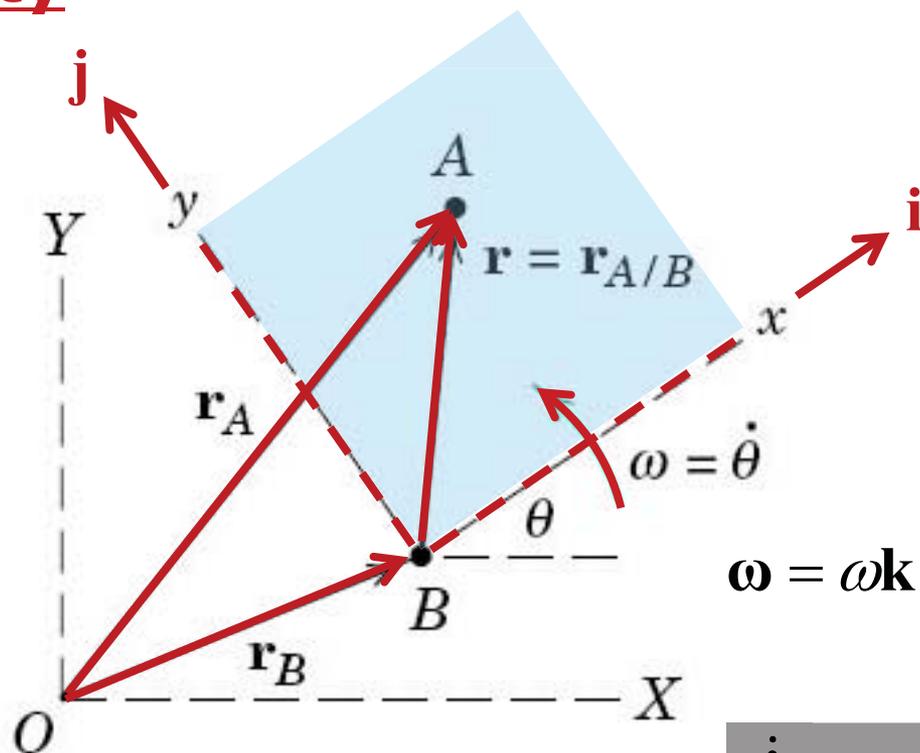
## Outline for Today

- Question of the day
- Relative acceleration
- Coriolis acceleration:  $2\boldsymbol{\omega} \times \mathbf{v}_{\text{rel}}$
- Rotating versus nonrotating systems
- Answer your questions!

## Recall: Relative Velocity

$$\mathbf{r}_A = \mathbf{r}_B + (x\mathbf{i} + y\mathbf{j})$$

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}$$



$$\boldsymbol{\omega} = \omega \mathbf{k}$$

$$\dot{\mathbf{i}} = \boldsymbol{\omega} \times \mathbf{i}$$

$$\dot{\mathbf{j}} = \boldsymbol{\omega} \times \mathbf{j}$$

- Absolute position of  $B$  is defined in an inertial coordinate system  $X-Y$
- Moving reference frame  $x-y$  has its origin at  $B$  and rotates with angular velocity  $\boldsymbol{\omega}$
- Define " $A$  relative to  $B$ " using unit vectors in  $x-y$

# Relative Acceleration

$$\mathbf{r}_A = \mathbf{r}_B + (x\mathbf{i} + y\mathbf{j})$$

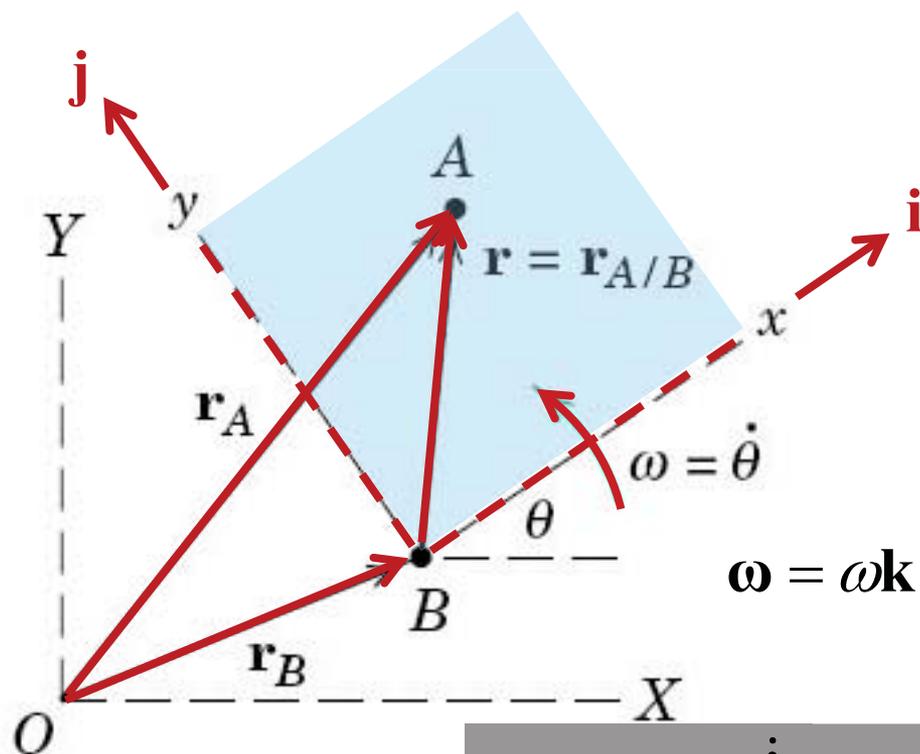
$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times \dot{\mathbf{r}} + \dot{\mathbf{v}}_{rel}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}) + \dot{\mathbf{v}}_{rel}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \boldsymbol{\omega} \times \mathbf{v}_{rel} + \dot{\mathbf{v}}_{rel}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$



$$\boldsymbol{\omega} = \omega \mathbf{k}$$

$$\dot{\mathbf{i}} = \boldsymbol{\omega} \times \mathbf{i}$$

$$\dot{\mathbf{j}} = \boldsymbol{\omega} \times \mathbf{j}$$

$$\dot{\mathbf{r}} = \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}$$

$$\dot{\mathbf{v}}_{rel} = \boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$

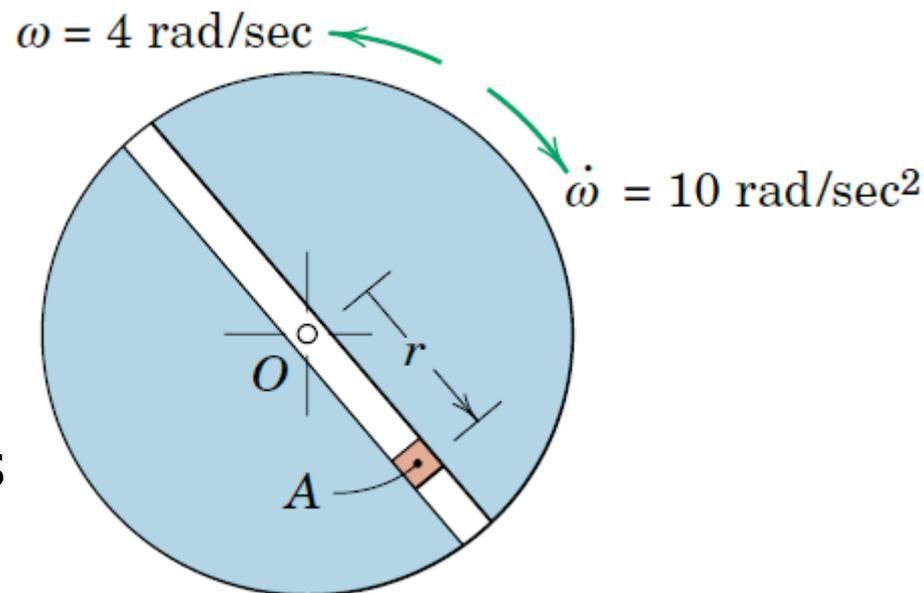
## Relative Acceleration: Exercise

A **disk** with the radial slot is **rotating** about  $O$  with  $\omega = 4 \text{ rad/s}$  and decreasing by  $10 \text{ rad/s}^2$ . The **slider**  $A$  motion is

$$r = 6 \text{ in}, \dot{r} = 5 \text{ in/s}, \text{ and}$$

$$\ddot{r} = 81 \text{ in/s}^2$$

Determine the **absolute acceleration** of  $A$  for this position.



$$\mathbf{a}_A = \mathbf{a}_O + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$

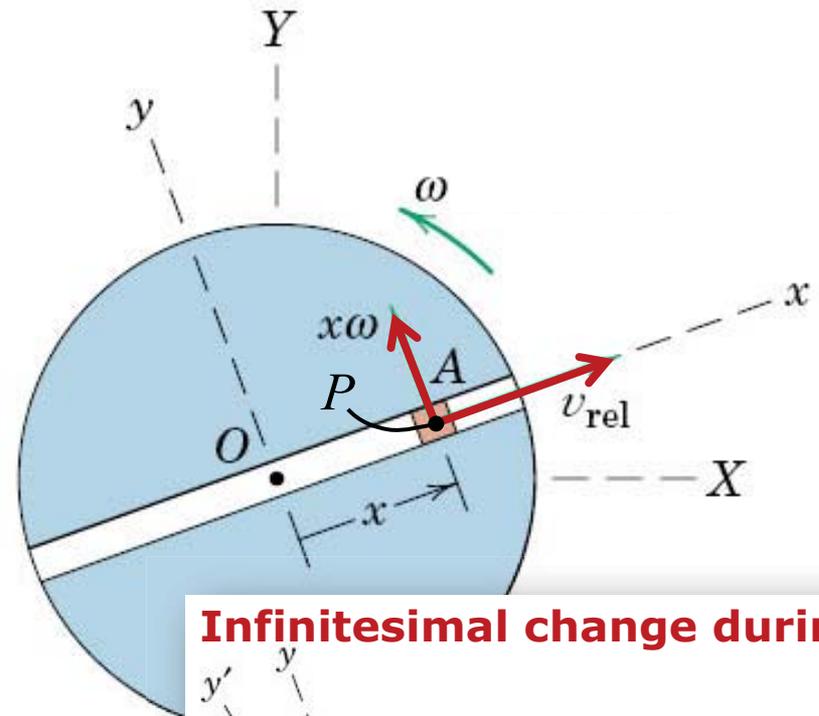
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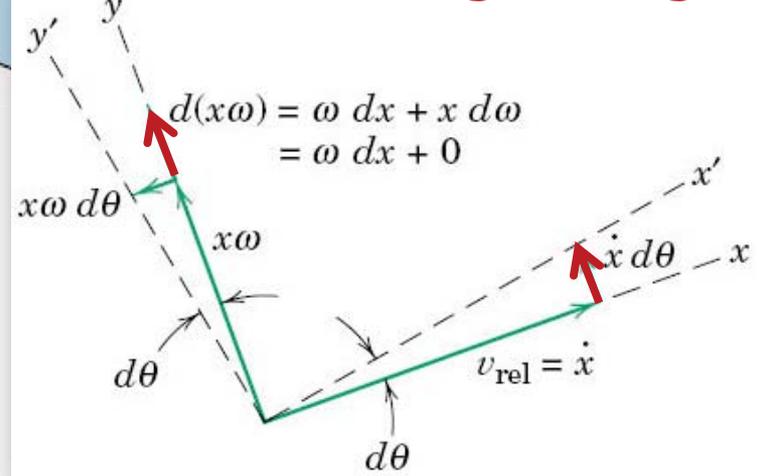
# Coriolis Acceleration:

$$2\boldsymbol{\omega} \times \mathbf{v}_{rel}$$

- Represents the **difference** between **acceleration** measured from **nonrotating** axes and **rotating axes**
- Composed of **two physical effects**
  1. Change in **direction** of  $\mathbf{v}_{rel}$
  2. Change in **magnitude** of  $x\omega$



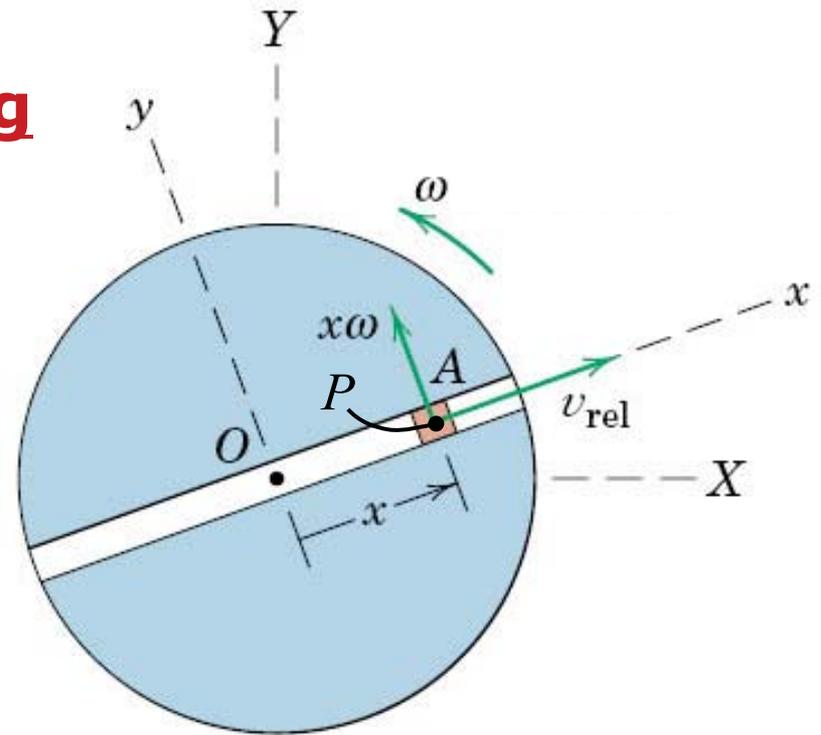
## Infinitesimal change during dt



$$\mathbf{a}_A = \mathbf{a}_O + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$

$$|2\boldsymbol{\omega} \times \mathbf{v}_{rel}| = \omega \dot{x} + \dot{x} \omega$$

# Rotating vs. Nonrotating Systems



$$\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}_{A/O} \quad \text{Relative acceleration for } A/O$$

$$\mathbf{a}_A = \mathbf{a}_P + \mathbf{a}_{A/P} \quad \text{Relative acceleration for } A/P$$

$$\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}_{P/O} + \mathbf{a}_{A/P} \quad \text{Tangent \& normal for } P/O \text{ \& } A/P$$

$$\mathbf{a}_A = \mathbf{a}_O + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$

## Outline for Today

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## For Next Time...

- Begin Homework #6 due next ***Monday***  
***(10/1)***
- Review Chapters 2 & 6