Kinematics of Rigid Bodies (Ch. 6) Review

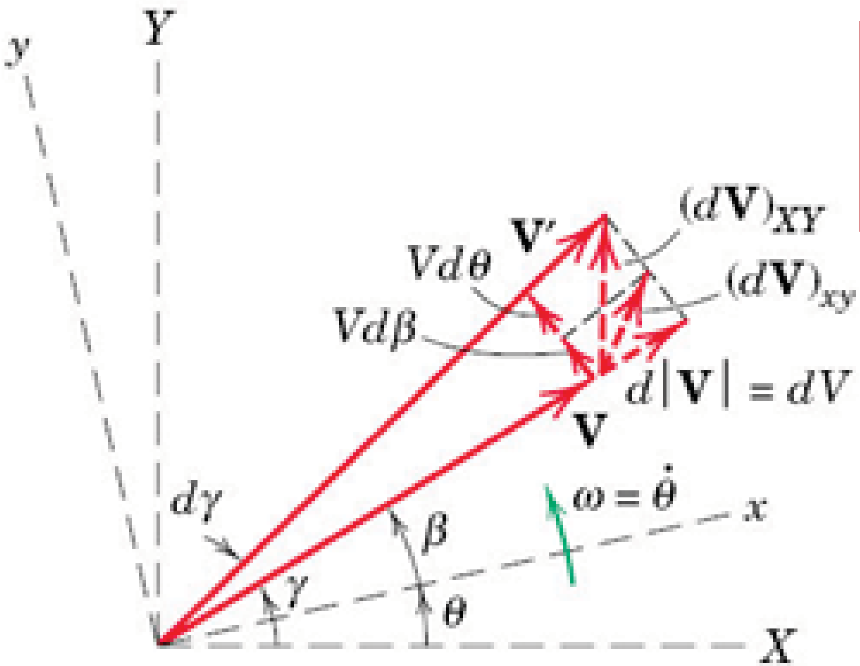
Lecture 17

ME 231: Dynamics

Question of the Day

What is the most important concept in Chapter 6? **Transformation of a Time Derivative**

$$\left(\frac{d\mathbf{V}}{dt}\right)_{XY} = \left(\dot{V}_x\mathbf{i} + \dot{V}_y\mathbf{j}\right) + \left(V_x\dot{\mathbf{i}} + V_y\dot{\mathbf{j}}\right)$$



$$\boxed{\left(\frac{d\mathbf{V}}{dt}\right)_{XY} = \left(\frac{d\mathbf{V}}{dt}\right)_{xy} + \boldsymbol{\omega}\times\mathbf{V}}$$

$$\dot{\mathbf{i}} = \boldsymbol{\omega}\times\mathbf{i} \qquad \dot{\mathbf{j}} = \boldsymbol{\omega}\times\mathbf{j}$$

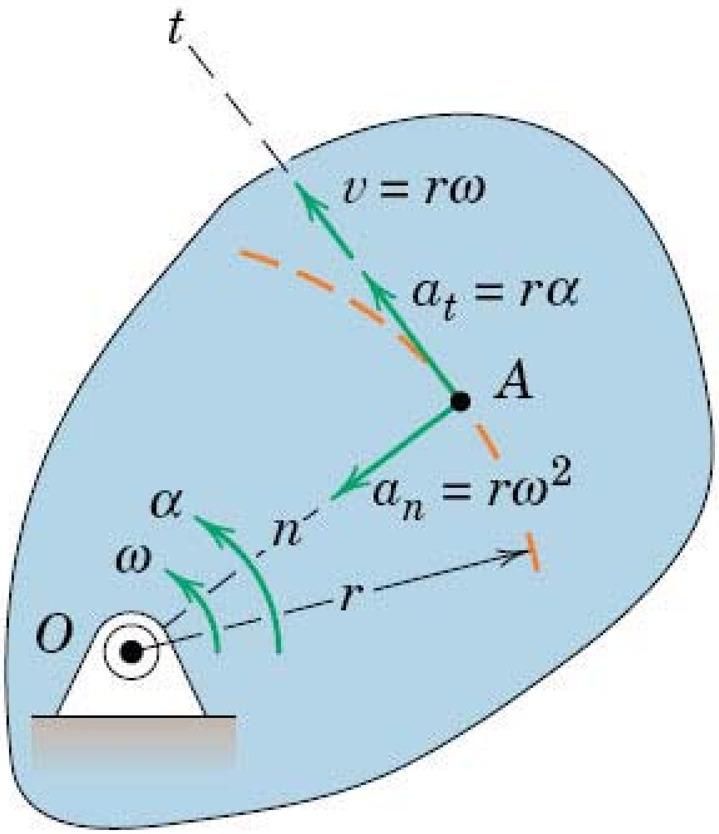
Outline for Today

- Question of the day
- Absolute-motion analysis
- Relative-motion analysis
- Locating the instantaneous center
- Rotating coordinate systems
- Velocity and acceleration
- Exam 1 breakdown (kinematics of rigid bodies)

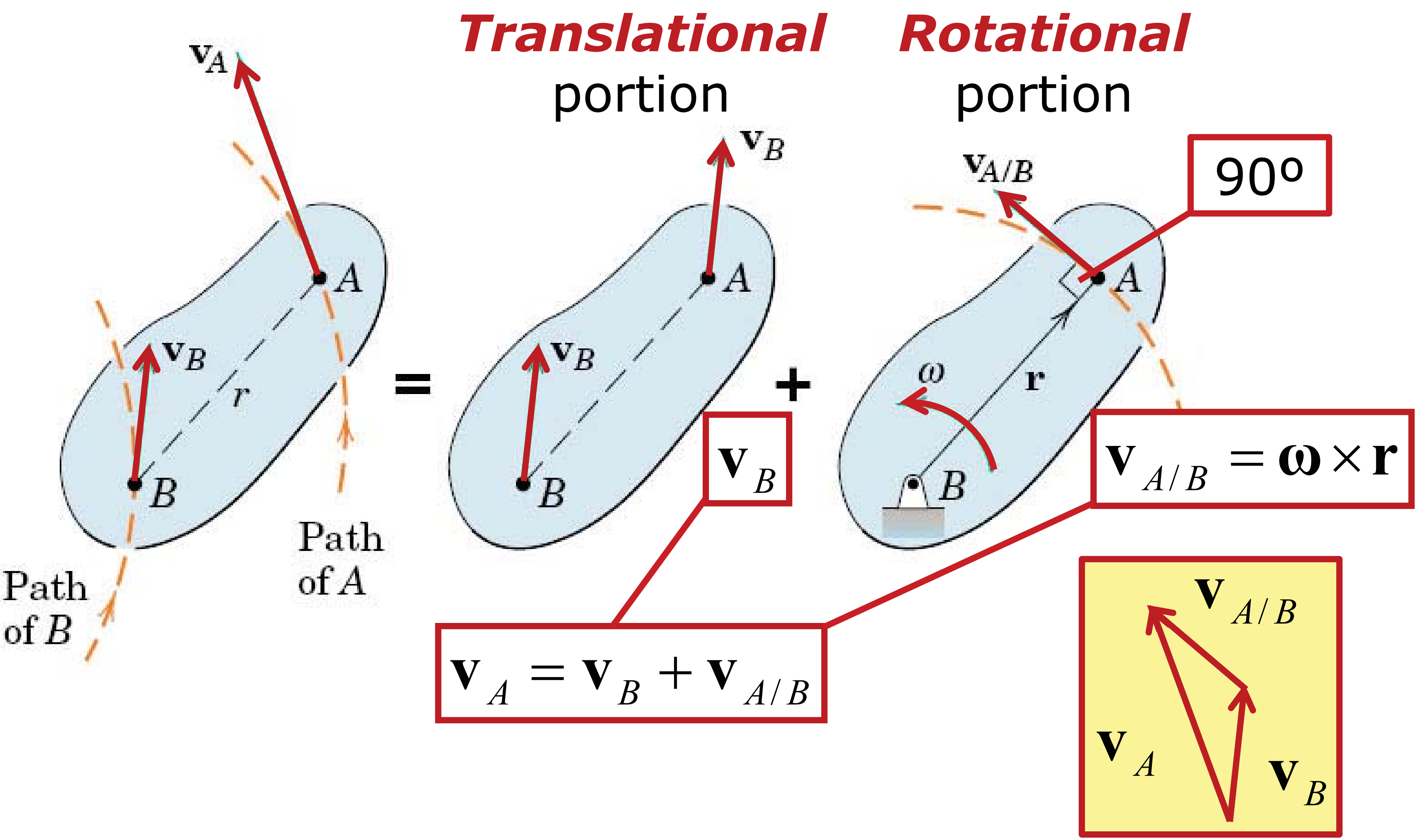
Absolute-Motion Analysis

- The method relates the ***position*** of a ***point***, A, on a rigid body to the ***angular position***, θ, of a ***line*** contained in the body

- The ***velocity*** and ***acceleration*** of ***point*** A are obtained in terms of the ***angular velocity***, ω, and ***angular acceleration***, α, of the rigid ***body***



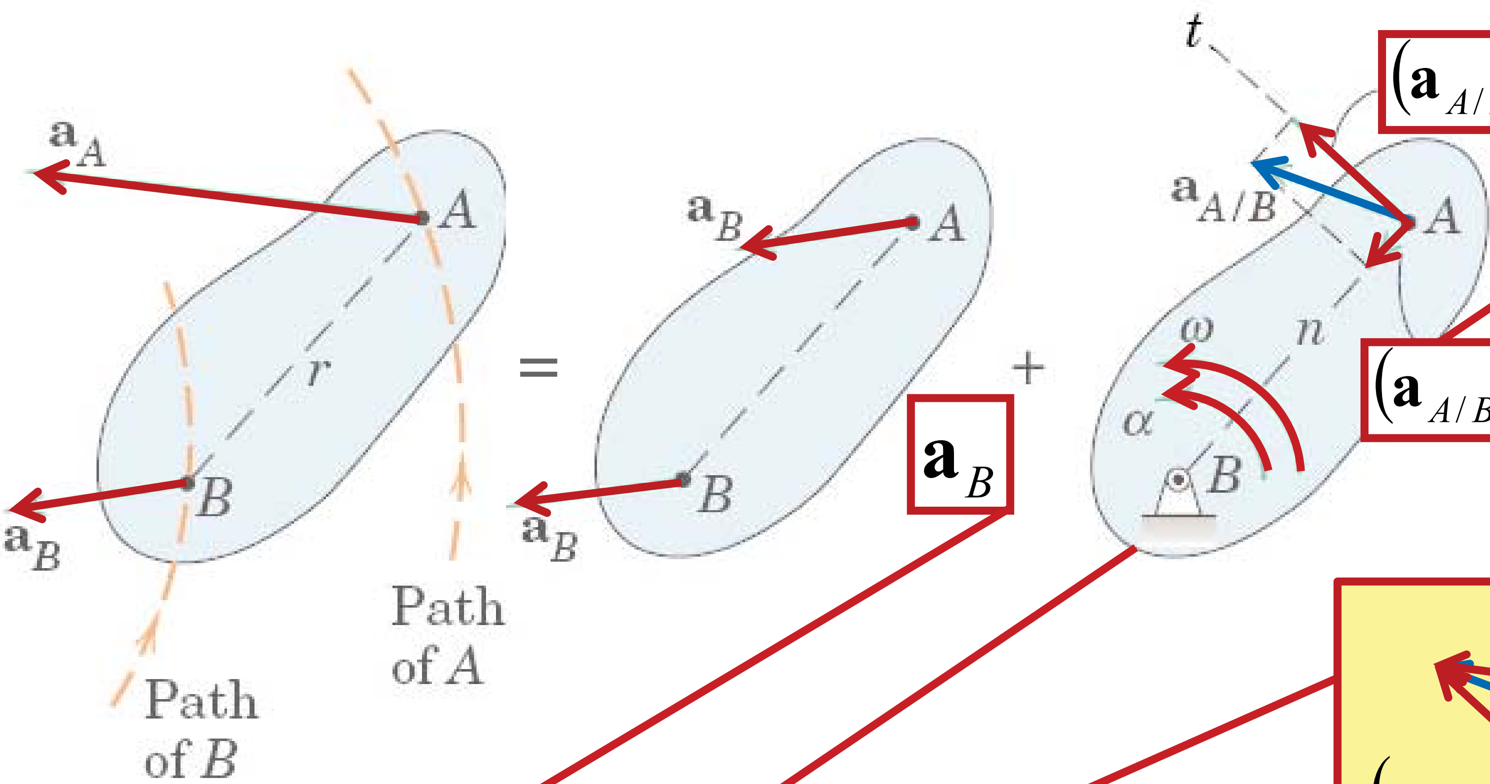
Relative-Motion Analysis: Velocity



Relative-Motion Analysis: Acceleration

Translational portion

Rotational portion

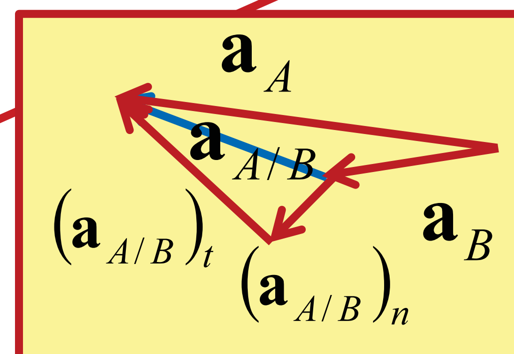


$$(\mathbf{a}_{A/B})_t = \boldsymbol{\alpha} \times \mathbf{r}$$

$$(\mathbf{a}_{A/B})_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

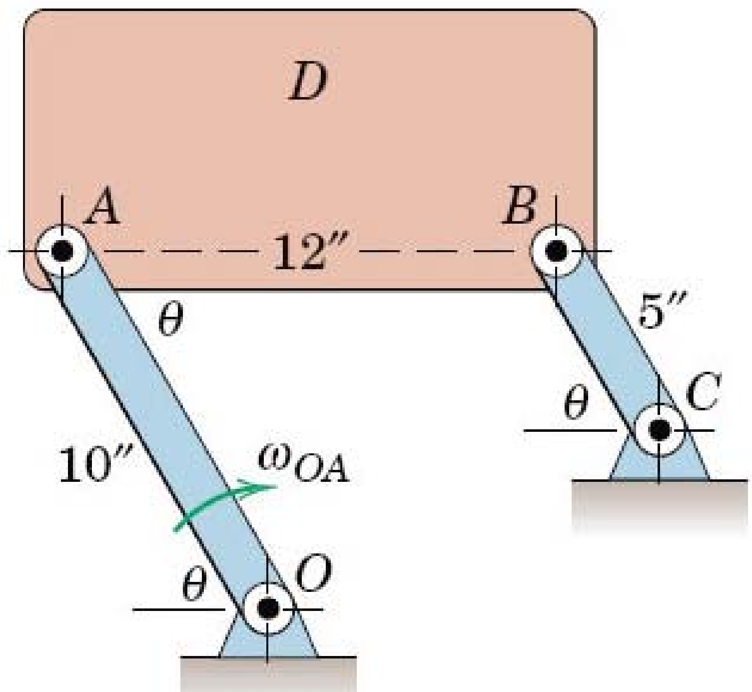
$$\mathbf{a}_B$$

$$\mathbf{a}_A = \mathbf{a}_B + (\mathbf{a}_{A/B})_t + (\mathbf{a}_{A/B})_n$$



Solution of Relative-Acceleration Eq.: Exercise

Calculate the ***angular acceleration*** of the plate, where AO has a constant ***angular velocity*** $\omega_{OA} = 4$ rad/s and $\theta = 60^{o}$ for both links.



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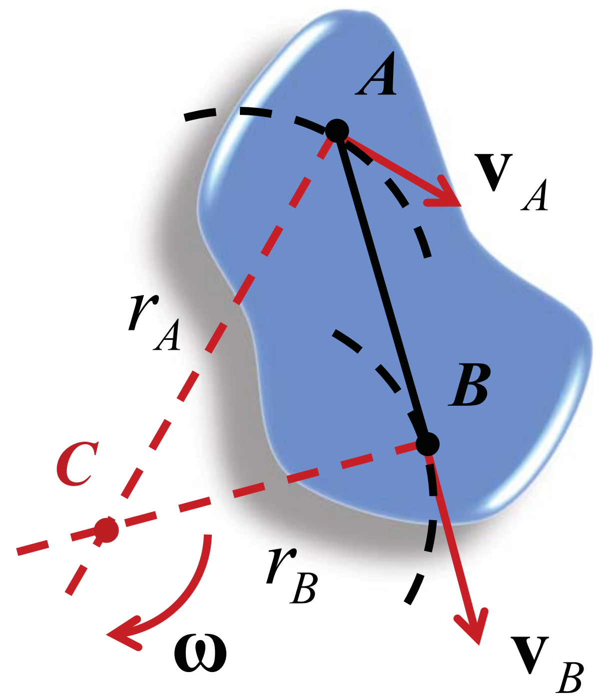
Locating the Instantaneous Center: Case #1

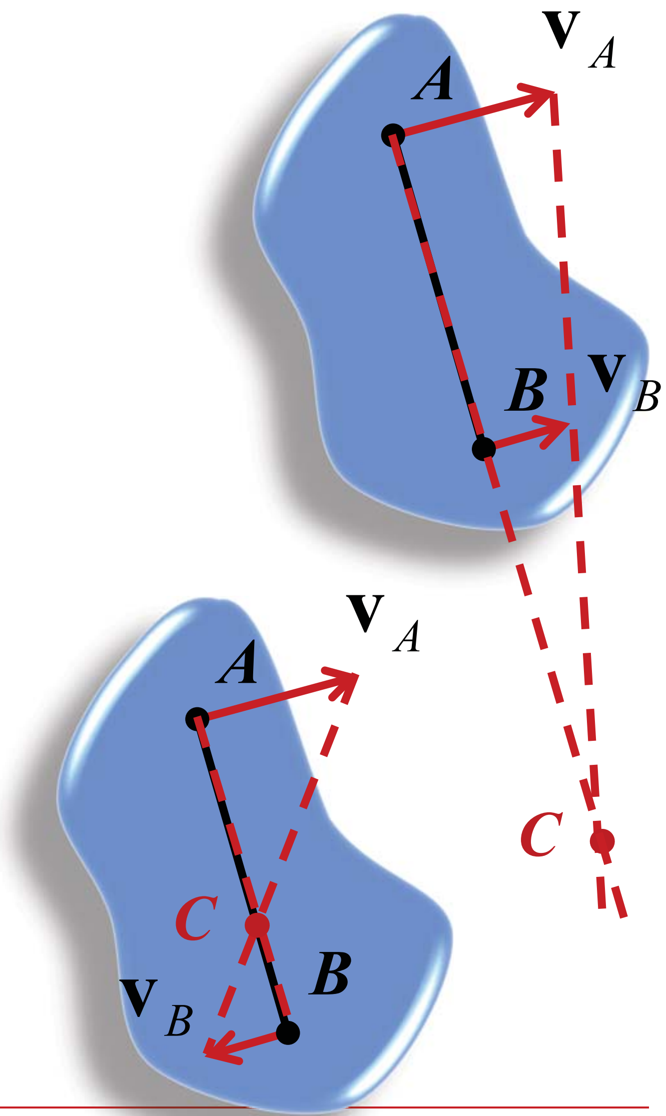
- Directions of absolute ***velocities*** for A and B are known (*and not parallel*)
- ***Point*** A has circular motion about some point on the ***line perpendicular*** to ***velocity*** $\mathbf{v}_A$
- ***Point*** B has a ***similar*** motion
- ***Point*** C is the ***instantaneous center*** of zero velocity (*may lie on or off the body*)



$$\omega = \frac{v_A}{r_A} = \frac{v_B}{r_B}$$

Locating the Instantaneous Center: Case #2

- Directions of absolute ***velocities*** for A and B are known AND ***parallel***
- The ***line*** joining the points is ***perpendicular*** to ***velocity*** $\mathbf{v}_A$ and $\mathbf{v}_B$
- ***Instantaneous center*** found by ***direct proportions***



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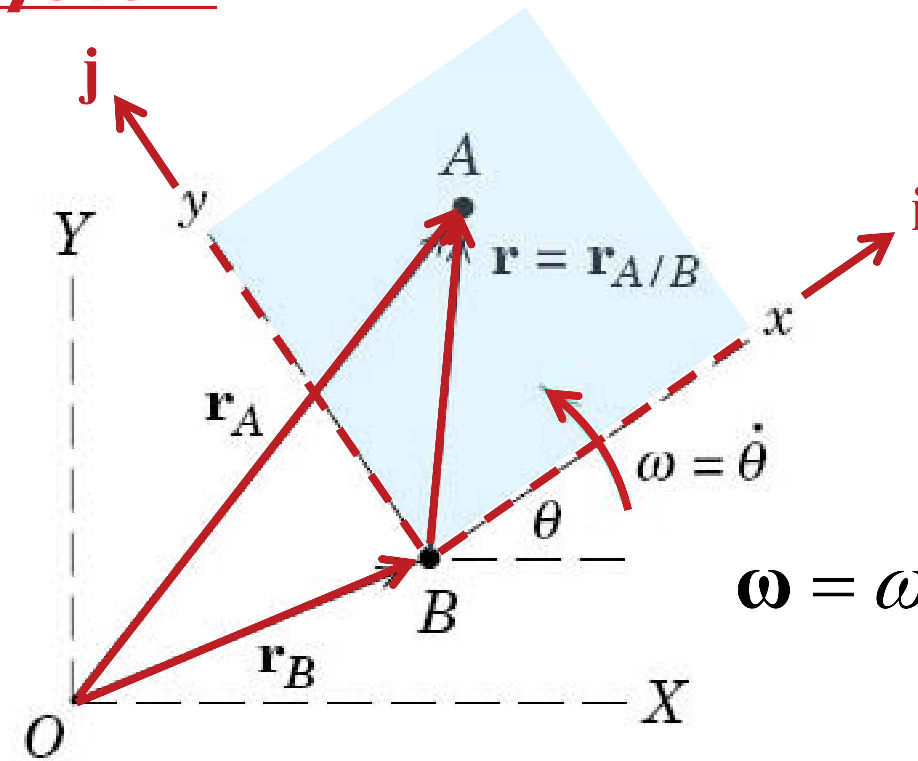
Rotating Coordinate System

$$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$$

$$\boxed{\mathbf{r}_A = \mathbf{r}_B + \left(x\mathbf{i} + y\mathbf{j}\right)}$$

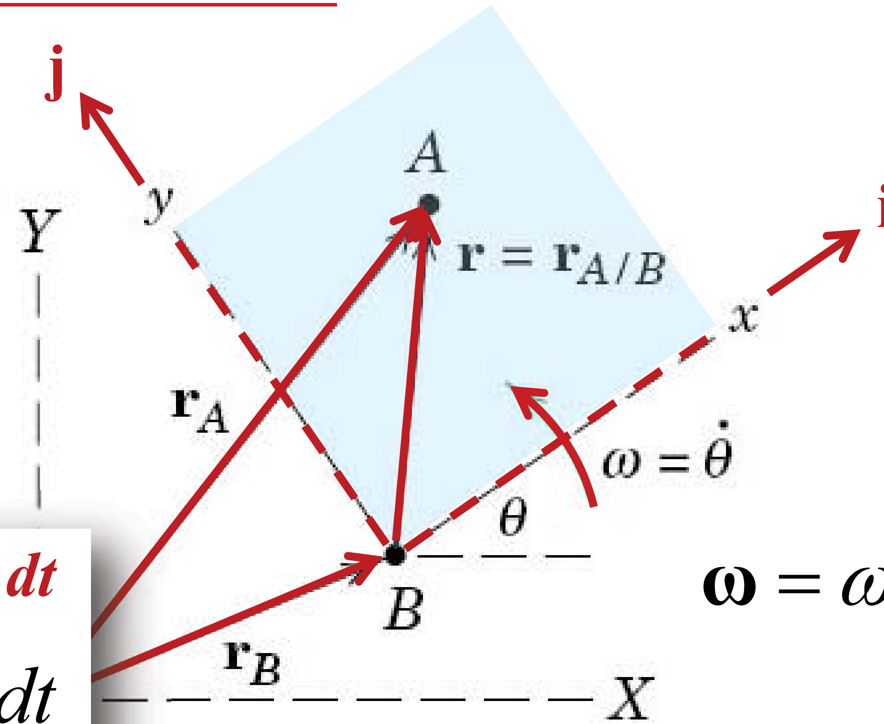


$$\boldsymbol{\omega} = \omega\mathbf{k}$$

- Absolute position of ***B*** is defined in an inertial coordinate system ***X***-***Y***
- Moving reference frame ***x***-***y*** has its origin at ***B*** and rotates with angular velocity ***ω***
- Define "***A relative to B***" using unit vectors in ***x***-***y***

Time Derivatives of Unit Vectors

$$\mathbf{r}_A = \mathbf{r}_B + \left(x\mathbf{i} + y\mathbf{j}\right)$$

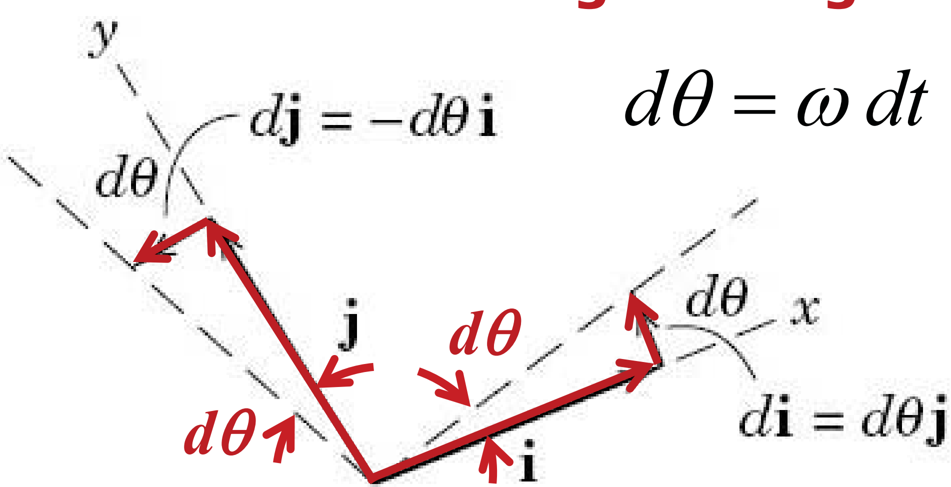


$$\boldsymbol{\omega} = \omega\mathbf{k}$$

$$\dot{\mathbf{i}} = \omega\mathbf{j} \qquad \dot{\mathbf{j}} = -\omega\mathbf{i}$$

Infinitesimal change during *dt*

$$d\theta = \omega\, dt$$



Relative Velocity

$$\mathbf{r}_A = \mathbf{r}_B + (x\mathbf{i} + y\mathbf{j})$$

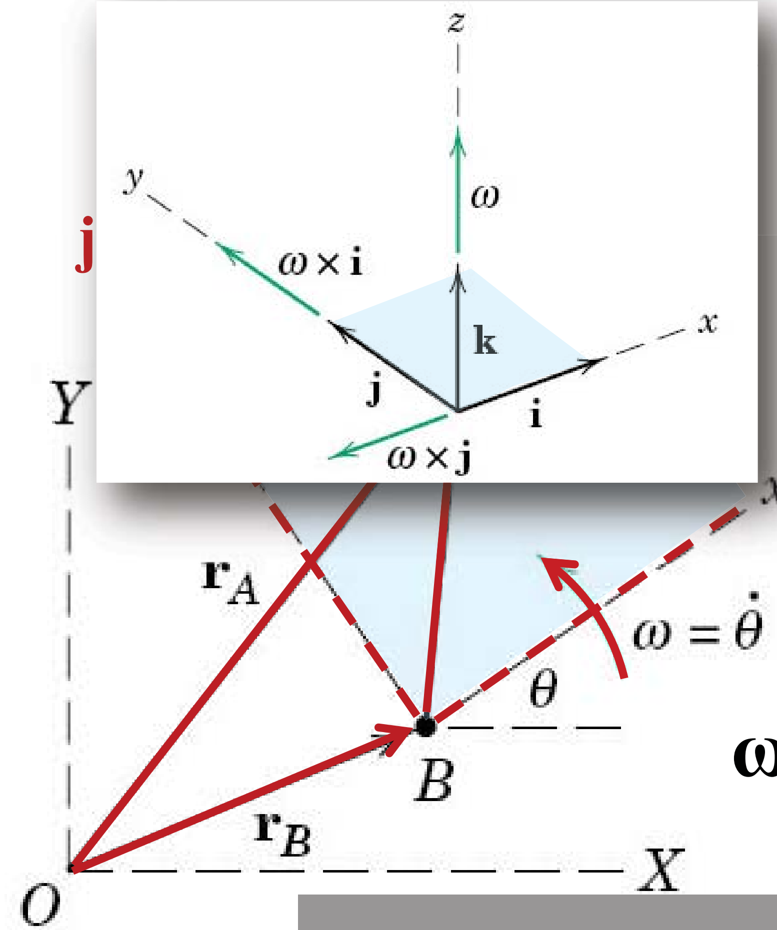
$$\boxed{\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}}$$

$$\dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \frac{d}{dt}(x\mathbf{i} + y\mathbf{j})$$

$$\dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + (x\dot{\mathbf{i}} + y\dot{\mathbf{j}}) + (\dot{x}\mathbf{i} + \dot{y}\mathbf{j})$$

$$\dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + (\boldsymbol{\omega} \times x\mathbf{i} + \boldsymbol{\omega} \times y\mathbf{j}) + (\dot{x}\mathbf{i} + \dot{y}\mathbf{j})$$

$$\dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + (\boldsymbol{\omega} \times (x\mathbf{i} + y\mathbf{j})) + (\dot{x}\mathbf{i} + \dot{y}\mathbf{j})$$

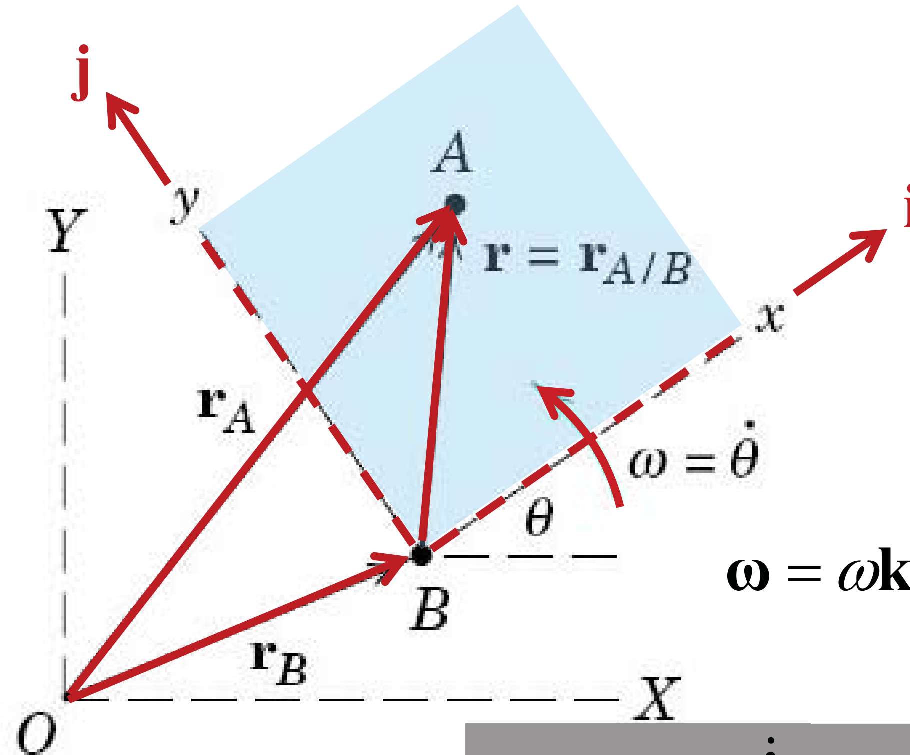


$$\boldsymbol{\omega} = \omega\mathbf{k}$$

$$\dot{\mathbf{i}} = \omega\mathbf{j} \qquad \dot{\mathbf{j}} = -\omega\mathbf{i}$$

$$\dot{\mathbf{i}} = \boldsymbol{\omega} \times \mathbf{i} \qquad \dot{\mathbf{j}} = \boldsymbol{\omega} \times \mathbf{j}$$

Relative Acceleration



$$\mathbf{r}_A = \mathbf{r}_B + (x\mathbf{i} + y\mathbf{j})$$

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}\times\mathbf{r} + \mathbf{v}_{rel}$$

$$\boldsymbol{\omega} = \omega\mathbf{k}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}}\times\mathbf{r} + \boldsymbol{\omega}\times\dot{\mathbf{r}} + \dot{\mathbf{v}}_{rel}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}}\times\mathbf{r} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r} + \mathbf{v}_{rel}) + \dot{\mathbf{v}}_{rel}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}}\times\mathbf{r} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) + \boldsymbol{\omega}\times\mathbf{v}_{rel} + \dot{\mathbf{v}}_{rel}$$

$$\boxed{\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}}\times\mathbf{r} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) + 2\boldsymbol{\omega}\times\mathbf{v}_{rel} + \mathbf{a}_{rel}}$$

$$\dot{\mathbf{i}} = \boldsymbol{\omega}\times\mathbf{i}$$

$$\dot{\mathbf{j}} = \boldsymbol{\omega}\times\mathbf{j}$$

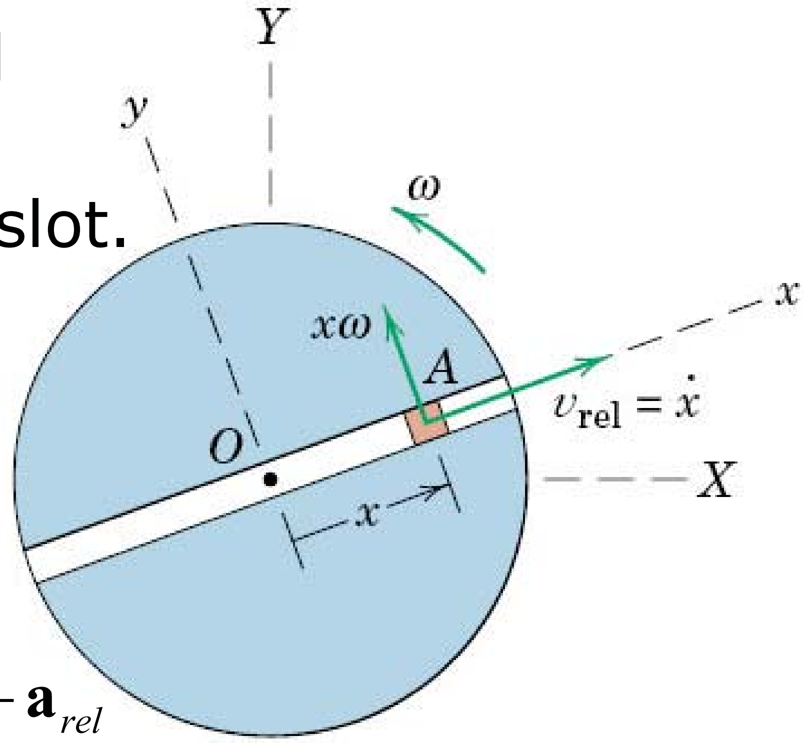
$$\dot{\mathbf{r}} = \boldsymbol{\omega}\times\mathbf{r} + \mathbf{v}_{rel}$$

$$\dot{\mathbf{v}}_{rel} = \boldsymbol{\omega}\times\mathbf{v}_{rel} + \mathbf{a}_{rel}$$

Relative Acceleration: Exercise

A ***disk*** with the radial slot is ***rotating*** about O with constant $\omega = 4$ rad/s. The ***slider*** A is positioned at $x = 0.25$ m and moves with constant speed $v_{\text{rel}} = 0.5$ m/s relative to the slot.

Determine the ***absolute acceleration*** of A for this position.



$$\mathbf{a}_A = \mathbf{a}_O + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$

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Velocity and Acceleration

Lecture	Velocity	Acceleration
9. Rotation	$\omega = \dot{\theta}$ $v = r\omega$ $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$	$\alpha = \dot{\omega} = \ddot{\theta}$ $a_t = r\alpha \qquad a_n = r\omega^2$ $\mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r} \qquad \mathbf{a}_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$
10. Absolute Motion	$\omega = \dot{\theta}$ $v = r\omega$ $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$	$\alpha = \dot{\omega} = \ddot{\theta}$ $a_t = r\alpha \qquad a_n = r\omega^2$ $\mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r} \qquad \mathbf{a}_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$
11. Relative Velocity	$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$ $\mathbf{v}_{A/B} = \boldsymbol{\omega} \times \mathbf{r}$	
13. Relative Acceleration		$\mathbf{a}_A = \mathbf{a}_B + (\mathbf{a}_{A/B})_t + (\mathbf{a}_{A/B})_n$ $(\mathbf{a}_{A/B})_t = \boldsymbol{\alpha} \times \mathbf{r} \quad (\mathbf{a}_{A/B})_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$
14. Rotating Axes	$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}$	$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ $+ 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$

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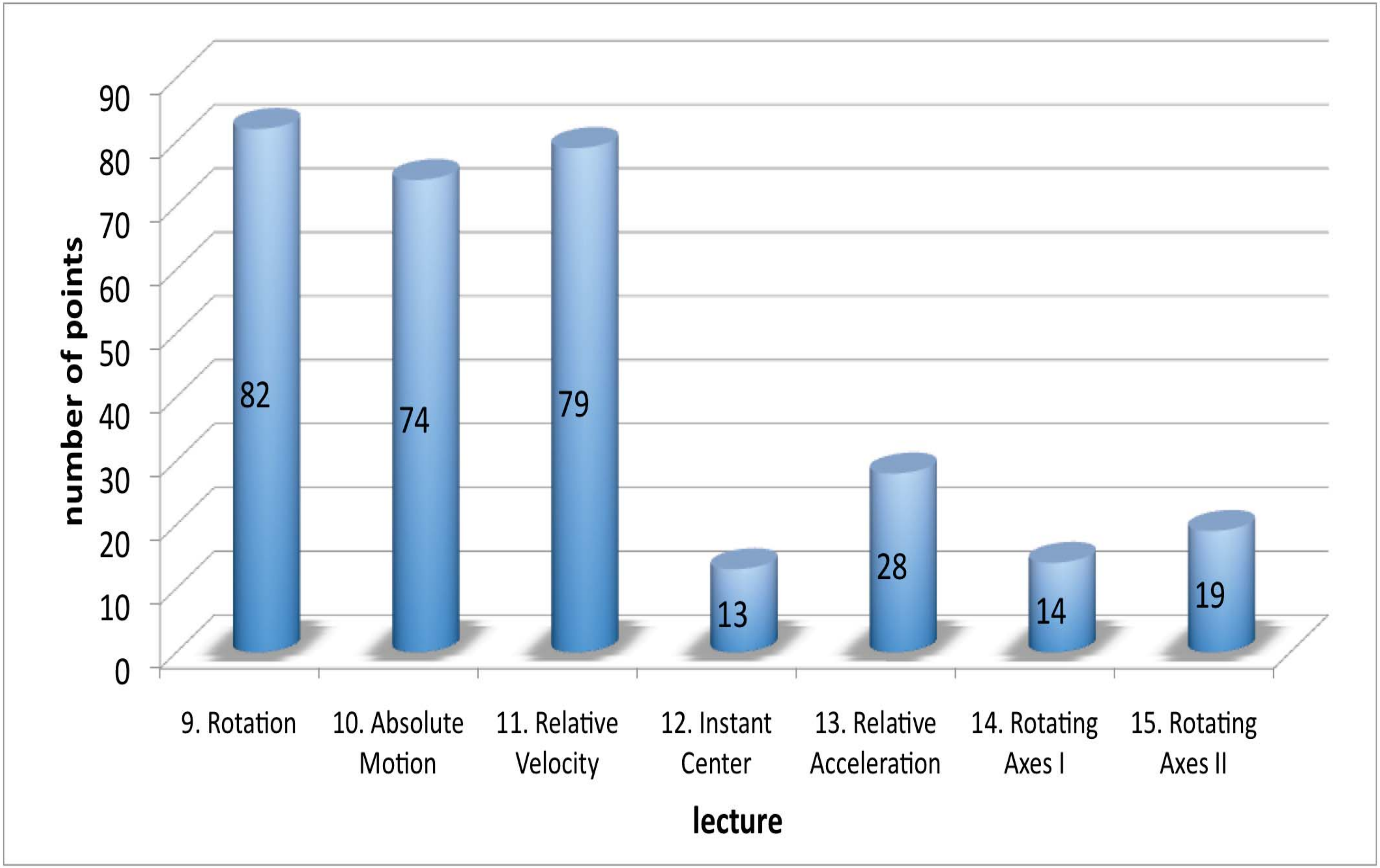
Exam 1 Breakdown (kinematics of rigid bodies)



For Next Time…

- Complete Homework #6 due on Tuesday (10/2)
- Review Lectures slides
  - http://connect.mcgraw-hill.com/class/me231
- Review Chapters 2 & 6
- Review Lectures slides
  - http://rrg.utk.edu/resources/ME231/lectures.html
- Review Examples from class
  - http://rrg.utk.edu/resources/ME231/examples.html
- *Exam #1 on Wednesday (10/3)*