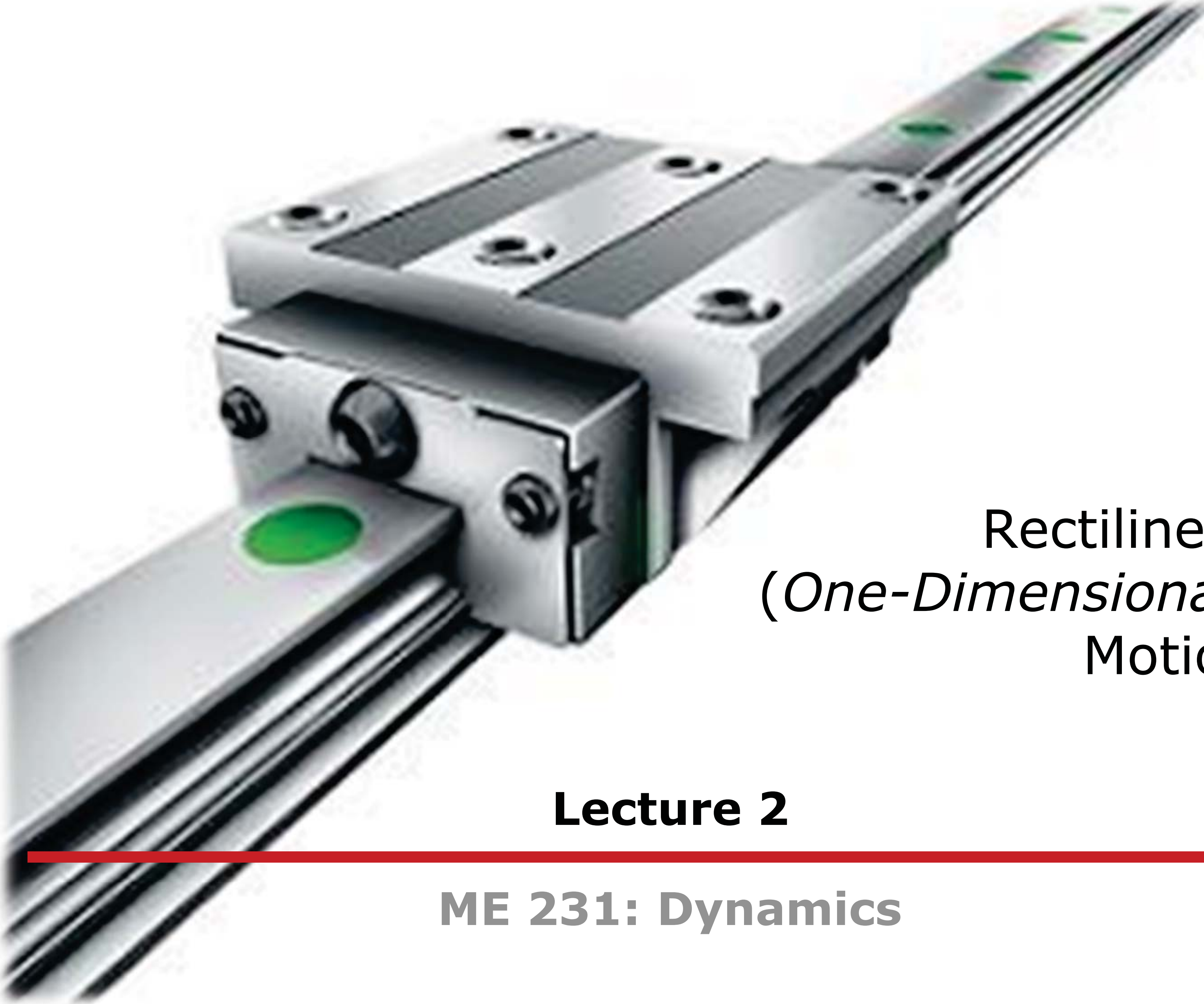


Rectilinear (*One-Dimensional*) Motion

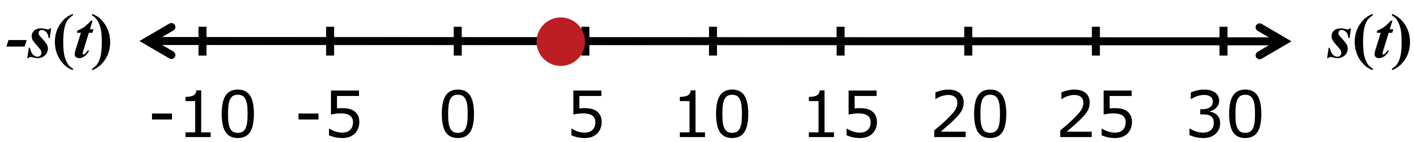
Lecture 2

ME 231: Dynamics

Question of the Day

A particle moving along a straight line has a ***position*** (s) as a function of ***time*** (t) given by

$$s(t) = t^3 - 10t + 4$$



Determine the ***velocity*** and ***acceleration*** as a function of ***time*** (t).

Outline for Today

- Question of the day
- Kinematics: geometry of motion
- Possible coordinate systems
- Velocity and acceleration
- Graphical interpretations
- Integrating acceleration
- Answer your questions!

Kinematics: Geometry of Motion

Concept: What is kinematics?

Main Entry: **ki·ne·mat·ics**

Pronunciation: \ˌki-nə-ˈma-tiks *also* ˌkī-\

Function: *noun plural but singular in construction*

Etymology: French *cinématique*, from Greek *kinēmat-*, *kinēma* motion, from *kinein* to move

Date: 1840

: a branch of dynamics that deals with aspects of motion apart from considerations of mass and force

— **ki·ne·mat·ic** \-tik\ or **ki·ne·mat·i·cal** \-ti-kəl\ *adjective*

— **ki·ne·mat·i·cal·ly** \-ti-k(ə-)lē\ *adverb*

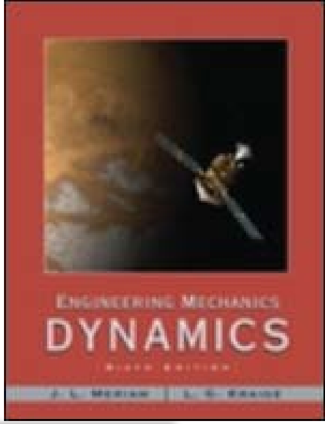
Kinematics: Geometry of Motion

Concept: What is kinematics?

Kinematics (from Greek κινεῖν, *kinein*, to move) is the branch of classical mechanics that describes the motion of objects without consideration of the causes leading to the motion.

Kinematics: Geometry of Motion

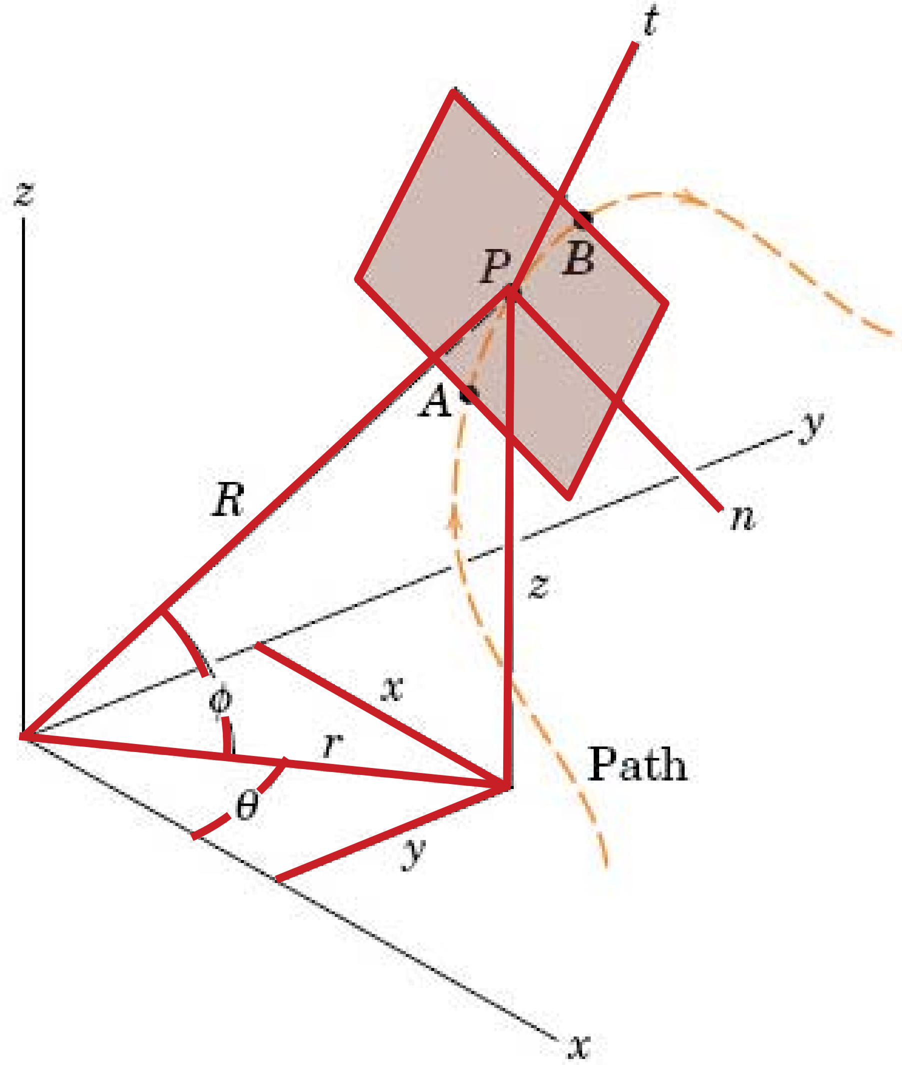
Concept: What is kinematics?



Kinematics: The description of motion (***position***, ***velocity***, and ***acceleration*** relationships) without reference to the forces which cause the motion.

Possible Coordinate Systems

- Rectangular (x, y, z)
- Polar (r, θ, z)
- Spherical (R, θ, ϕ)
- Normal and Tangential (n, t)



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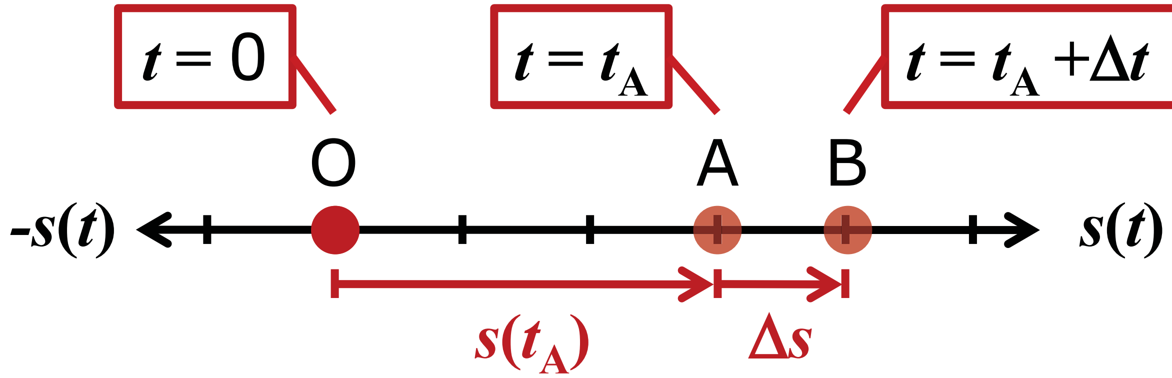
Velocity and Acceleration



- Position (s)
- Velocity (v)

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

- Acceleration (a)

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$

DE relating s, v, a

$$a = \frac{dv}{ds/v} = v\frac{dv}{ds}$$

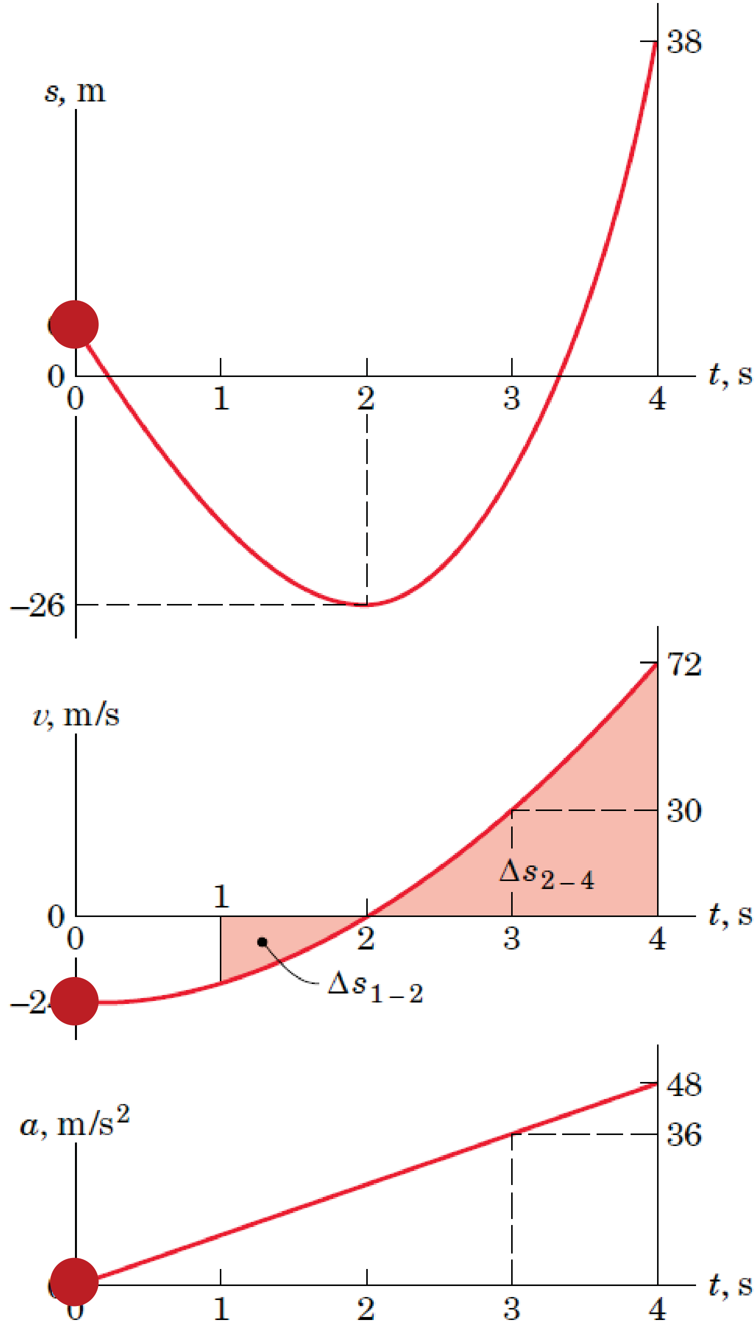
Velocity and Acceleration: Exercise

A particle moving along a straight line has a position (***s***) a function of time (***t***) given by

$$s(t) = 2t^3 - 24t + 6$$

where ***s*** is measured in meters from a convenient origin and ***t*** is in seconds.

Determine (a) the time required for the particle to reach a ***velocity*** of 72 m/s, (b) the ***acceleration*** of the particle when it's velocity is 30 m/s, and (c) the net ***position*** change of the particle during the interval from 1s to 4s.



Graphical Interpretations

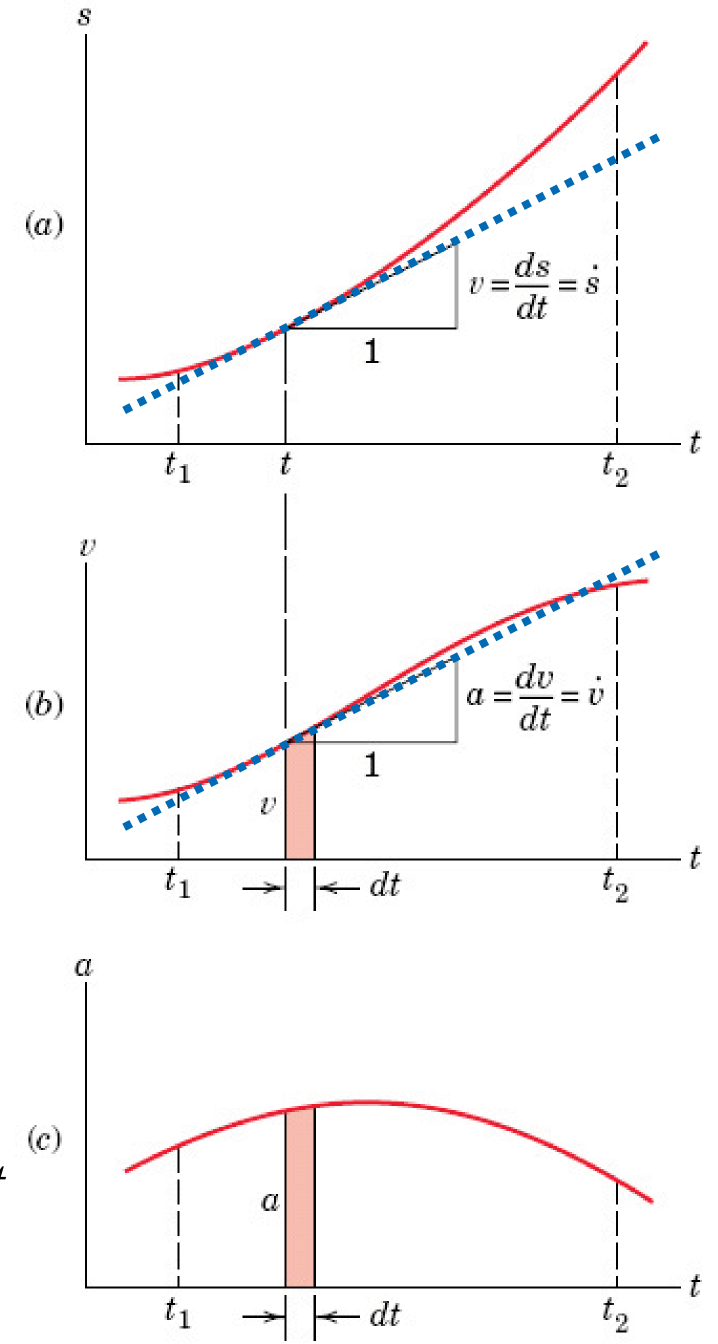
Clarifying relationships among *position*, *velocity*, and *acceleration*

- ***Velocity*** is ***slope*** of ***s-t*** curve
- ***Acceleration*** is ***slope*** of ***v-t*** curve
- Net ***position*** change (***ds***) is area under ***v-t*** curve

$$\int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} v\,dt$$

- Net ***velocity*** change (***dv***) is area under ***a-t*** curve

$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt$$



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Integrating Acceleration

- Case #1: constant ***acceleration***

$$a = \frac{dv}{dt} \qquad \int_{v_0}^{v} dv = a \int_{0}^{t} dt \qquad v = v_0 + at$$

$$a = v\frac{dv}{ds} \qquad \int_{v_0}^{v} v\, dv = a \int_{s_0}^{s} ds \qquad v^2 = v_0{}^2 + 2a(s - s_0)$$

$$v = \frac{ds}{dt} \qquad \int_{s_0}^{s} ds = \int_{0}^{t} (v_0 + at)\, dt \qquad s = s_0 + v_0 t + \frac{1}{2}at^2$$

CAUTION: Equations only for <u>constant acceleration</u>

Integrating Acceleration

- Case #2: $a = f(t)$

$$a = \frac{dv}{dt} \qquad \int_{v_0}^{v} dv = \int_0^t f(t)\,dt \qquad v = v_0 + \int_0^t f(t)\,dt$$

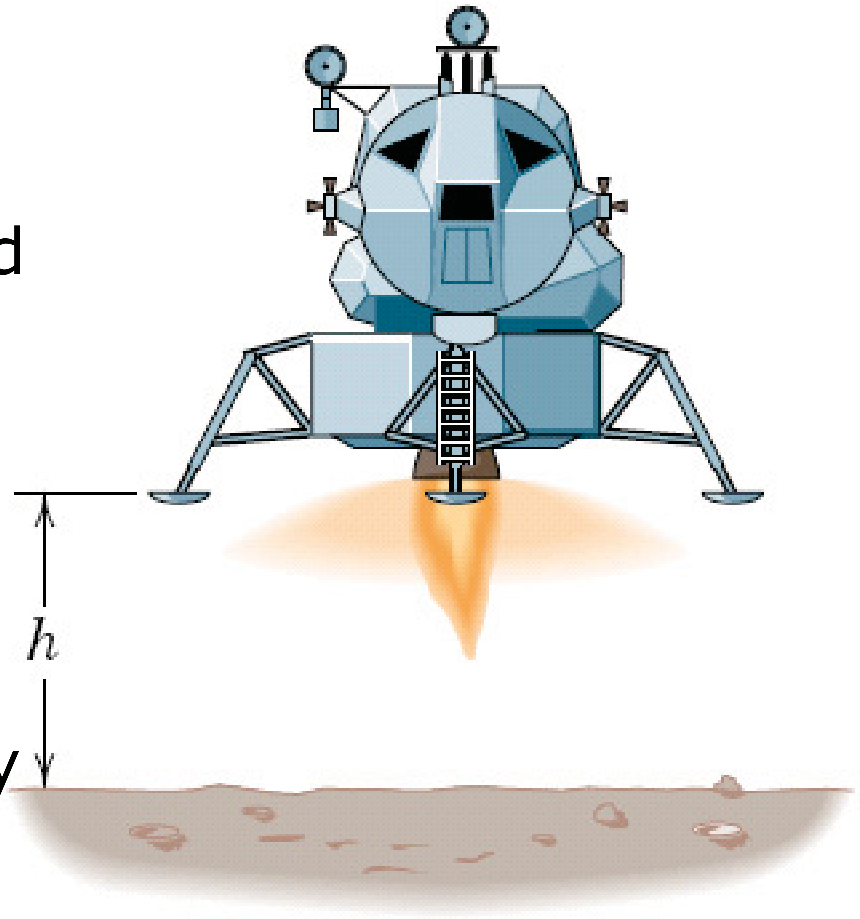
$$v = \frac{ds}{dt} \qquad \int_{s_0}^{s} ds = \int_0^t v\,dt \qquad s = s_0 + v_0 t + \int \int_0^t f(t)\,dt^2$$

Integrating Acceleration: Exercise

Case #1: constant *acceleration*

A lunar module is ***positioned*** 5 m above the surface and has a downward ***velocity*** of 2 m/s when its engine stops.

Compute the impact ***velocity*** (v) of the module with the moon (hint: gravity is 1/6 earth's gravity).



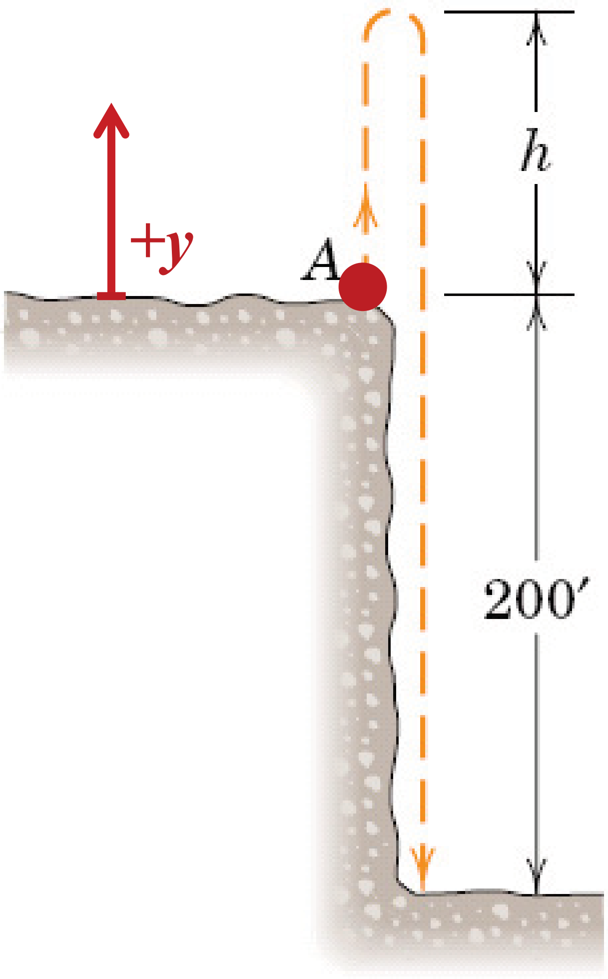
Integrating Acceleration: Exercise

Case #1: constant *acceleration*

A ball is thrown vertically with a ***velocity*** of 80 ft/s at the edge of a 200-ft cliff.

Calculate the ***height*** (h) the ball rises and the total ***time*** (t) to reach the bottom of the cliff.

Neglect air resistance and take ***acceleration*** as a = 32.2 ft/s².



Integrating Acceleration: Exercise

Case #2: $a = f(t)$

Acceleration of a particle is given by

$$a(t) = 2t - 10$$

where a is in meters per second squared and t is in seconds.

Determine the ***velocity*** and ***position*** as functions of time. The initial ***position*** is s_0 = -4 m at t = 0, and the initial ***velocity*** is v_0 = 3 m/s.

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For Next Time…

- Continue Homework #1 due next Wednesday (8/29)
- Read Chapter 2, Sections 2.3 and 2.4