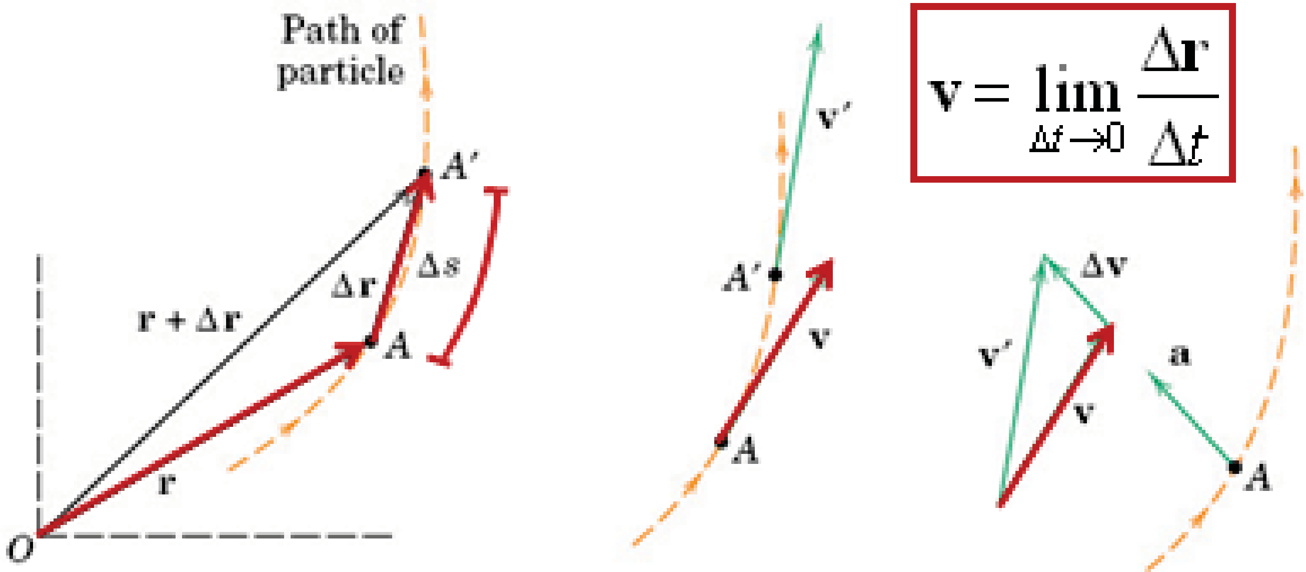
Kinematics of Particles (Ch. 2) Review

Lecture 16

ME 231: Dynamics

Question of the Day

What is the most important concept in Chapter 2? **Time Derivative of a Vector**



$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t}$$

- Δs is the scalar displacement along the path ($A \rightarrow A'$)
- **Magnitude** and **direction** of $\mathbf{r}$ are known at time t
- $\Delta \mathbf{r}$ is the vector (*not scalar*) change of position at $t + \Delta t$
- $\mathbf{v}$ has direction of $\Delta \mathbf{r}$ (*tangent*) and magnitude $|\Delta \mathbf{r} / \Delta t|$

Outline for Today

- Question of the day
- Where are we in the course?
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- Inertial versus moving coordinate systems
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Where are we in the course?

Concept: What is dynamics?

Chapters 1, 2, 6

Chapters 3, 5, 7, 8

Relationship among ***position***, ***velocity***, and ***acceleration***

Relationship among ***forces*** and ***acceleration***

Where are we in the course?

Calculation: How do we use dynamics?

Newton's 2nd Law

Force. A push or pull exerted on a body, characterized by:

- magnitude
- direction
- point of application

$$F = m\,a$$

Mass. Measure of the resistance of a body to linear acceleration.

Acceleration. Velocity rate of change with respect to time

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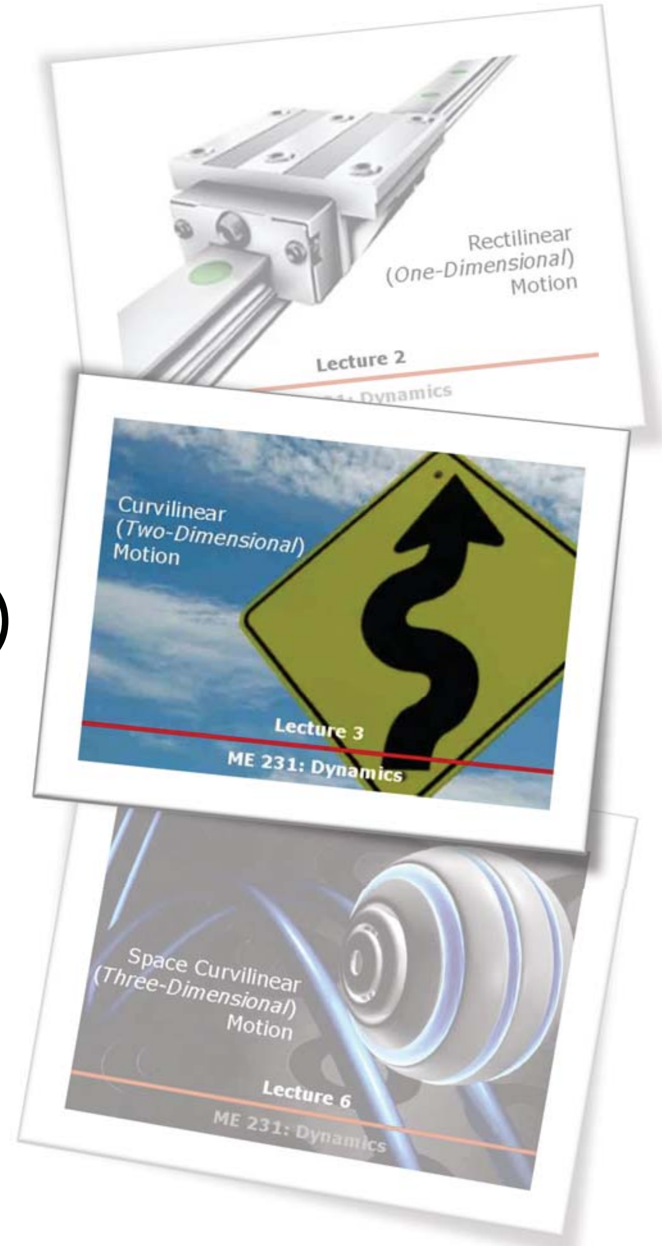
Categories of Motion

Geometry of a problem identifies the category.

- Lecture 2: rectilinear (1D)
- Lecture 3: curvilinear (2D)
- Lecture 6: space (3D)

Many of our motion problems involve curvilinear (2D), or plane motion.



Curvilinear (2D) Time Derivative: Exercise

A particle moving in two-dimensions has a position vector ($\mathbf{r}$) as a function of time (t) with coordinates given by

$$x(t) = t^2 - 4t + 20 \quad , \quad y(t) = 3\sin(2t)$$

where $\mathbf{r}$ is measured in inches and t is in seconds.

Determine the ***velocity*** ($\mathbf{v}$) and the ***acceleration*** ($\mathbf{a}$).

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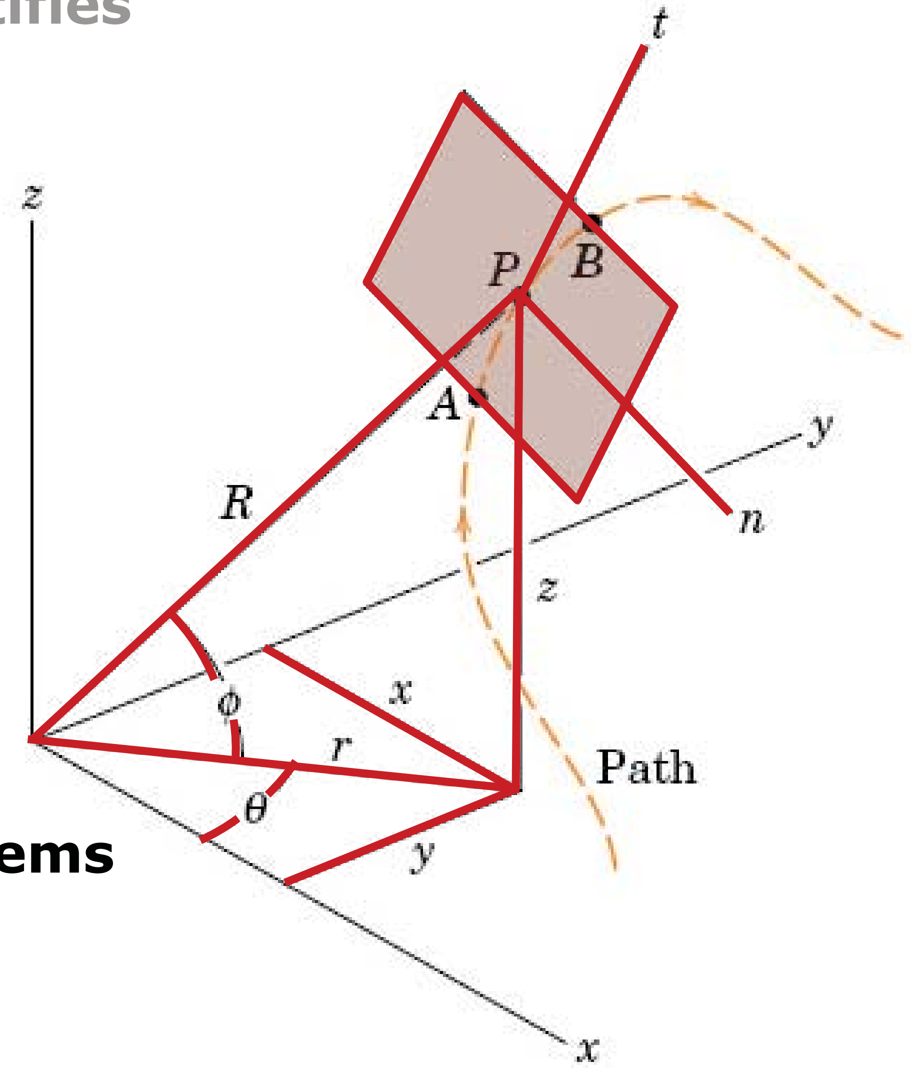
Choice of Coordinate Systems

Motion of a problem identifies the coordinate system.

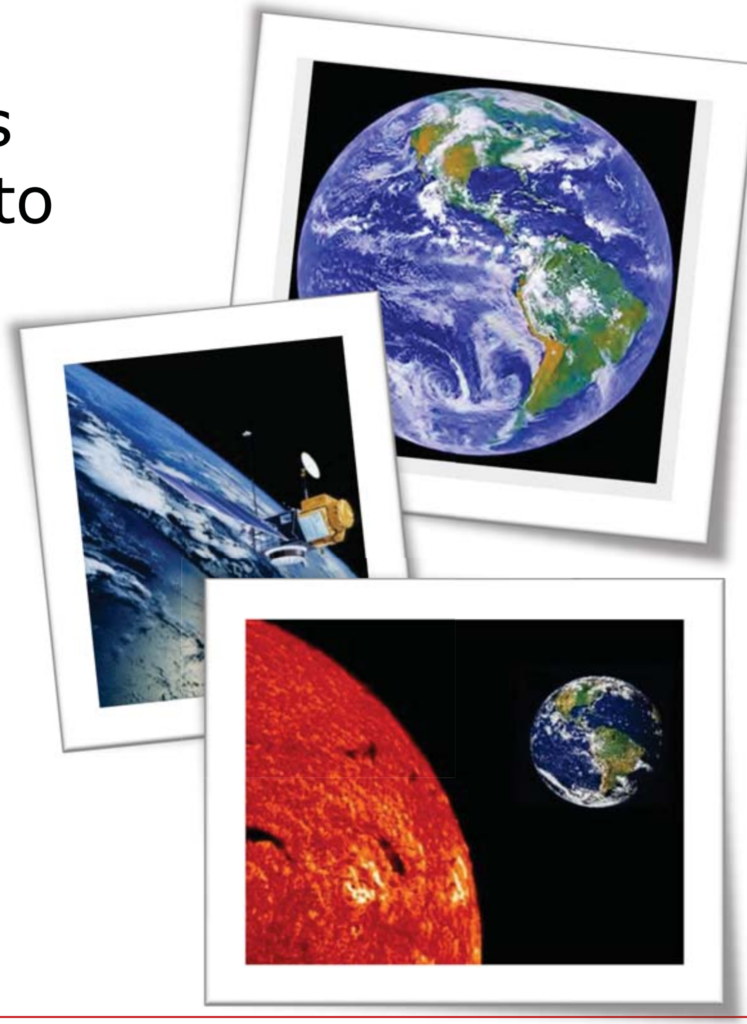
- Rectangular (x, y, z)
- Polar (r, θ, z)
- Spherical (R, θ, ϕ)
- Normal and Tangential (n, t)



Many of our motion problems involve 2D rectangular coordinates (x, y).

Inertial Versus Moving Coordinate Systems

Moving coordinate systems are measured with respect to an ***inertial*** coordinate system whose ***motion is negligible***.

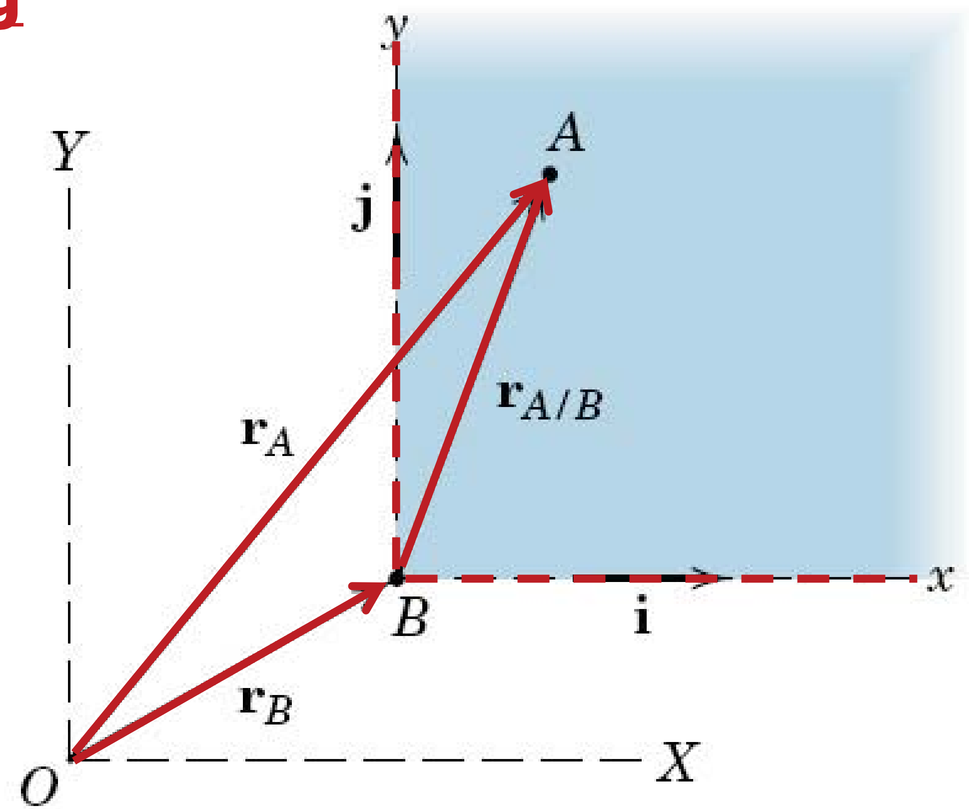


Inertial Versus Moving Coordinate Systems

$$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$$

$$\mathbf{v}_A = \dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \dot{\mathbf{r}}_{A/B}$$

$$\mathbf{a}_A = \dot{\mathbf{v}}_A = \ddot{\mathbf{r}}_A = \ddot{\mathbf{r}}_B + \ddot{\mathbf{r}}_{A/B}$$



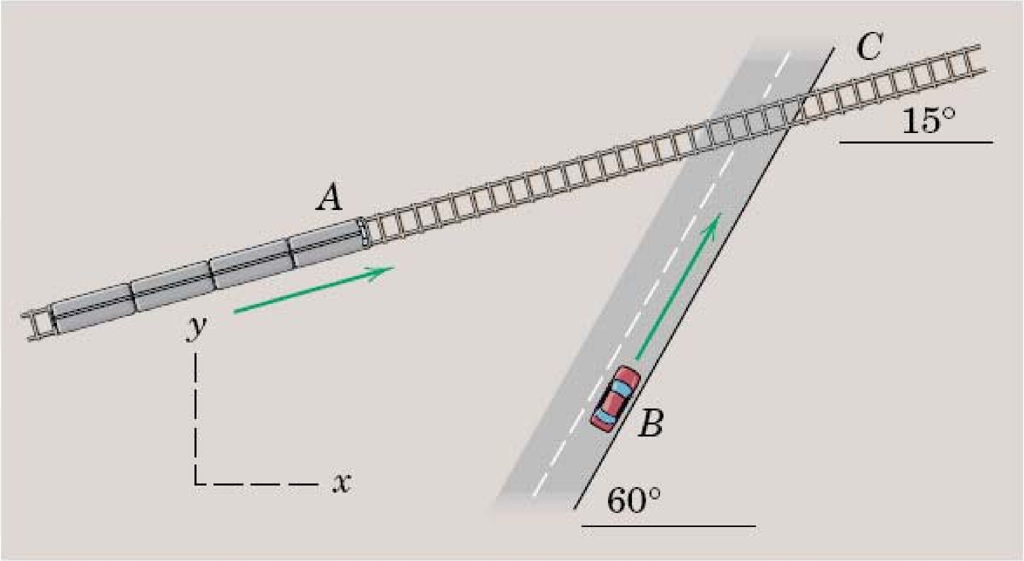
- Absolute position of ***B*** is defined in an inertial coordinate system ***X***-***Y***
- Attach a set of translating (*non-rotating*) axes ***x***-***y*** to particle ***B*** and define the position of ***A***
- Define position of "***A relative to B***" ("***A/B***") in ***x***-***y***

Vector Representation: Exercise

$$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$$

$$\mathbf{v}_A = \dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \dot{\mathbf{r}}_{A/B}$$

$$\mathbf{a}_A = \dot{\mathbf{v}}_A = \ddot{\mathbf{r}}_A = \ddot{\mathbf{r}}_B + \ddot{\mathbf{r}}_{A/B}$$



Train A travels with constant ***speed*** v_A = 120 km/h. Anticipating the need to stop, car B decreases its ***speed*** of 90 km/h at the rate of 3 m/s^2.

Determine the ***velocity*** and ***acceleration*** of the train relative to the car.

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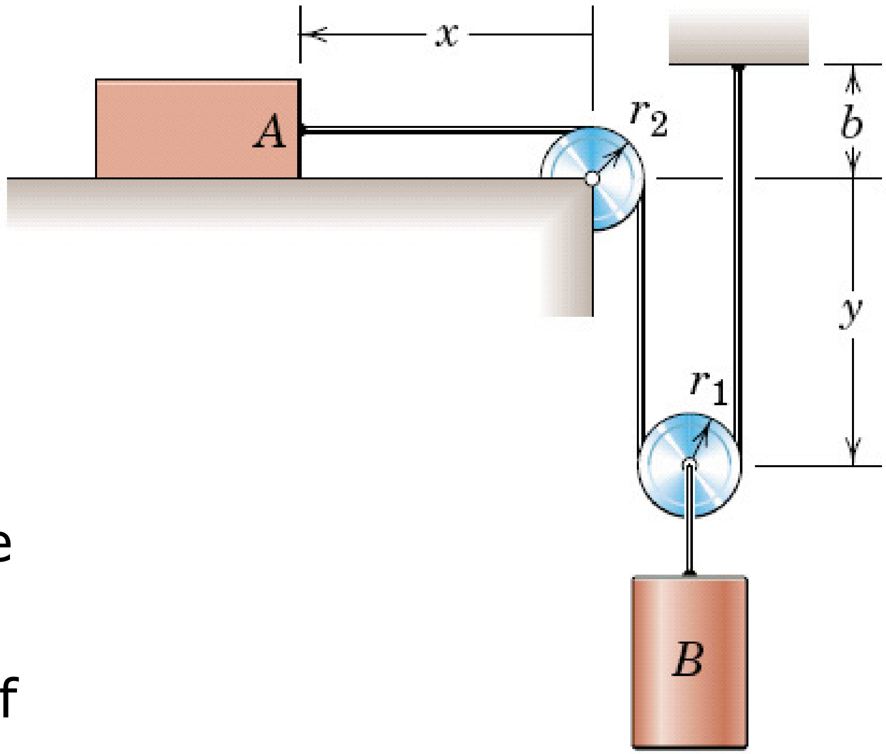
Degrees of Freedom



- Simple system of ***two interconnected particles***
- With L, r_2, r_1, and b are constant
- Horizontal motion (x) of A is twice the vertical motion (y) of B
- Only ***one variable*** (x or y) is needed to specify the ***positions of all parts*** of the system

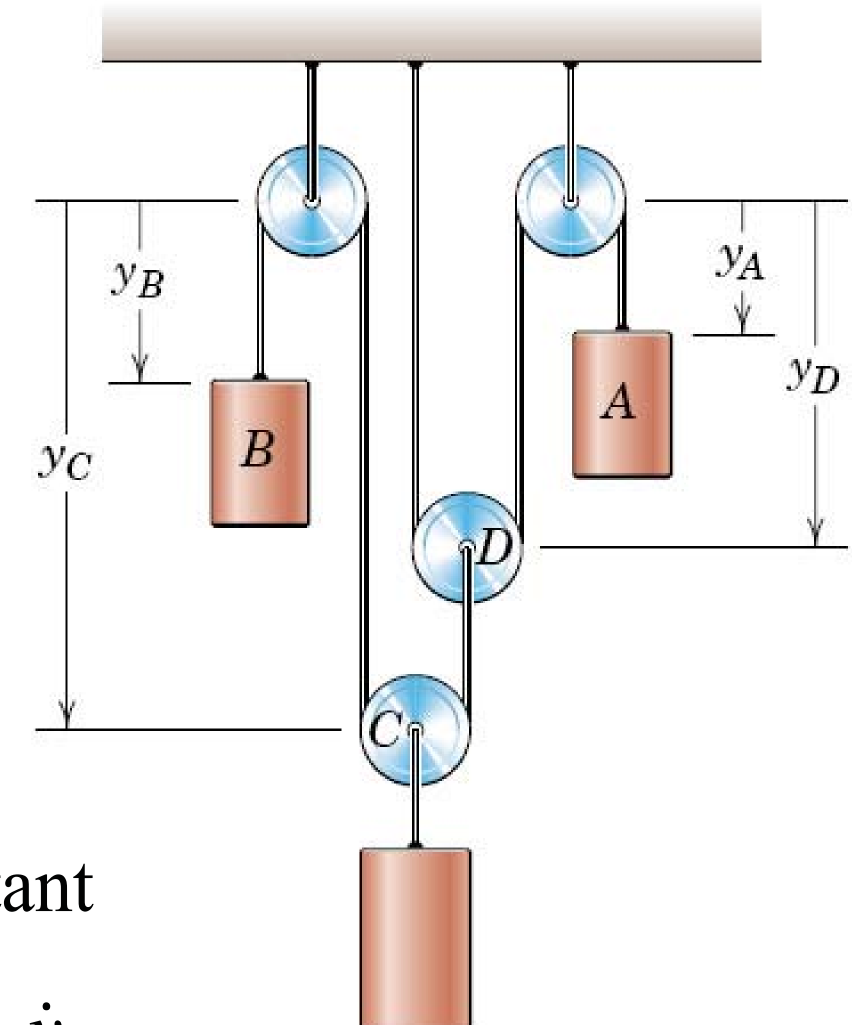
Constraint Equations

$$L = x + \frac{\pi}{2} r_2 + 2y + \pi r_1 + b$$

$$0 = \dot{x} + 2\dot{y} \qquad 0 = v_A + 2v_B$$

$$0 = \ddot{x} + 2\ddot{y} \qquad 0 = a_A + 2a_B$$

Degrees of Freedom



Position of lower cylinder depends on ***two variables*** (y_A and y_B)

Constraint Equations

$$L_A = y_A + 2y_D + \text{constant}$$

$$L_B = y_B + y_C + \left(y_C - y_D\right) + \text{constant}$$

$$0 = \dot{y}_A + 2\dot{y}_D \qquad 0 = \dot{y}_B + 2\dot{y}_C - \dot{y}_D$$

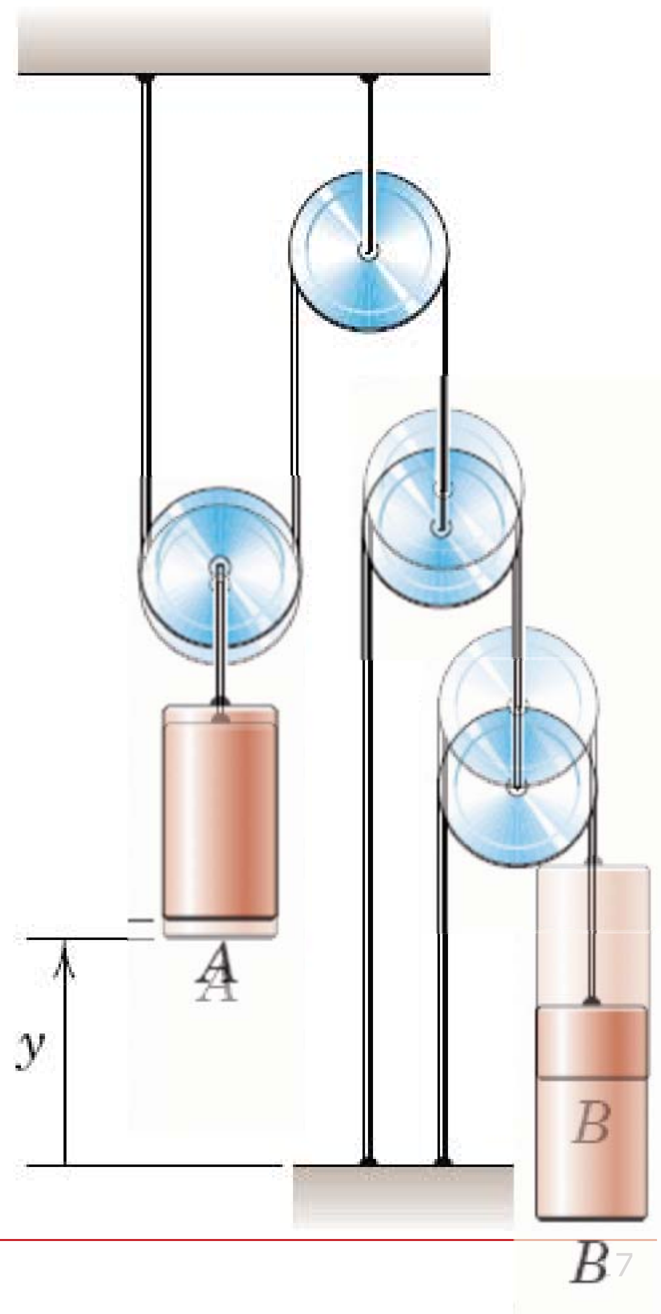
$$0 = \ddot{y}_A + 2\ddot{y}_D \qquad 0 = \ddot{y}_B + 2\ddot{y}_C - \ddot{y}_D$$

$$0 = \dot{y}_A + 2\dot{y}_B + 4\dot{y}_C$$

$$0 = \ddot{y}_A + 2\ddot{y}_B + 4\ddot{y}_C$$

Degrees of Freedom: Exercise

How many ***degrees of freedom*** are necessary to specify the position of all parts of the system of ***interconnected particles***?



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Velocity and Acceleration

Lecture	Velocity	Acceleration
2. Rectilinear	$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$	$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$
3. Curvilinear	$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\,\mathbf{i} + \dot{y}\,\mathbf{j}$	$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \ddot{x}\,\mathbf{i} + \ddot{y}\,\mathbf{j}$
4. Normal & Tangential	$\mathbf{v} = v\,\mathbf{e_t}$	$\mathbf{a} = \frac{v^2}{\rho}\mathbf{e_n} + \dot{v}\,\mathbf{e_t}$
5. Polar Coordinates	$\mathbf{v} = \dot{r}\,\mathbf{e_r} + r\dot{\theta}\,\mathbf{e}_\theta$	$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e_r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta$
7. Relative Motion	$\mathbf{v}_A = \dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \dot{\mathbf{r}}_{A/B}$	$\mathbf{a}_A = \dot{\mathbf{v}}_A = \ddot{\mathbf{r}}_A = \ddot{\mathbf{r}}_B + \ddot{\mathbf{r}}_{A/B}$

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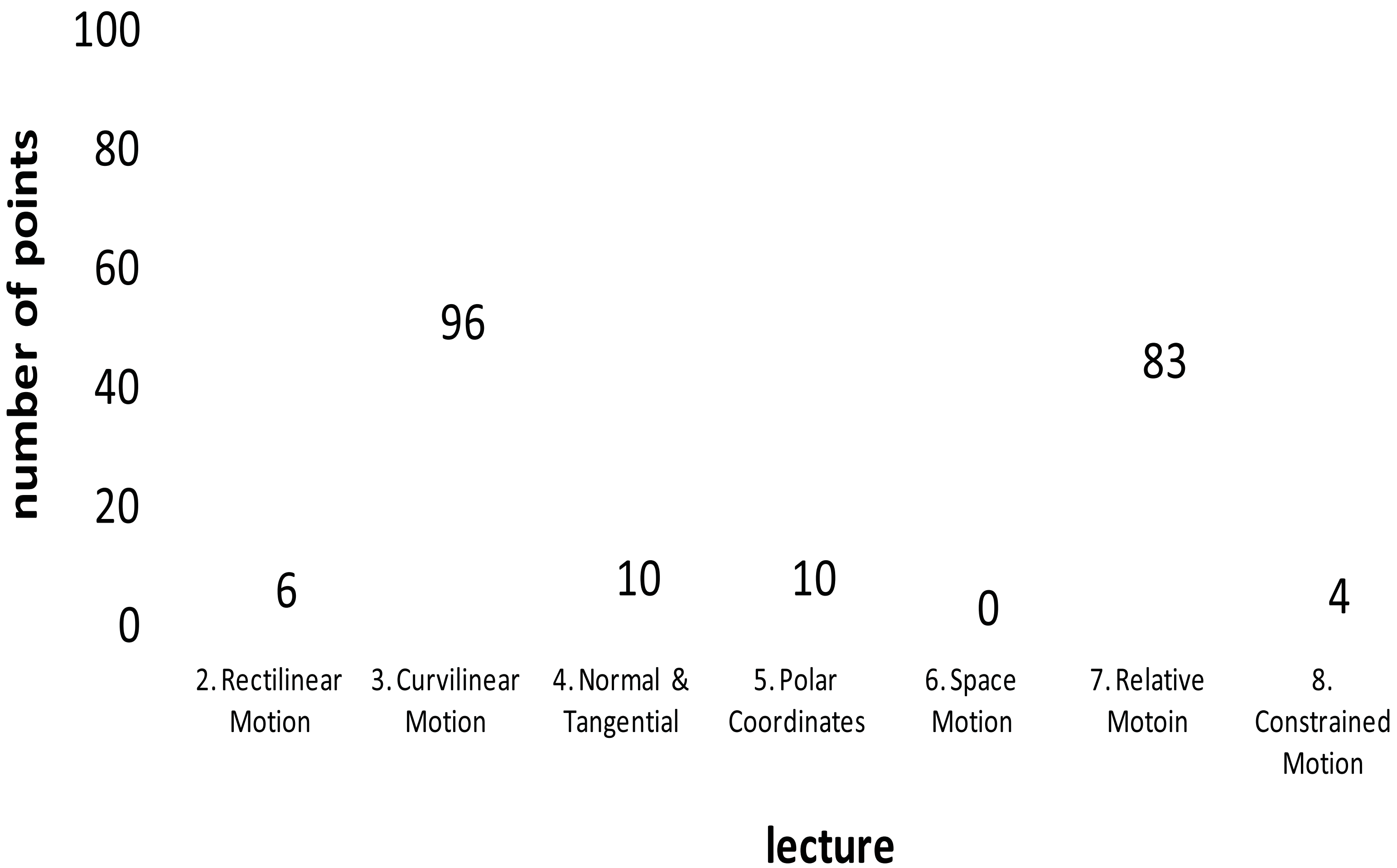
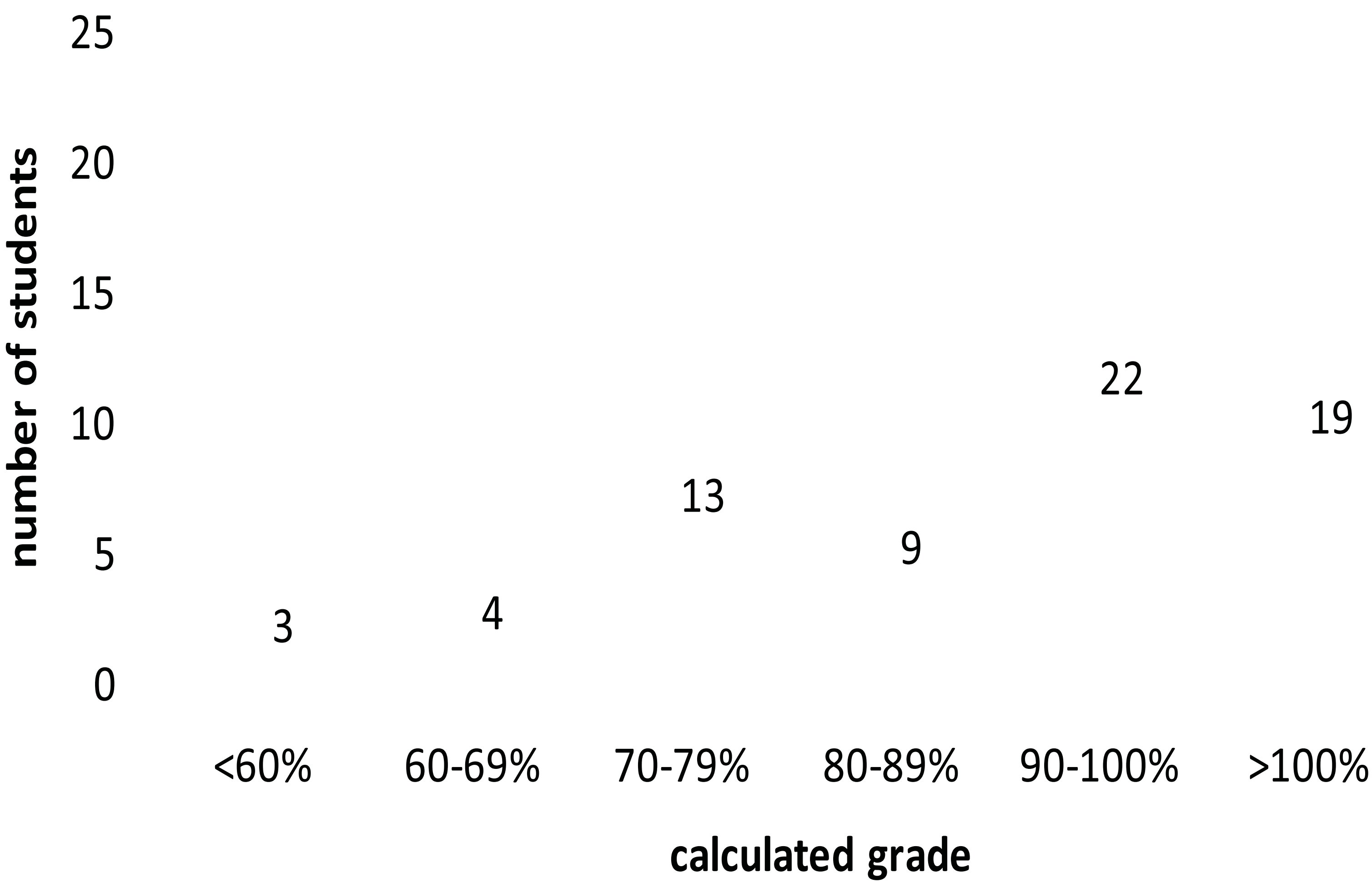
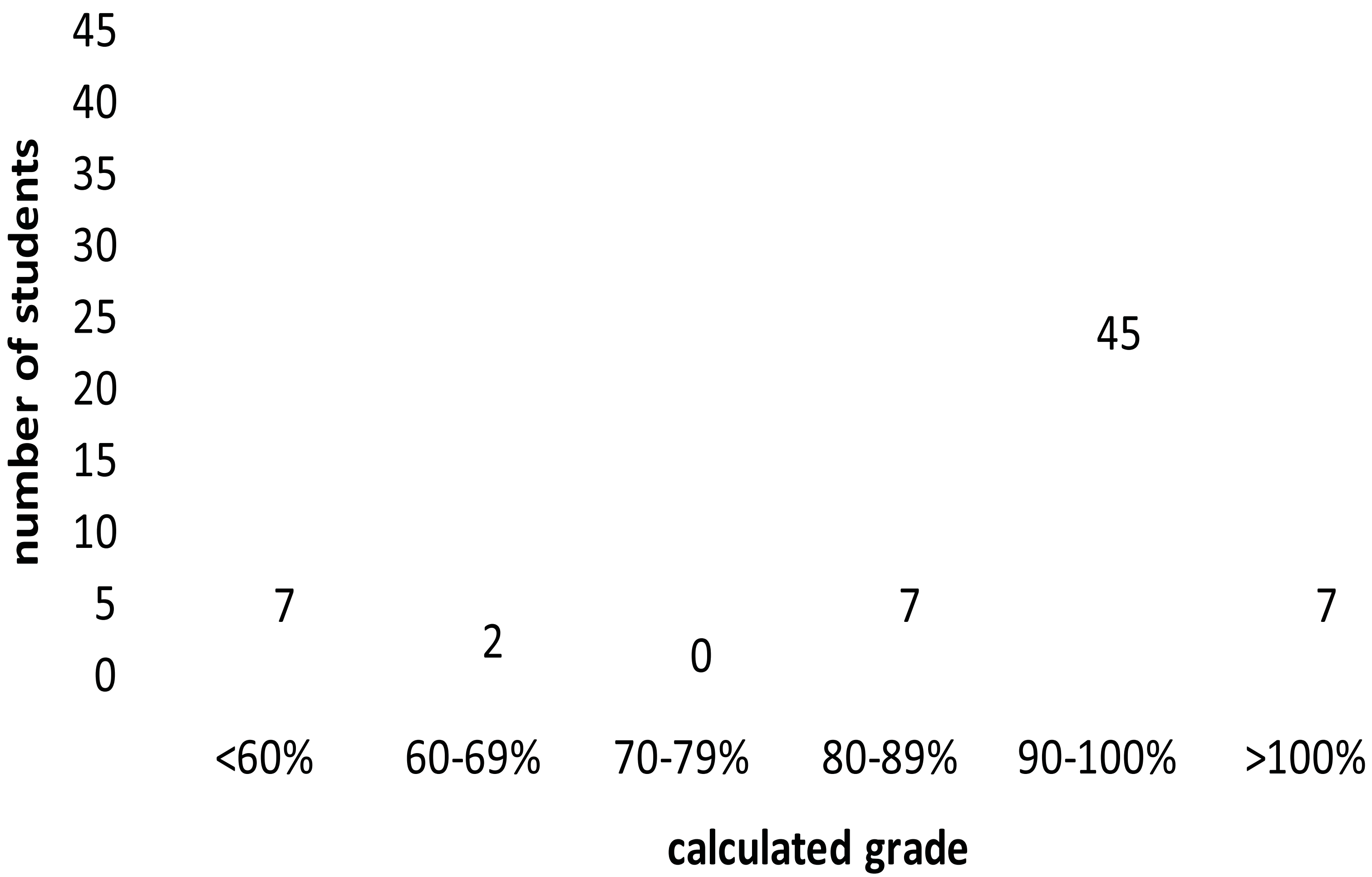
Exam 1 Breakdown (kinematics of particles)



"Final" Course Grades (thru HW #5 LAST YEAR)



“Final” Course Grades (thru HW #5 THIS YEAR)



For Next Time…

- Complete Homework #6 due on Tuesday (10/2)
- Review Chapter 6