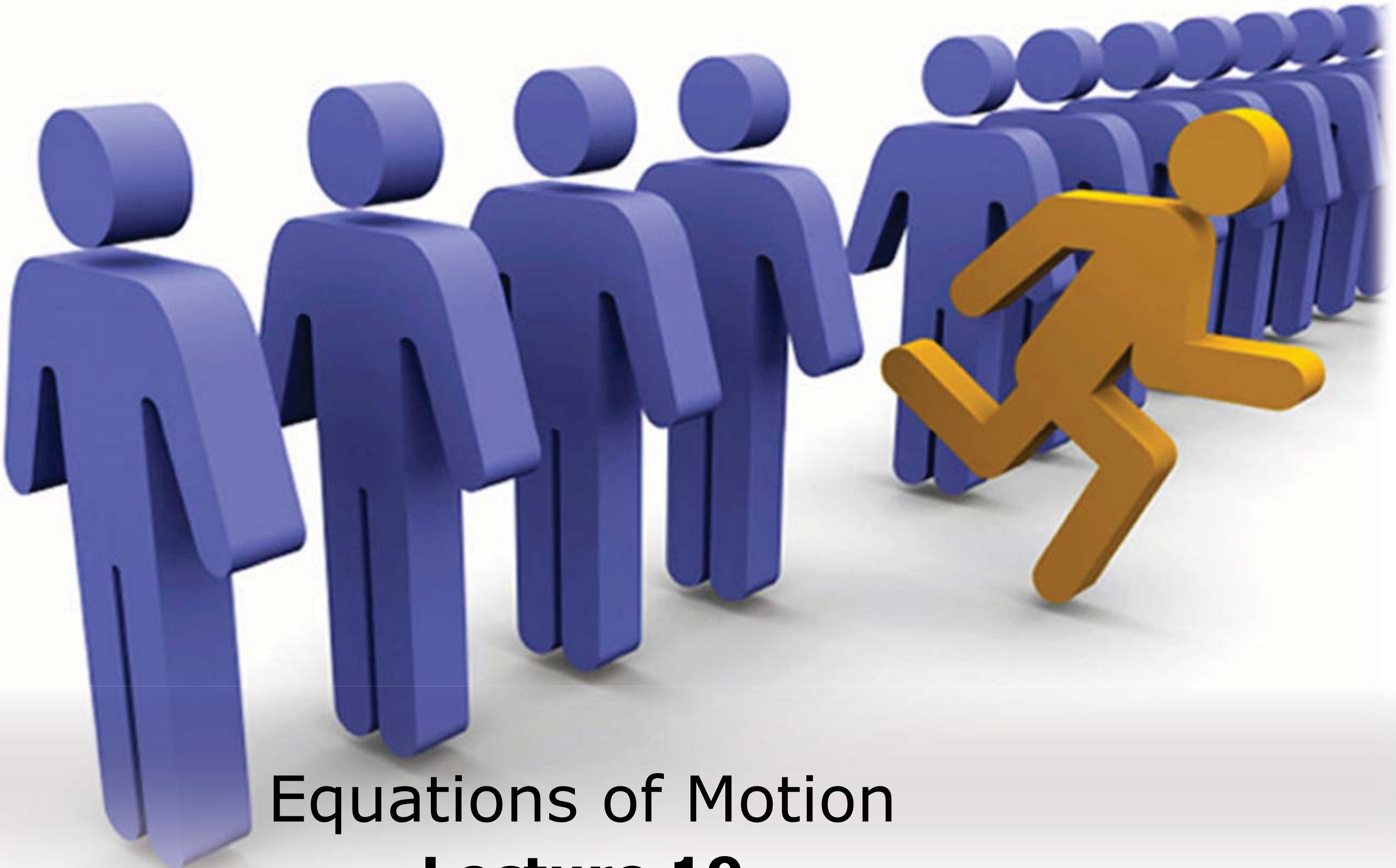
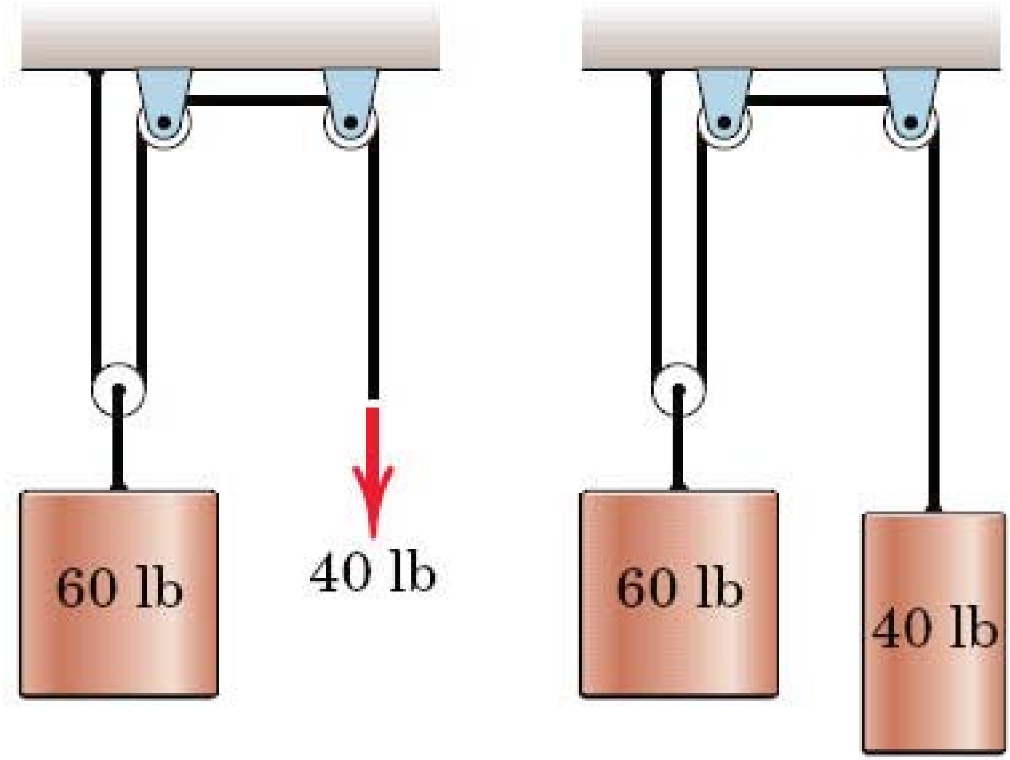


Equations of Motion

Lecture 19

ME 231: Dynamics

Question of the Day



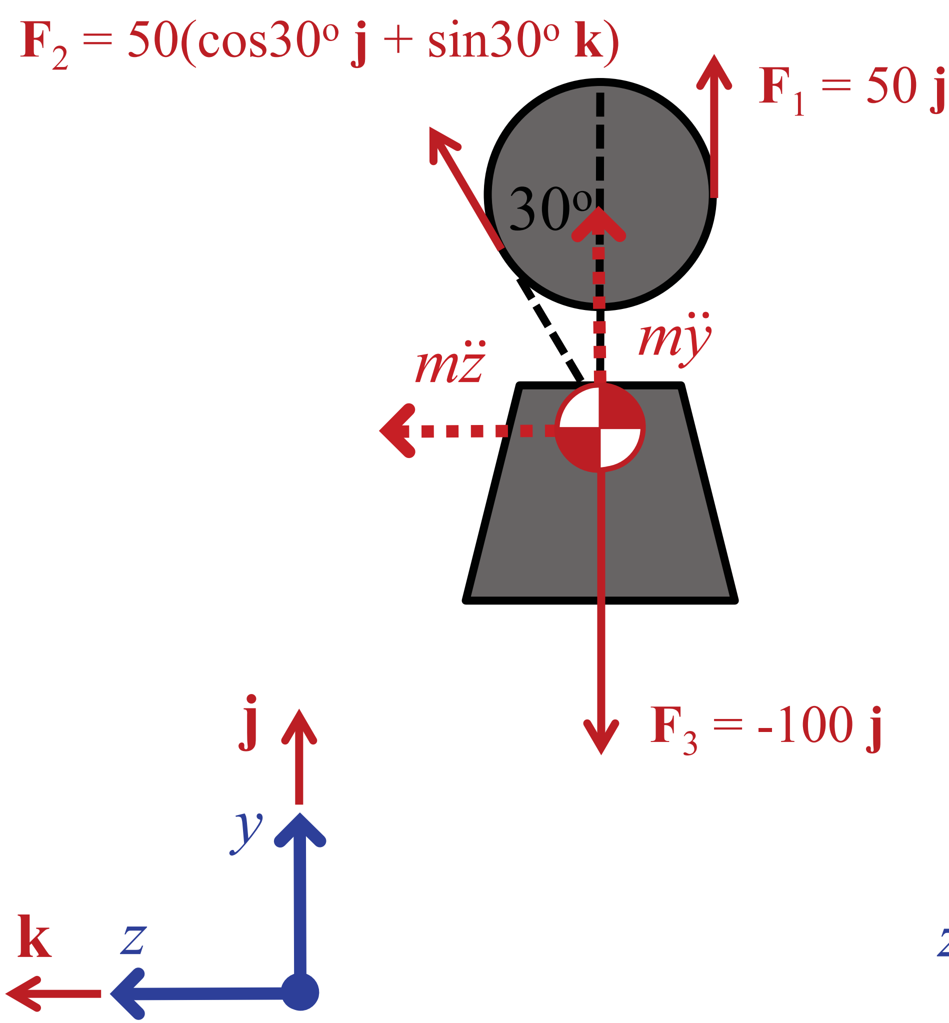
Determine the vertical ***acceleration*** of the ***60-lb*** cylinder for each of the two cases.

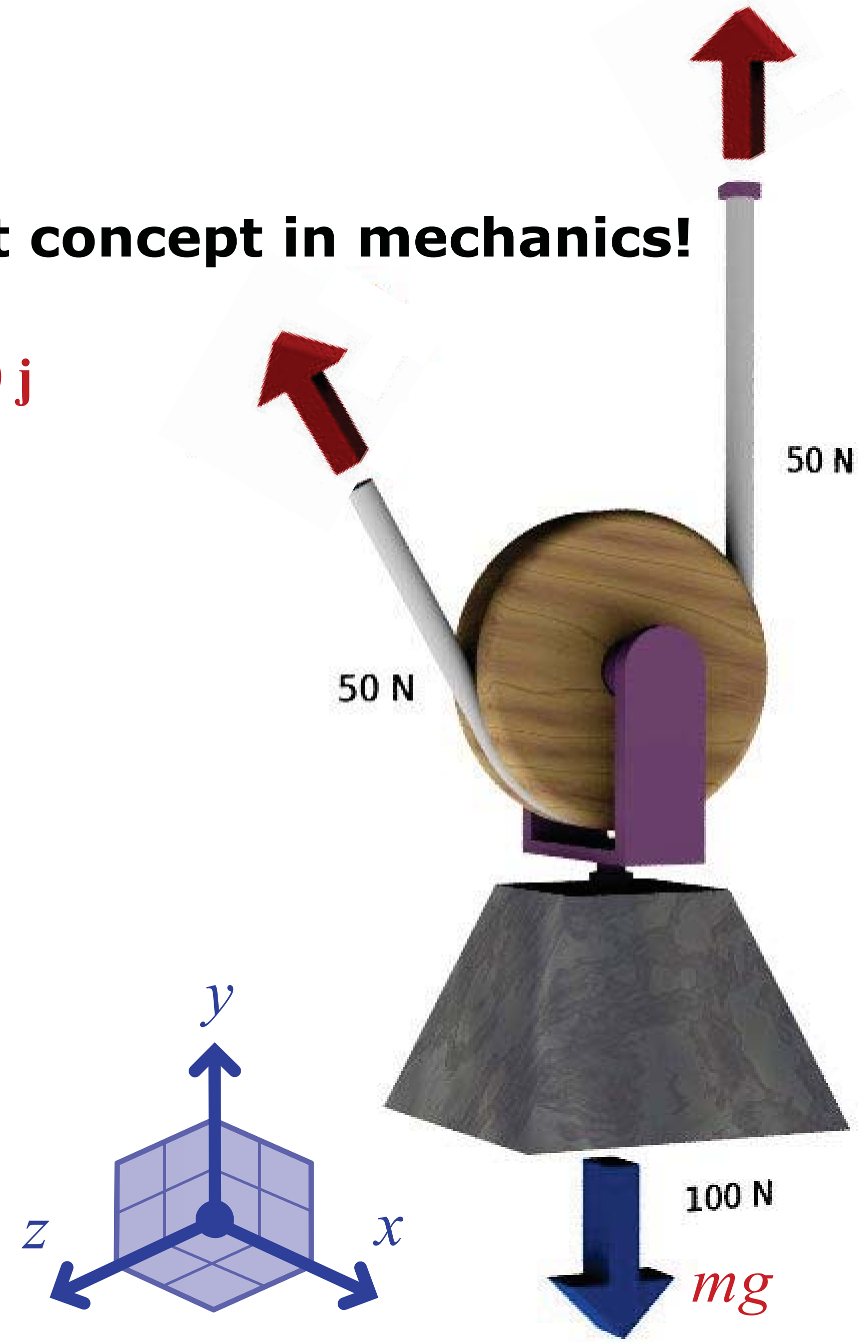
Outline for Today

- Question of the day
- Free body diagram
- Equations of motion
- Two types of problems
  - Inverse dynamics
  - Forward dynamics
- Constrained vs. unconstrained motion
- Rectilinear motion
- Answer your questions!

Free Body Diagram

Perhaps the most important concept in mechanics!





Equations of Motion

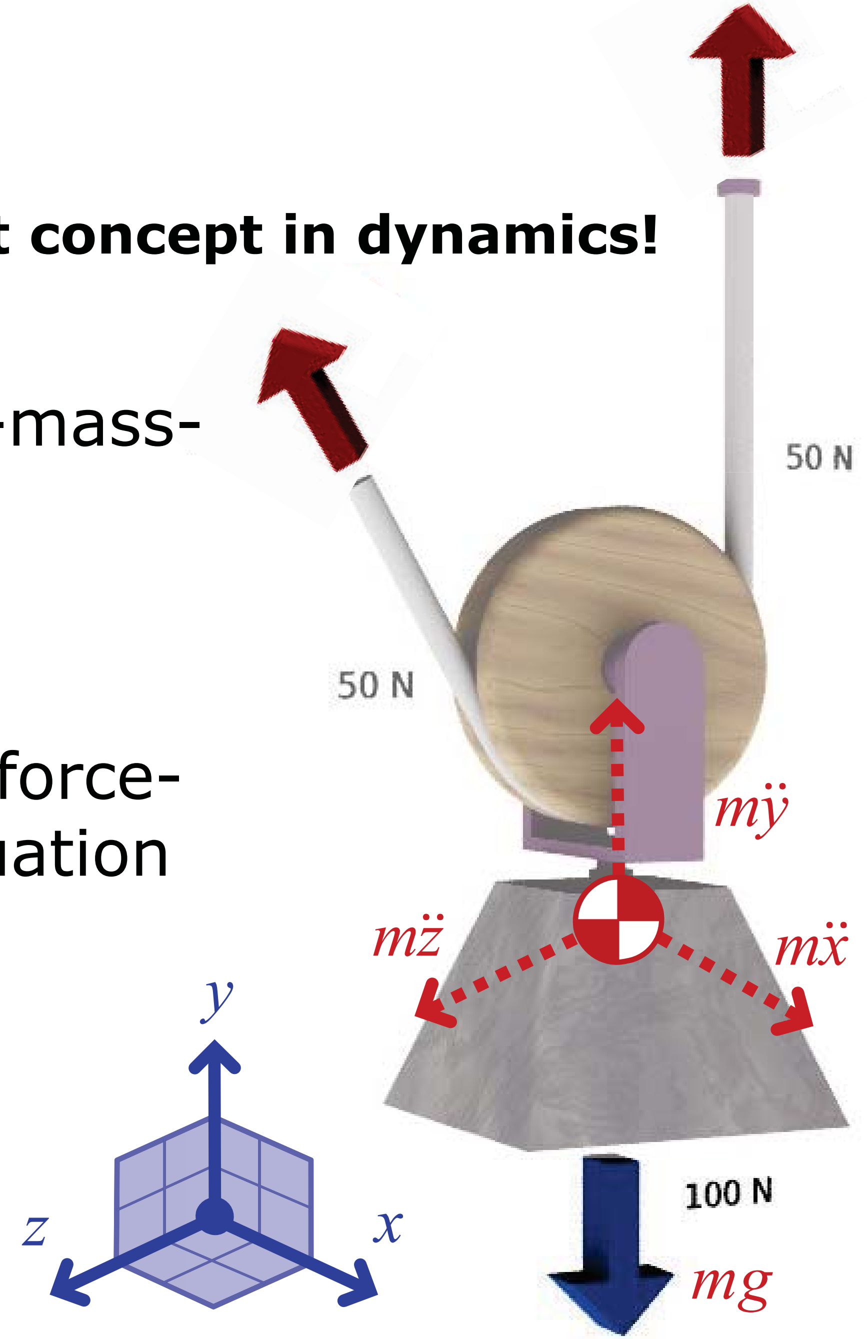
Perhaps the most important concept in dynamics!

- ***Vector form*** of force-mass-acceleration equation

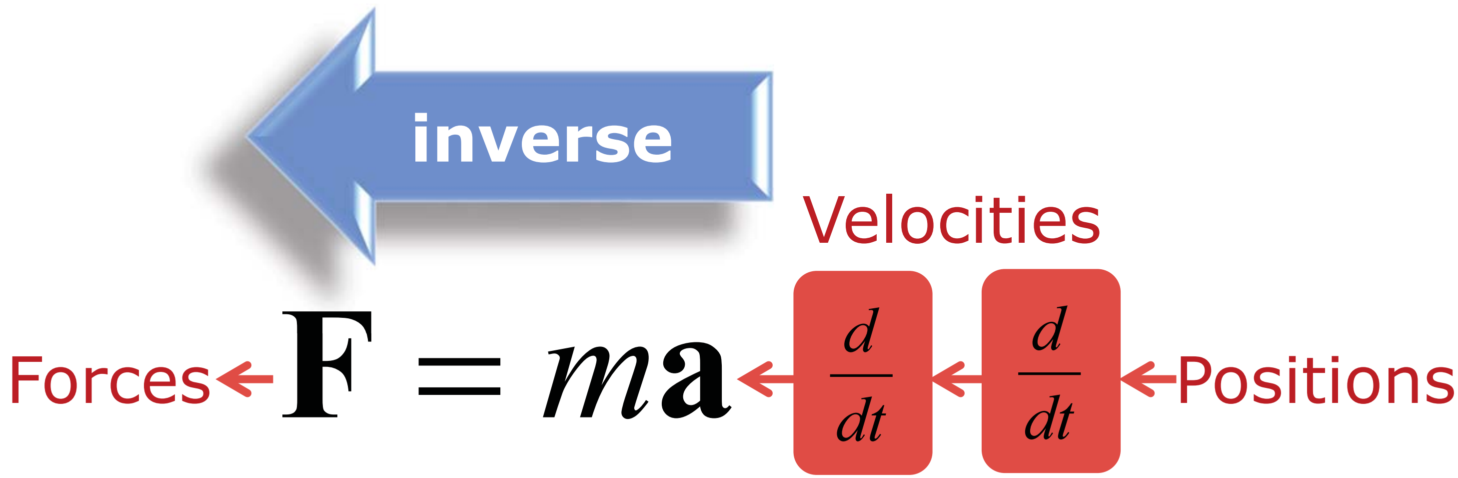
$$\sum \mathbf{F} = m\mathbf{a}$$

- ***Component form*** of force-mass-acceleration equation

$$\sum F_x = ma_x = m\ddot{x}$$
$$\sum F_y = ma_y = m\ddot{y}$$
$$\sum F_z = ma_z = m\ddot{z}$$

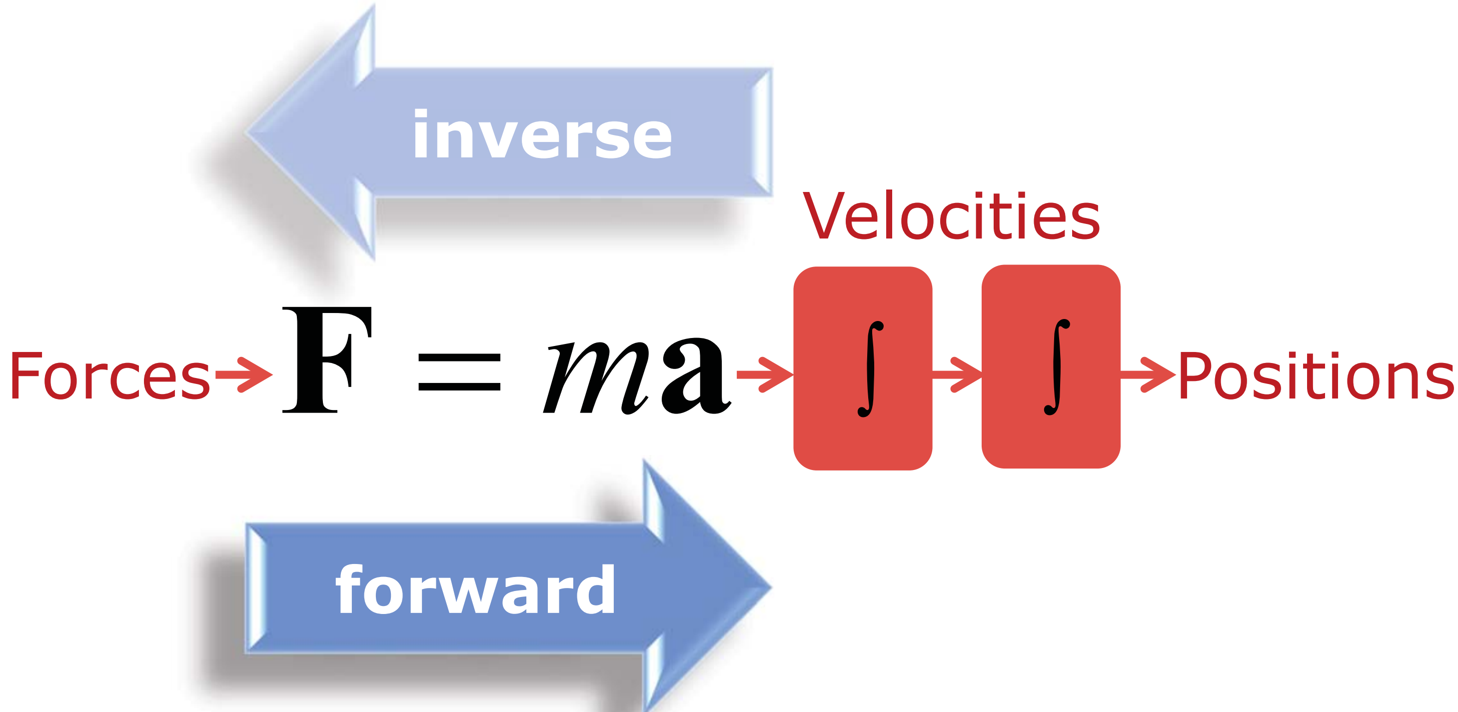


Inverse Dynamics Problem



- ***Accelerations*** are specified or determined directly from kinematic conditions
- Determine the corresponding ***forces***

Forward Dynamics Problem



- ***Forces*** are specified or determined from a FBD
- Determine the corresponding ***acceleration***
- If ***forces*** are functions of ***time***, ***position***, or ***velocity***, then ***integrate*** differential equation to determine ***velocity*** and ***position***

Constrained vs. Unconstrained Motion

- Unconstrained
  - ***Free*** of mechanical guides
  - Follows a ***path*** determined by its ***initial motion*** and applied ***external forces***
- Constrained
  - Path is partially or totally determined by ***restraining guides***
  - All forces (***applied*** and ***reactive***) must be accounted for in **F**=*m***a**

Outline for Today

- Question of the day
- Free body diagram
- Equations of motion
- Two types of problems
  - Inverse dynamics
  - Forward dynamics
- Constrained vs. unconstrained motion
- Rectilinear motion
- Answer your questions!

Rectilinear Motion

- ***Coordinate*** direction (x) ***along*** the ***motion***

$$\sum F_x = ma_x$$
$$\sum F_y = 0$$
$$\sum F_z = 0$$

- ***Coordinate*** direction ***NOT along*** the ***motion***

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$
$$\sum F_z = ma_z$$

- ***Acceleration***

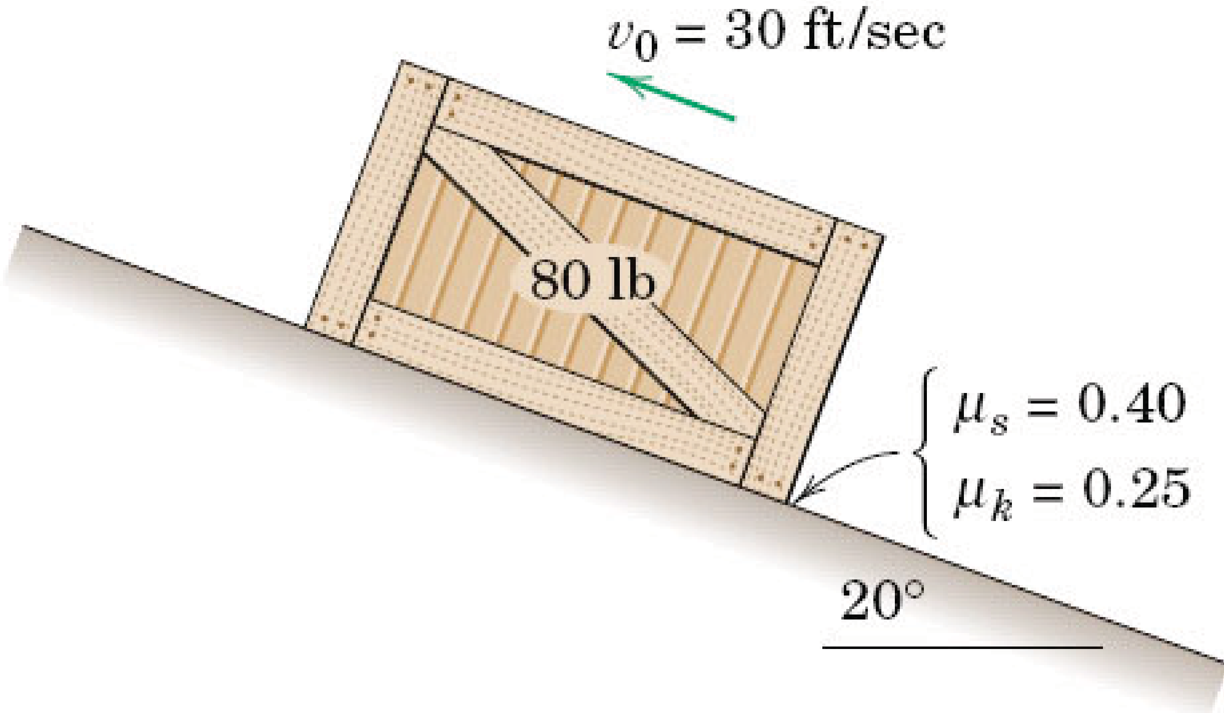
$$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$
$$|\mathbf{a}| = \sqrt{a_x^{\ 2} + a_y^{\ 2} + a_z^{\ 2}}$$

- ***Resultant force***

$$\sum \mathbf{F} = \sum F_x\mathbf{i} + \sum F_y\mathbf{j} + \sum F_z\mathbf{k}$$
$$\left|\sum \mathbf{F}\right| = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2 + \left(\sum F_z\right)^2}$$

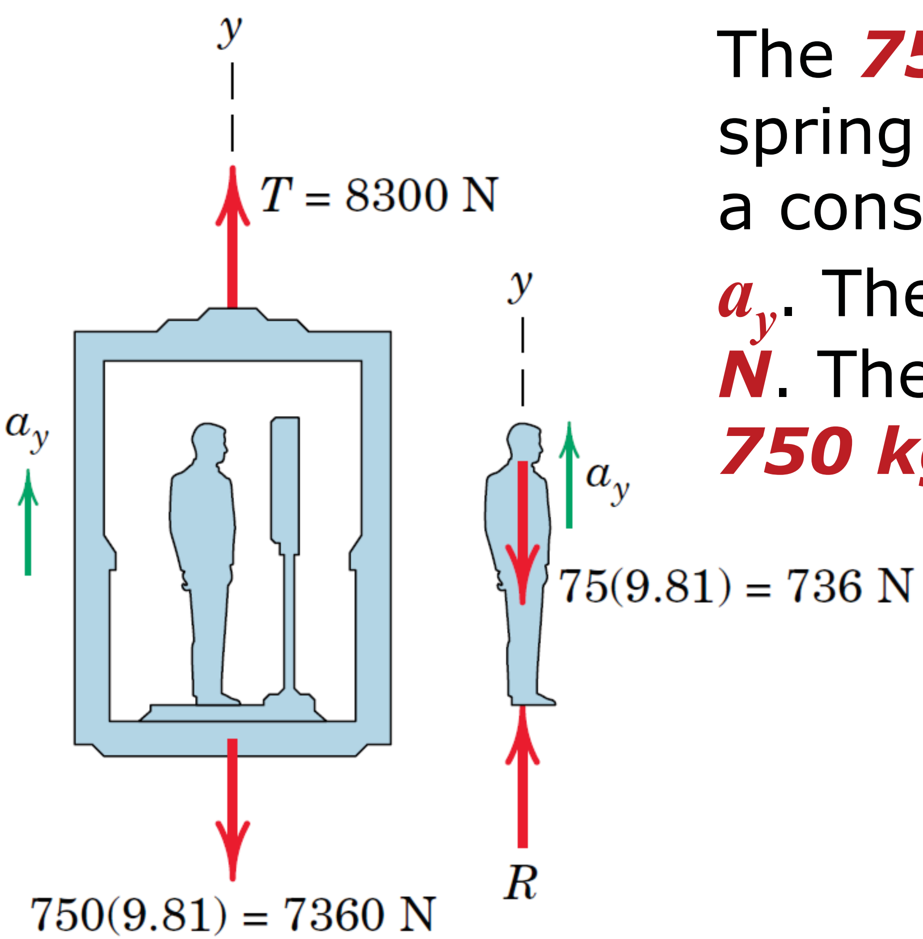
Rectilinear Motion: Exercise

The ***80-lb crate*** has a ***velocity*** of ***30 ft/s*** up the incline.



Calculate the ***time*** required for the crate to come to ***rest***.

Rectilinear Motion: Another Exercise

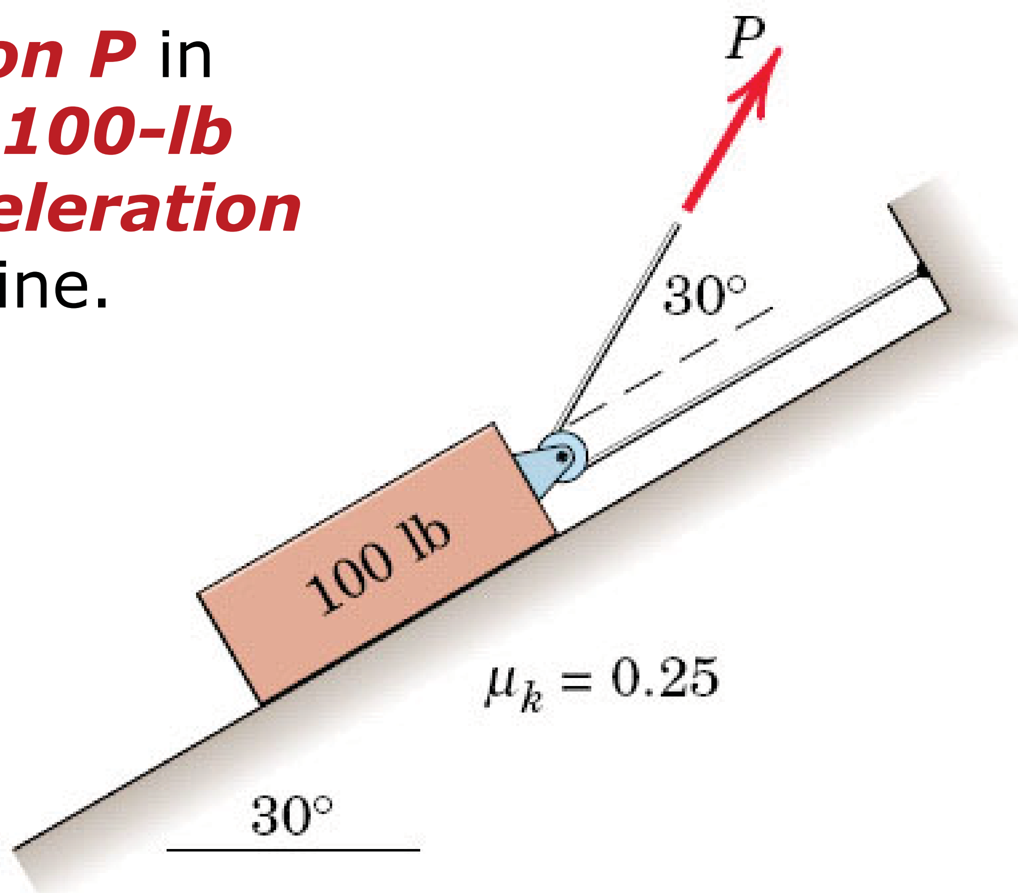


The ***75-kg*** man stands on a spring scale in an elevator with a constant upward ***acceleration*** a_y. The cable ***tension*** T is ***8300 N***. The total system ***mass*** is ***750 kg***.

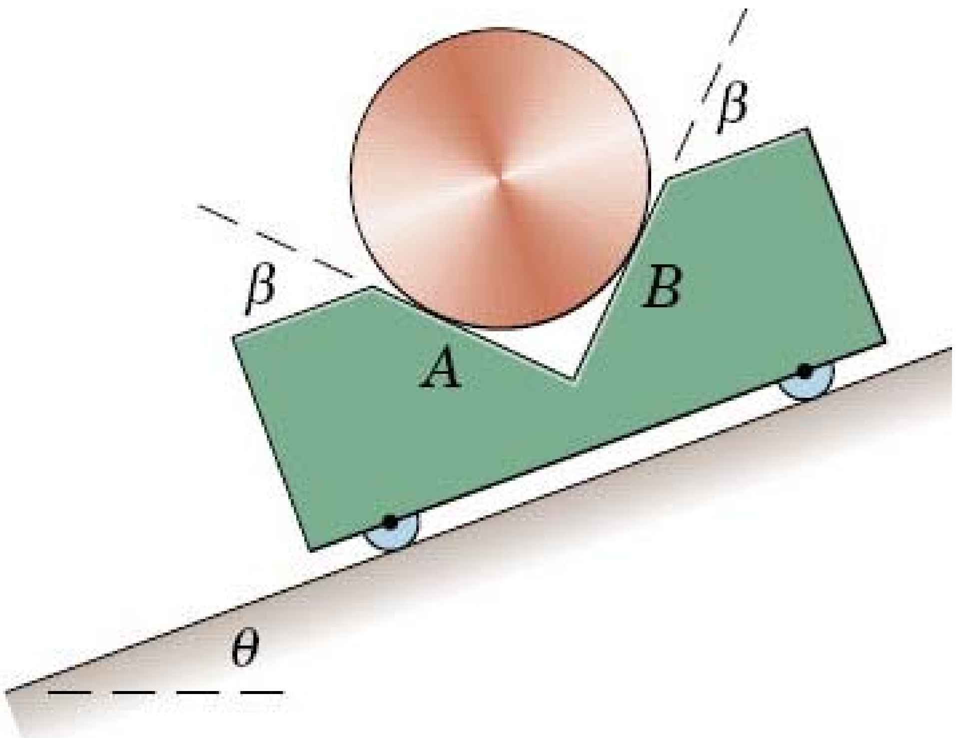
Find the ***reading*** R (in Newtons) of the scale.

Rectilinear Motion: Yet Another Exercise

Determine the ***tension P*** in the cable to give the ***100-lb*** block a constant ***acceleration*** of ***5 ft/s²*** up the incline.



Rectilinear Motion: One More Exercise



A cylinder rests in a supporting carriage where ***β* = 45º** and ***θ* = 30º**.

Calculate the maximum ***acceleration a*** up the incline so that the cylinder does not lose contact with the carriage.

Outline for Today

- Question of the day
- Free body diagram
- Equations of motion
- Two types of problems
  - Inverse dynamics
  - Forward dynamics
- Constrained vs. unconstrained motion
- Rectilinear motion
- Answer your questions!

For Next Time...

- Continue Homework #7 due ***next Wednesday*** (10/17)
- Read Chapter 3, Sections 3.1 & 3.2