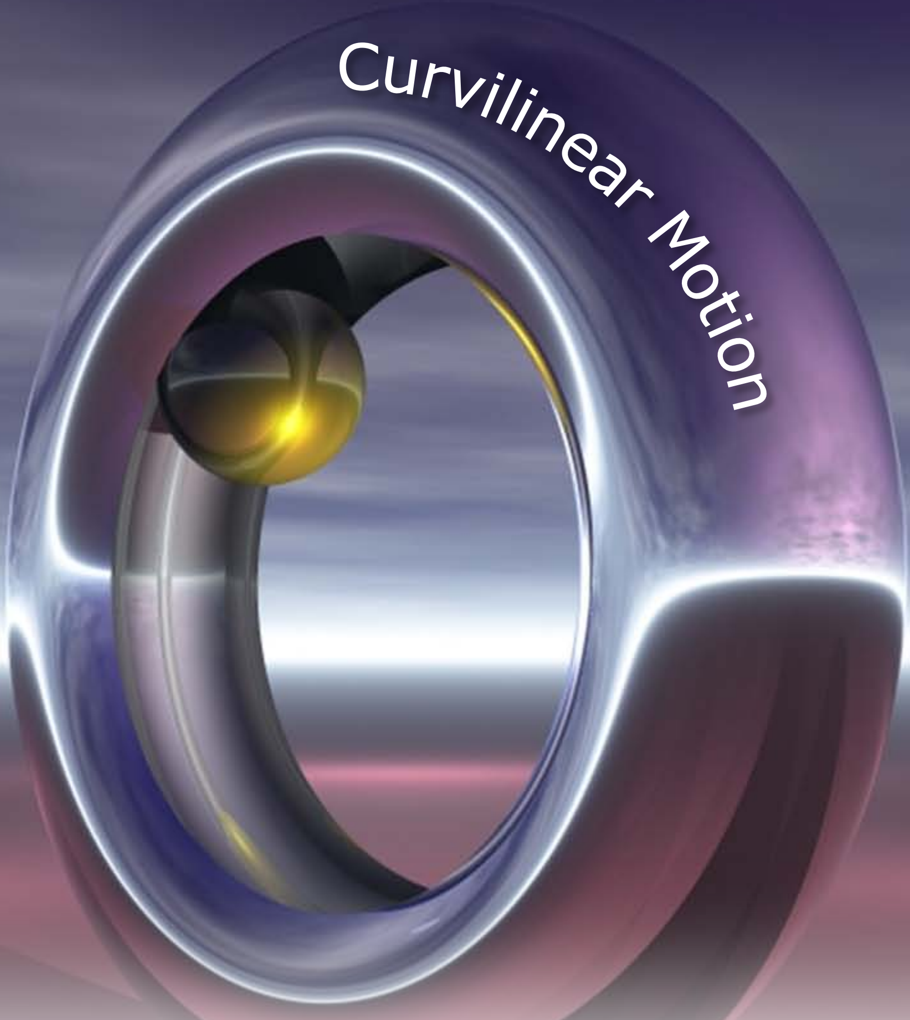
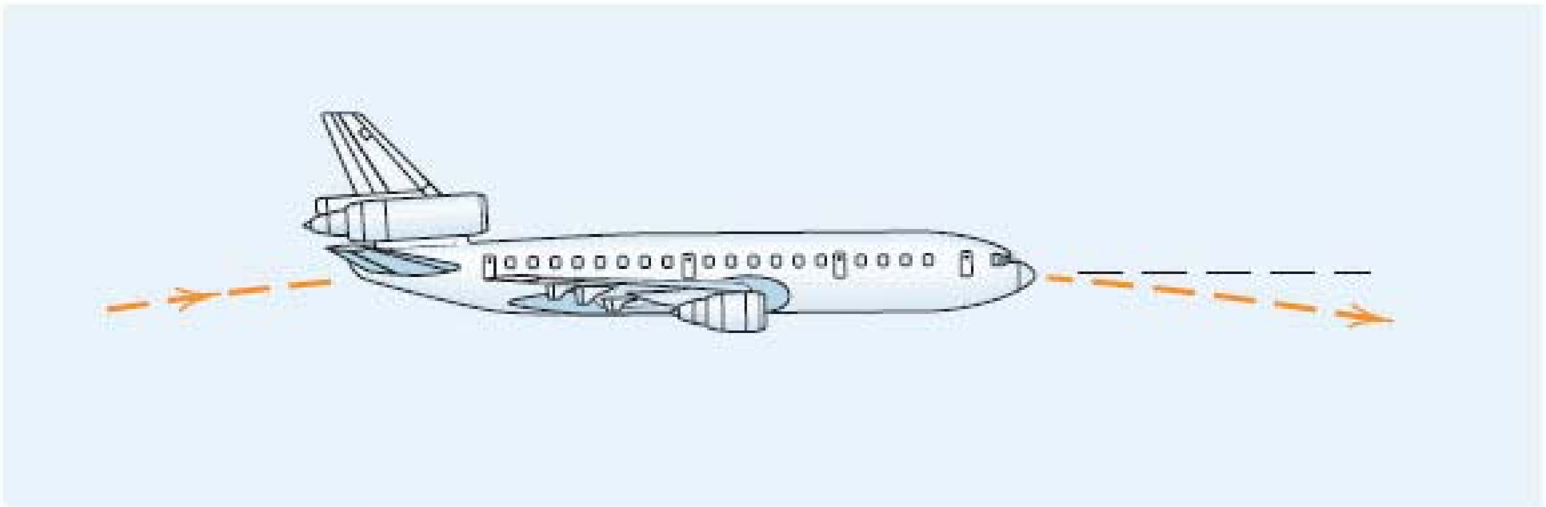


Curvilinear Motion

Lecture 21

Question of the Day



A jet flies in a trajectory to allow astronauts experience a weightless condition. The ***speed*** at the highest point is ***600 mi/hr***.

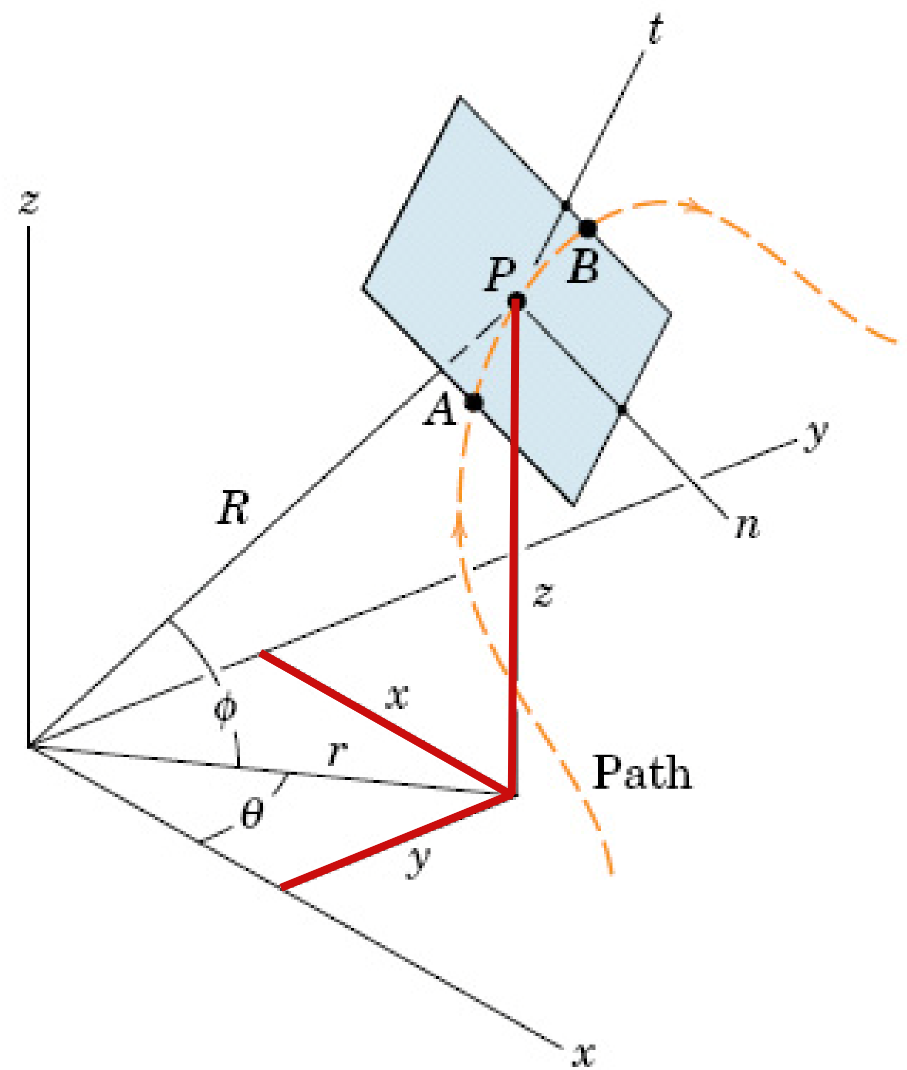
What is the ***radius of curvature*** ρ necessary to simulate weightlessness?

Outline for Today

- Question of the day
- Rectangular (x-y) coordinates
- Polar (r-θ) coordinates
- Normal and tangential (n-t) coordinates
- Answer your questions!

Recall: Possible Coordinate Systems

- Rectangular (x, y, z)
- Polar (r, θ, z)
- Spherical (R, θ, ϕ)
- Normal and Tangential (n, t)

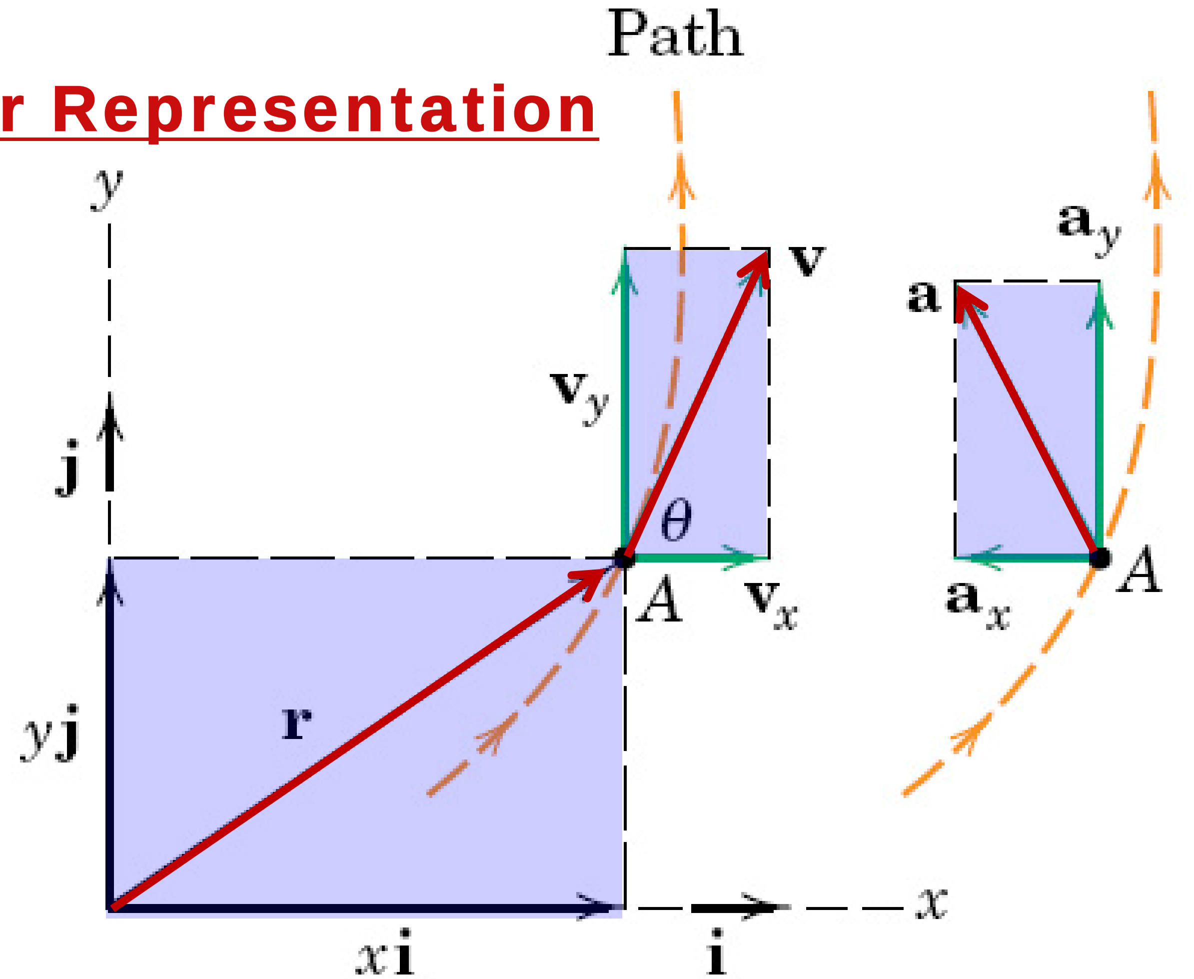


Recall: *x*-*y* Vector Representation

$$\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j}$$

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\,\mathbf{i} + \dot{y}\,\mathbf{j}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \ddot{x}\,\mathbf{i} + \ddot{y}\,\mathbf{j}$$

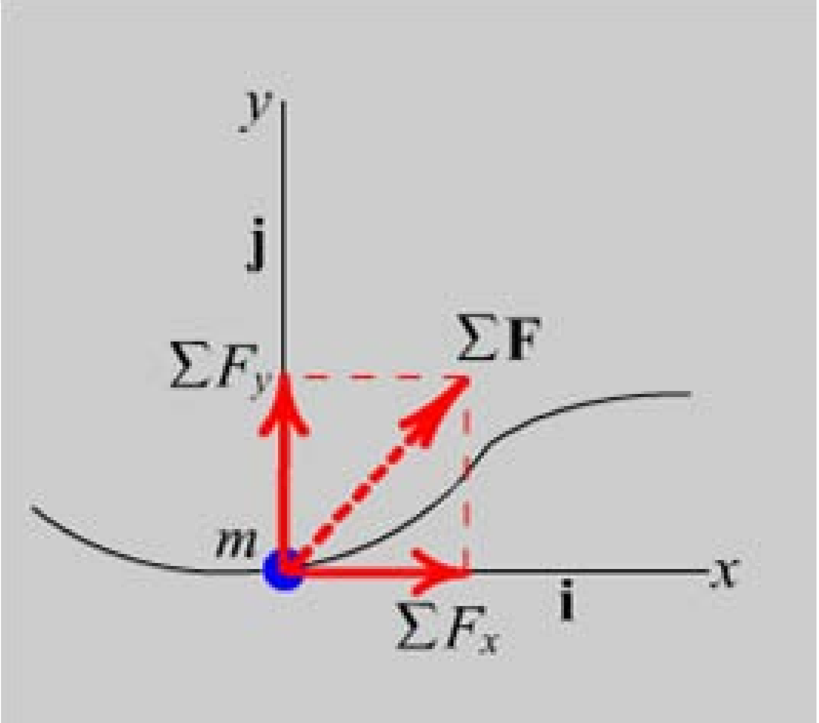


- The *x*- and *y*-components are independent
- Resulting motion is a vector combination of *x*- and *y*-components

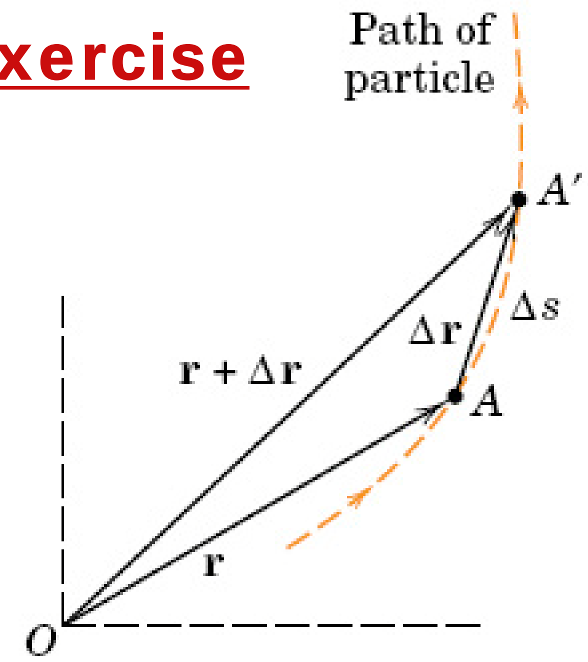
Rectangular (x-y) Coordinates

$$\sum \mathbf{F} = m\mathbf{a}$$

vector	*x-coord*	*y-coord*
$\mathbf{a} =$	$a_x \mathbf{i} +$	$a_y \mathbf{j}$
$\sum \mathbf{F} =$	$\sum F_x \mathbf{i} +$	$\sum F_y \mathbf{j}$
	$\sum F_x = m a_x$	$\sum F_y = m a_y$



Rectangular (x-y) Coordinates: Exercise



A particle with ***mass*** of ***10 slugs*** moving in two-dimensions has a position vector ($\mathbf{r}$) as a function of time (t) with coordinates given by

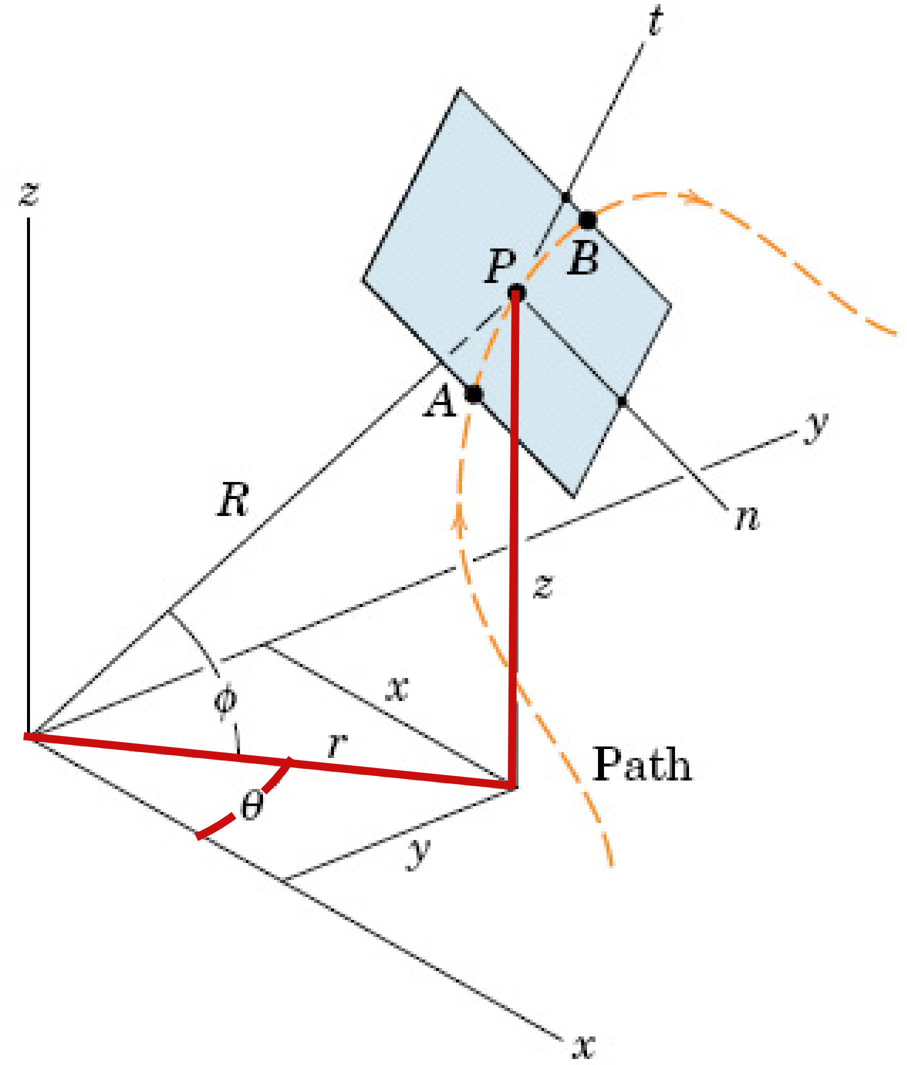
$$x(t) = t^2 - 4t + 20 \quad , \quad y(t) = 3\sin(2t)$$

where $\mathbf{r}$ is measured in feet and t is in seconds.

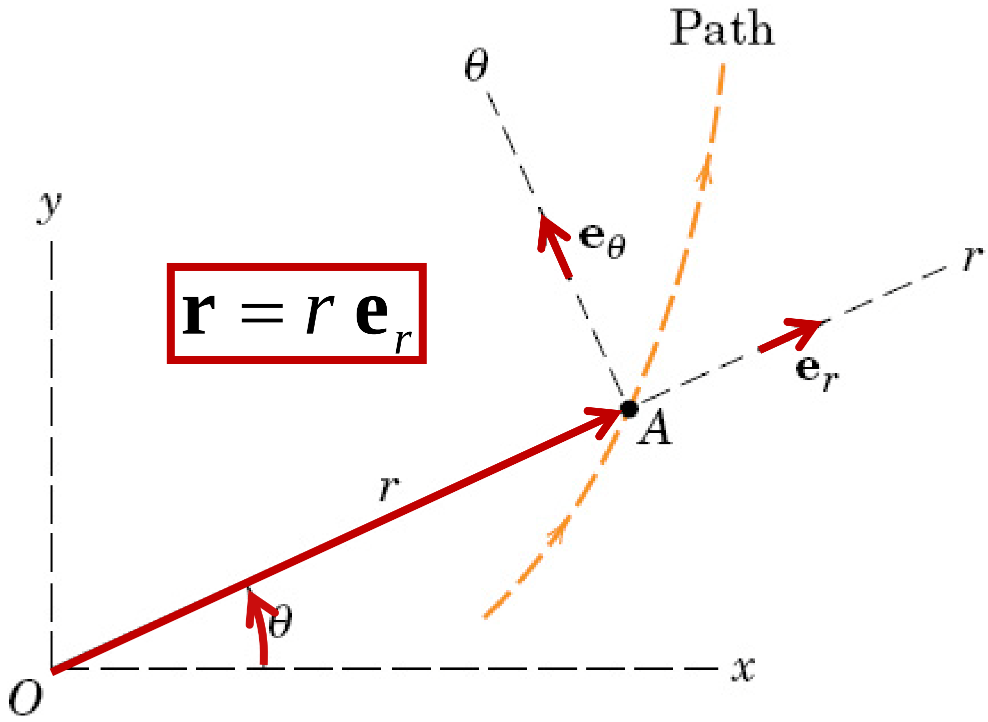
Determine the magnitude of the net ***force*** ($\mathbf{F}$) ***accelerating*** the particle at time $t = 3$ s.

Recall: Possible Coordinate Systems

- Rectangular (x, y, z)
- **Polar (r, θ, z)**
- Spherical (R, θ, ϕ)
- Normal and Tangential (n, t)

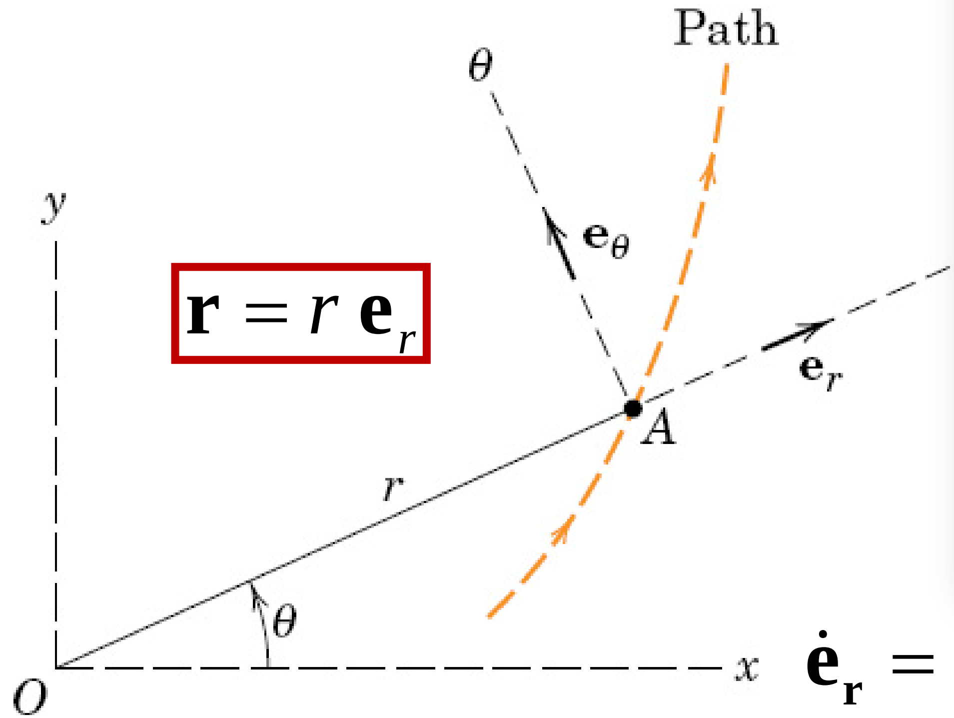


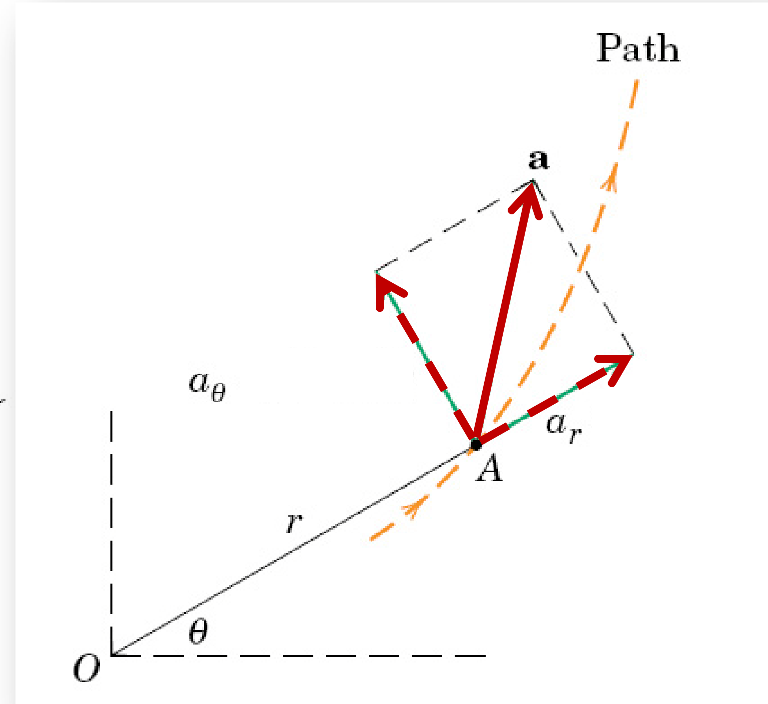
Recall: r-θ Vector Representation



- Useful when motion is measured by a ***radial distance*** (r) and an ***angular position*** (θ)
- $\mathbf{e}_r$ is the unit vector in the r-direction
- $\mathbf{e}_\theta$ is the unit vector in the θ-direction

Recall: r-θ Acceleration





$$\mathbf{r} = r\,\mathbf{e}_r$$

$$\dot{\mathbf{e}}_\mathbf{r} = \dot{\theta}\,\mathbf{e}_\theta$$

$$\mathbf{v} = \dot{r}\,\mathbf{e}_\mathbf{r} + r\dot{\theta}\,\mathbf{e}_\theta$$

$$\mathbf{a} = \dot{\mathbf{v}} = \left(\ddot{r}\,\mathbf{e}_\mathbf{r} + \dot{r}\,\dot{\mathbf{e}}_\mathbf{r}\right) + \left(\dot{r}\dot{\theta}\,\mathbf{e}_\theta + r\ddot{\theta}\,\mathbf{e}_\theta + r\dot{\theta}\,\dot{\mathbf{e}}_\theta\right)$$

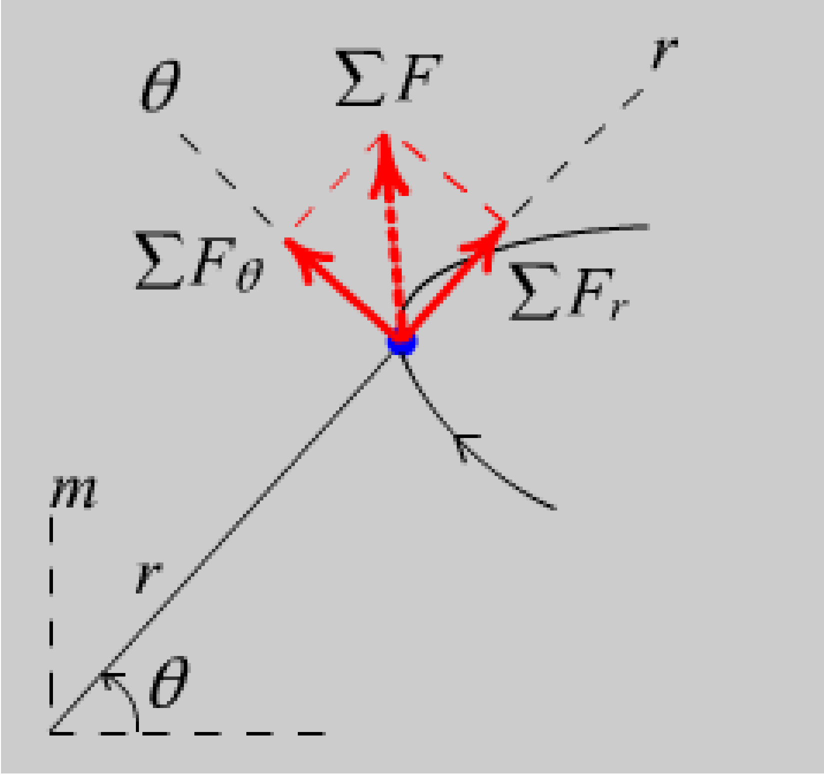
$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e}_\mathbf{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta$$

$$\dot{\mathbf{e}}_\theta = -\dot{\theta}\,\mathbf{e}_r$$

Polar (r-θ) Coordinates

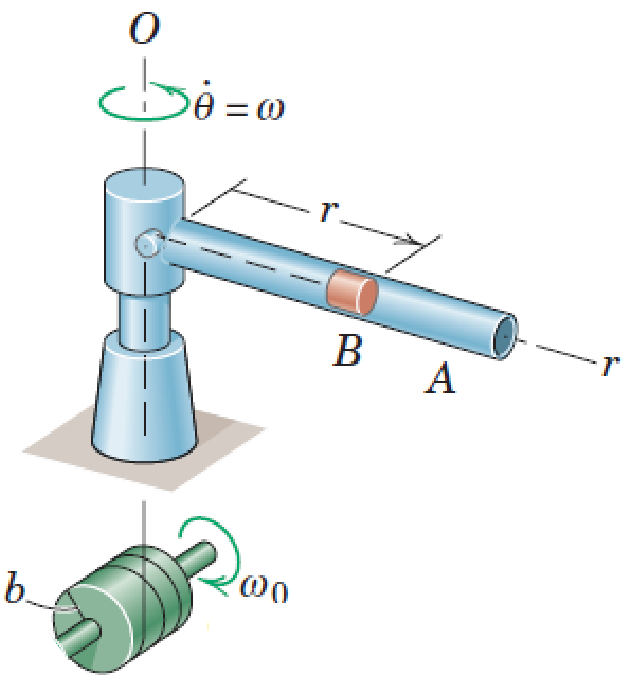
$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e_r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta$$

$$\sum \mathbf{F} = m\mathbf{a}$$



vector	*r-coord*	*θ-coord*
$\mathbf{a} =$	$a_r \mathbf{e}_r +$	$a_\theta \mathbf{e}_\theta$
$\sum \mathbf{F} =$	$\sum F_r \mathbf{e}_r +$	$\sum F_\theta \mathbf{e}_\theta$
	$\sum F_r = m a_r$	$\sum F_\theta = m a_\theta$

Polar (r-θ) Coordinates: Exercise



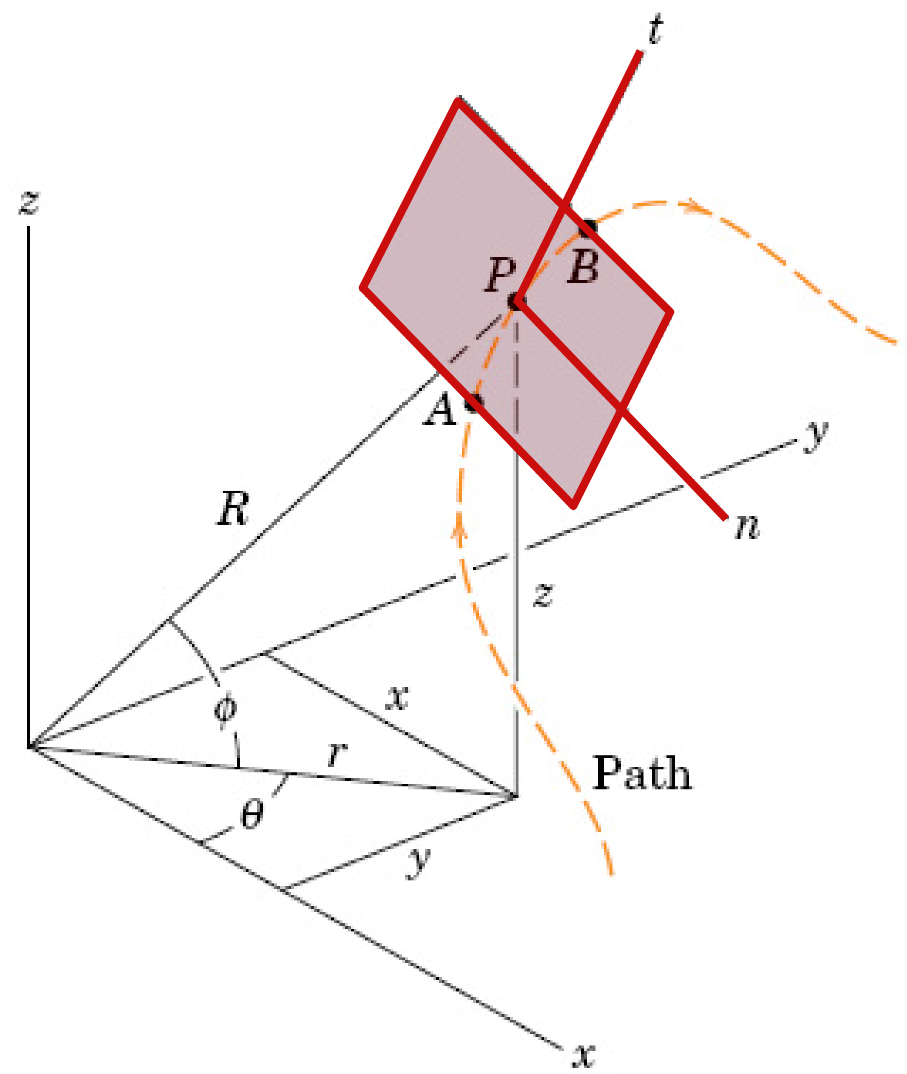
$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e_r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta$$

Tube A rotates about the vertical ***O-axis*** with constant ***angular velocity*** ω and contains a small ***cylinder B*** of ***mass*** m whose radial position is controlled by a cord passing through the tube and wound around a ***drum*** of ***radius*** b.

Determine the ***tension*** T in the cord and ***θ-component*** of ***force*** F_θ if the drum has a constant angular rate of rotation of ω_0 as shown.

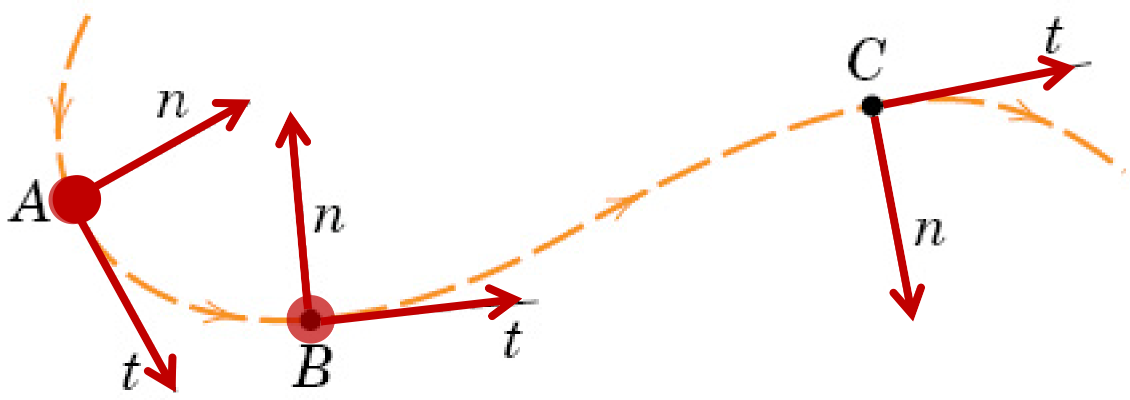
Recall: Possible Coordinate Systems

- Rectangular (x, y, z)
- Polar (r, θ, z)
- Spherical (R, θ, ϕ)
- Normal and Tangential (n, t)



Recall: *n-t* Vector Representation

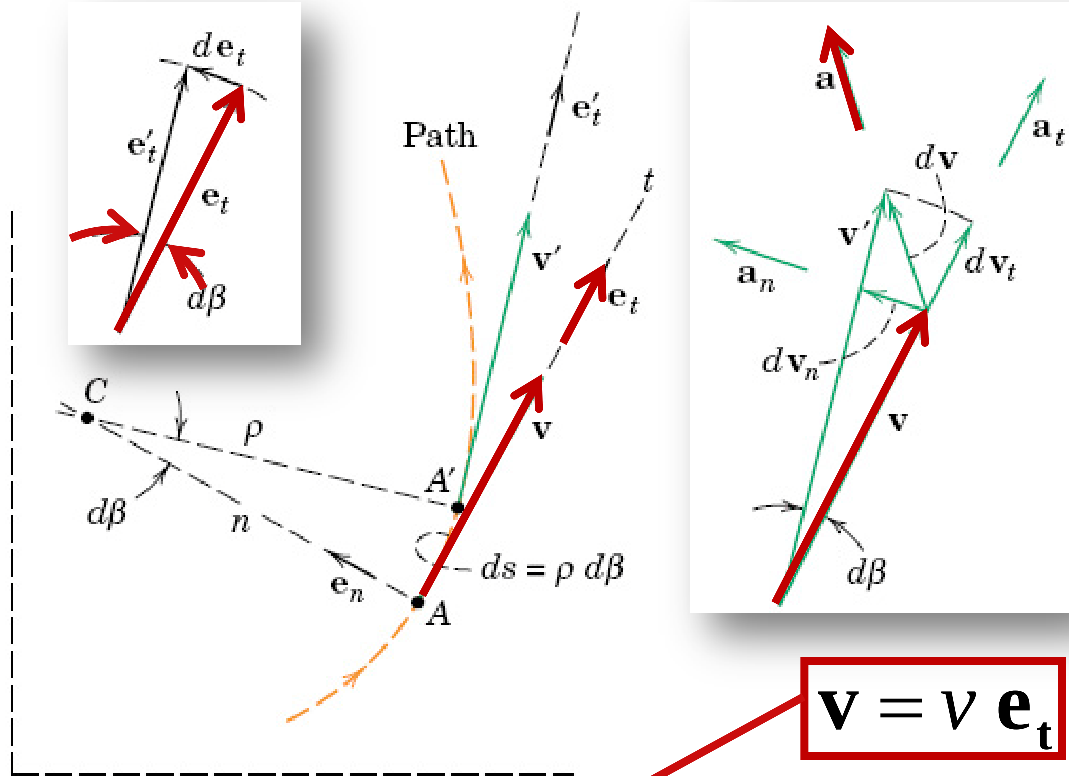
Path variables along the tangent (*t*) and normal (*n*)



- The ***n***- and ***t***-coordinates move along the path with the particle
- ***Tangential*** coordinate is parallel to the ***velocity***
- The positive direction for the ***normal*** coordinate is toward the center of curvature

Recall: *n-t* Acceleration

$$ds = \rho \, d\beta$$



$$\mathbf{v} = \frac{ds}{dt}\mathbf{e_t} = v\,\mathbf{e_t} = \rho\dot{\beta}\,\mathbf{e_t}$$

$$\mathbf{v} = v\,\mathbf{e_t}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d\left(v\,\mathbf{e_t}\right)}{dt} = v\,\dot{\mathbf{e}}_\mathbf{t} + \dot{v}\,\mathbf{e_t}$$

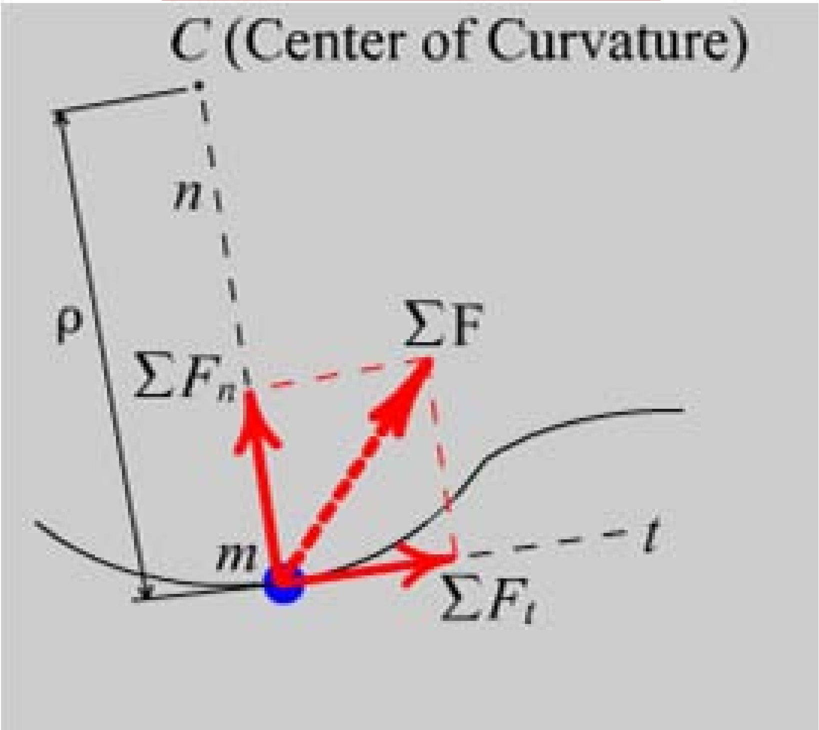
$$\dot{\mathbf{e}}_\mathbf{t} = \frac{d\,\mathbf{e_t}}{dt} = \left(\frac{d\beta}{dt}\right)\mathbf{e_n} = \dot{\beta}\,\mathbf{e_n} = \frac{v}{\rho}\mathbf{e_n}$$

$$\mathbf{a} = \frac{v^2}{\rho}\mathbf{e_n} + \dot{v}\,\mathbf{e_t}$$

Normal and Tangential (n-t) Coordinates

$$\mathbf{a} = \frac{v^2}{\rho}\mathbf{e_n} + \dot{v}\,\mathbf{e_t}$$

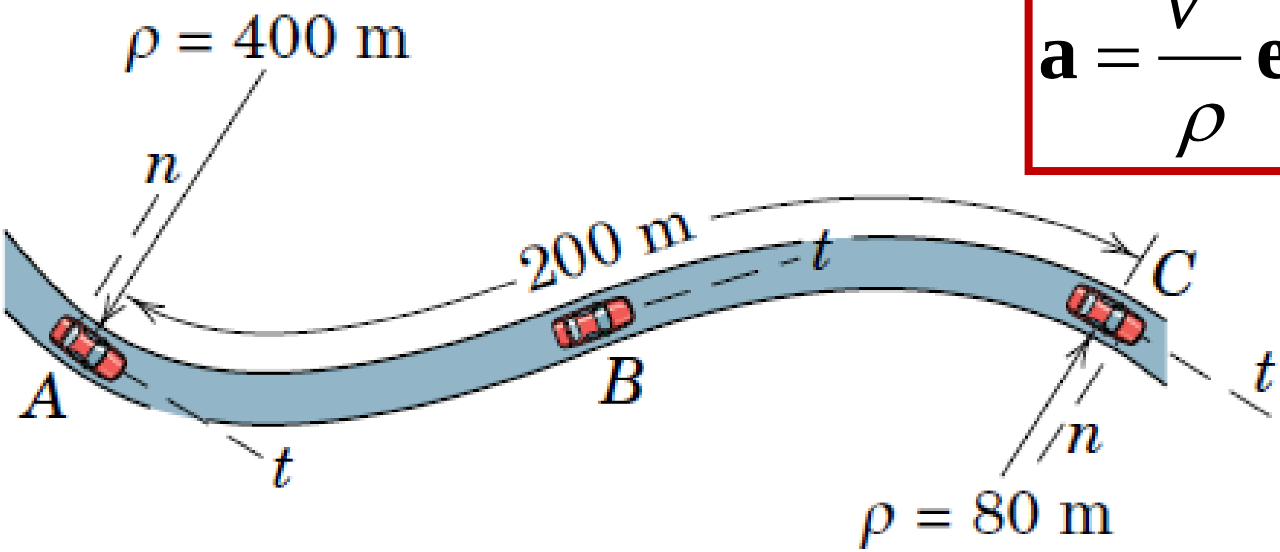
$$\sum \mathbf{F} = m\mathbf{a}$$



vector	***n-coord***	***t-coord***
$\mathbf{a} =$	$a_n\mathbf{e_n} +$	$a_t\mathbf{e_t}$
$\sum \mathbf{F} =$	$\sum F_n\mathbf{e_n} +$	$\sum F_t\mathbf{e_t}$
	$\sum F_n = ma_n$	$\sum F_t = ma_t$

Normal and Tangential (*n*-*t*) Coordinates: Exercise

$$\mathbf{a} = \frac{v^2}{\rho}\mathbf{e_n} + \dot{v}\,\mathbf{e_t}$$



A ***1500-kg*** car enters an s-curve and slows down from ***100 km/h*** at ***A*** to a speed of ***50 km/h*** as it passes ***C***.

Determine the total ***horizontal force*** exerted by the road on the tires at ***positions A***, ***B***, and ***C***.

Outline for Today

- Question of the day
- Rectangular (x-y) coordinates
- Polar (r-θ) coordinates
- Normal and tangential (n-t) coordinates
- Answer your questions!

For Next Time…

- Complete Homework #7 due on Wednesday (10/12) at the ***beginning of class***
- Read Chapter 3, Articles 3/5