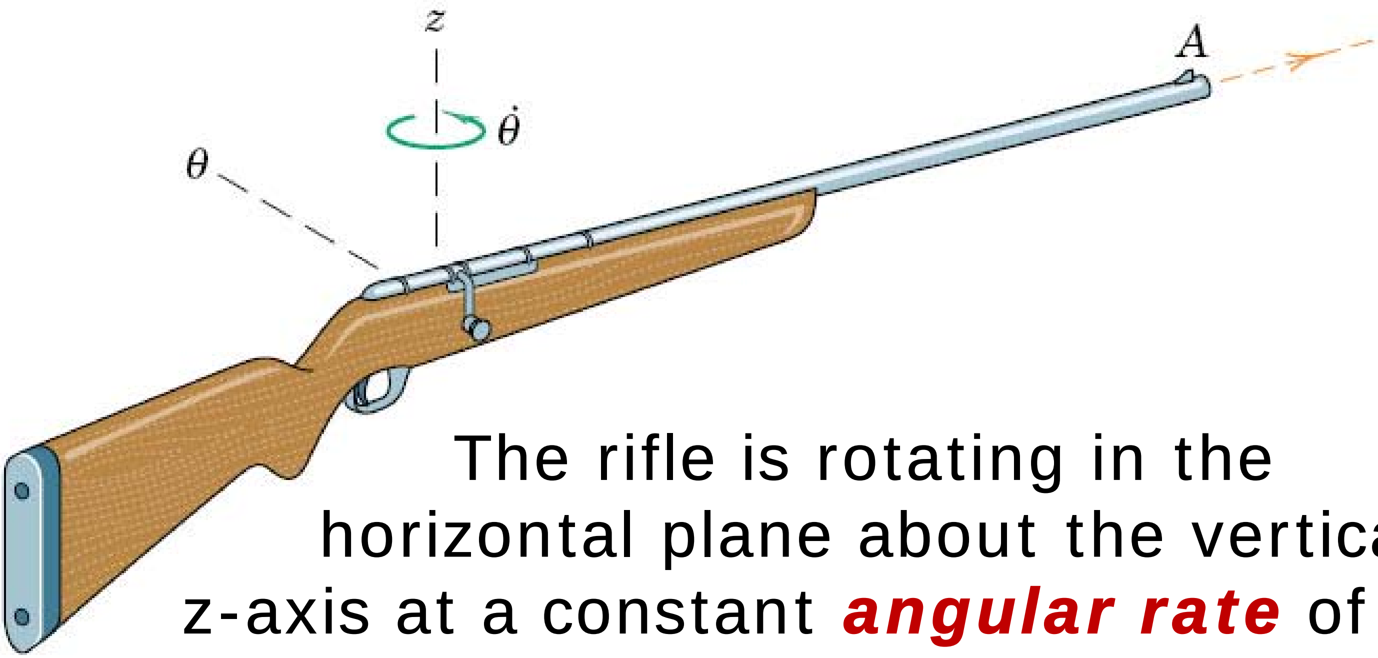
Rigid Body Equations of Motion

Lecture 22

ME 231: Dynamics

Question of the Day



The rifle is rotating in the horizontal plane about the vertical z-axis at a constant ***angular rate*** of ***0.5 rad/s*** when a ***60-g*** bullet is fired and reaches ***point*** A with a ***velocity*** of ***600 m/s*** relative to the barrel.

Determine the ***horizontal force*** exerted by the barrel on the bullet?

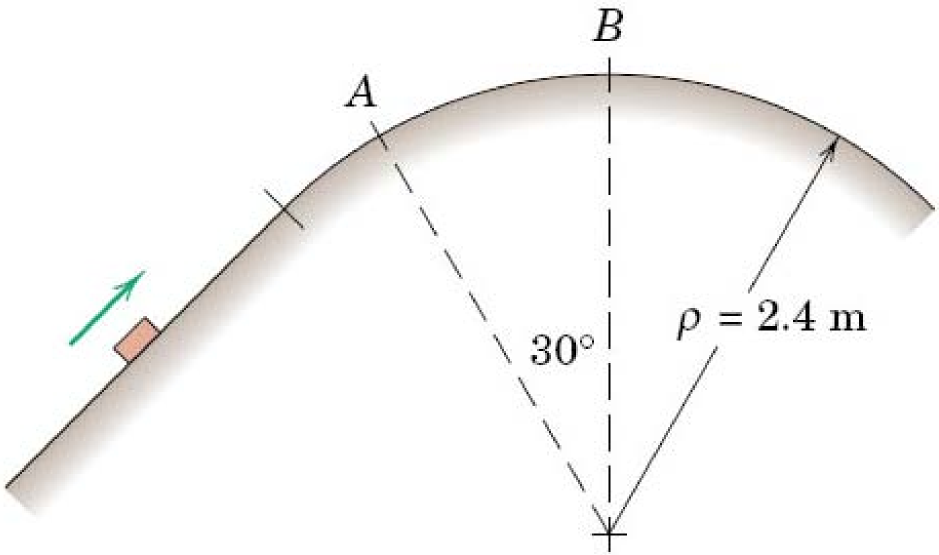
Outline for Today

- Question of the day
- Curvilinear motion exercises
- General equations of motion
- Plane-motion equations
- Answer your questions!

Curvilinear Motion: Exercise 1

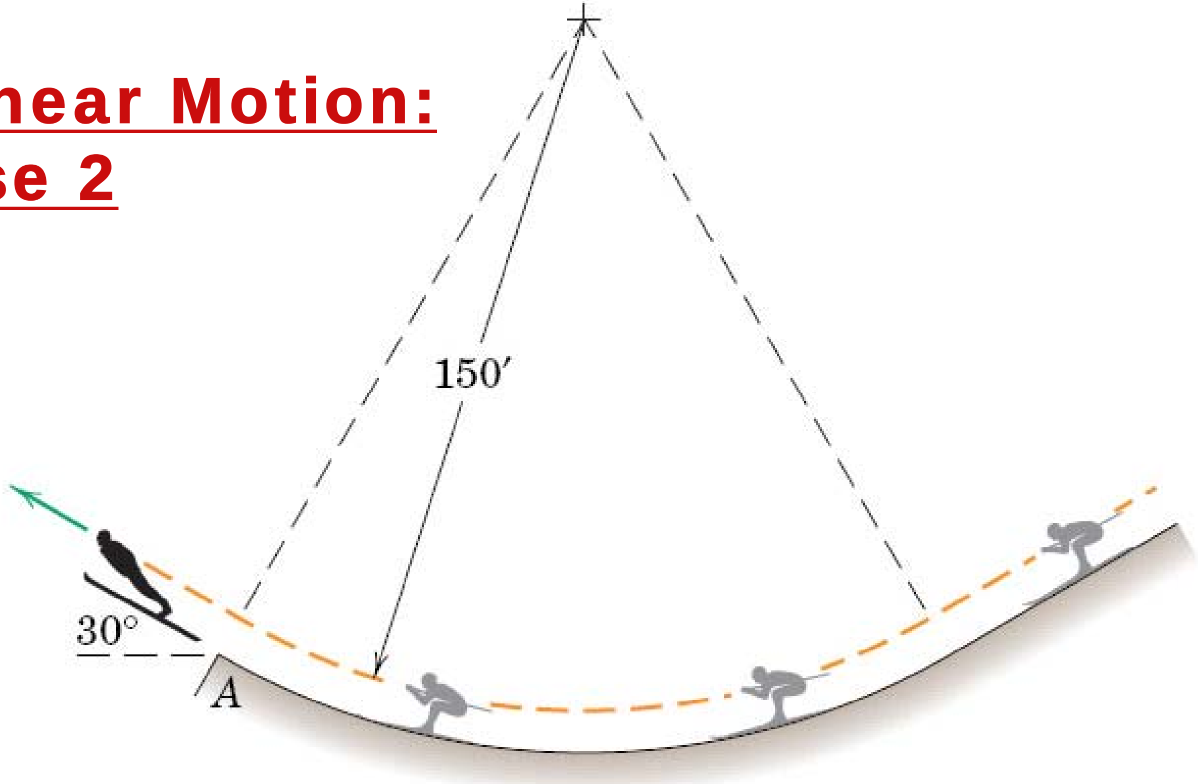
A ***2-kg*** block passes over the ***top*** $\boldsymbol{B}$ with a ***speed*** of ***3.5 m/s***.

Calculate the ***normal force*** $\boldsymbol{N_B}$ exerted by the path on the block.



Determine the ***maximum speed*** $\boldsymbol{v}$ which the block can have at $\boldsymbol{A}$ without losing contact with the path.

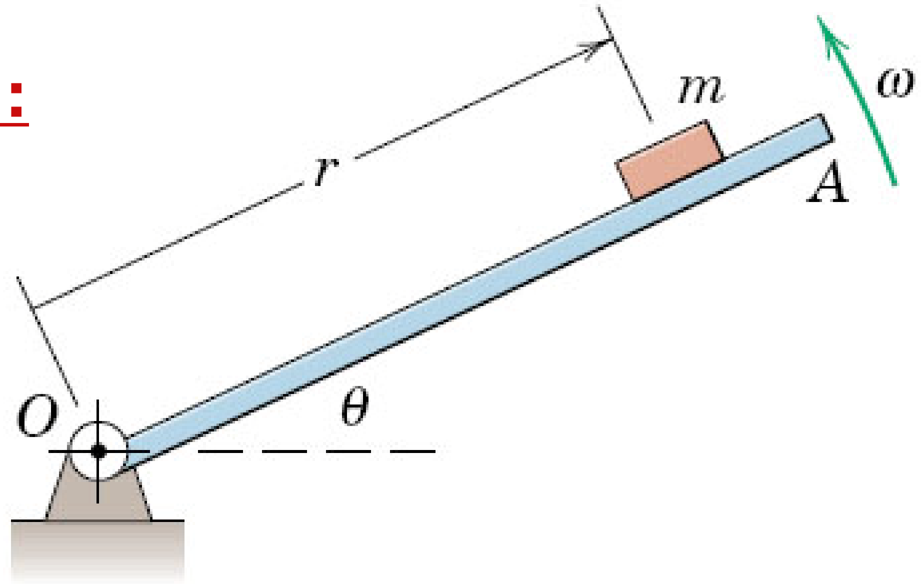
Curvilinear Motion: Exercise 2



A ***180-lb*** skier has a ***speed*** of ***80 ft/s*** as she approaches the takeoff ***position*** A.

Determine the magnitude N of the ***normal force*** exerted by the snow on her skies just before she reaches A.

Curvilinear Motion: Exercise 3



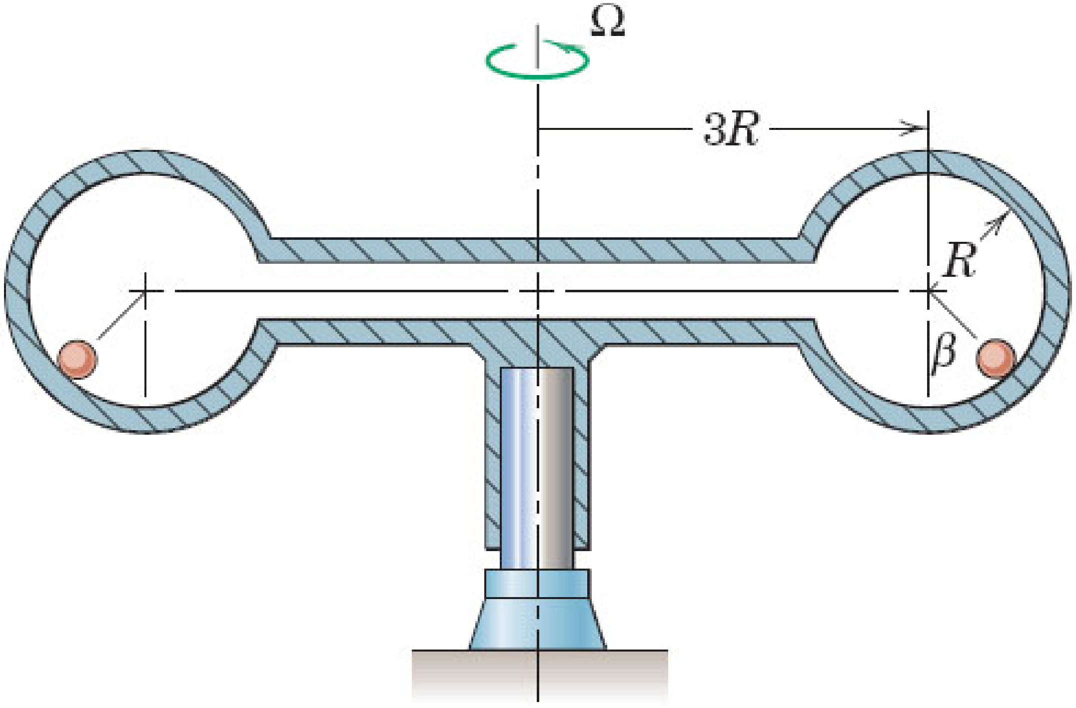
Link OA rotates about a horizontal axis through ***O*** with constant ***angular velocity ω = 3 rad/s***. When ***θ = 0º***, a small block of ***mass m*** is placed on it at a radial ***distance r = 18 in***. When ***θ = 50º***, the block begins to slip.

Determine the ***coefficient of static friction μ_s*** between the block and link.

Curvilinear Motion: Exercise 4

Two small spheres are free to move inside rotating spherical chambers with ***radius R = 200 mm***.

Determine the ***angular velocity Ω*** of the device when the spheres reach a steady-state ***angular position*** of ***β = 45º***.



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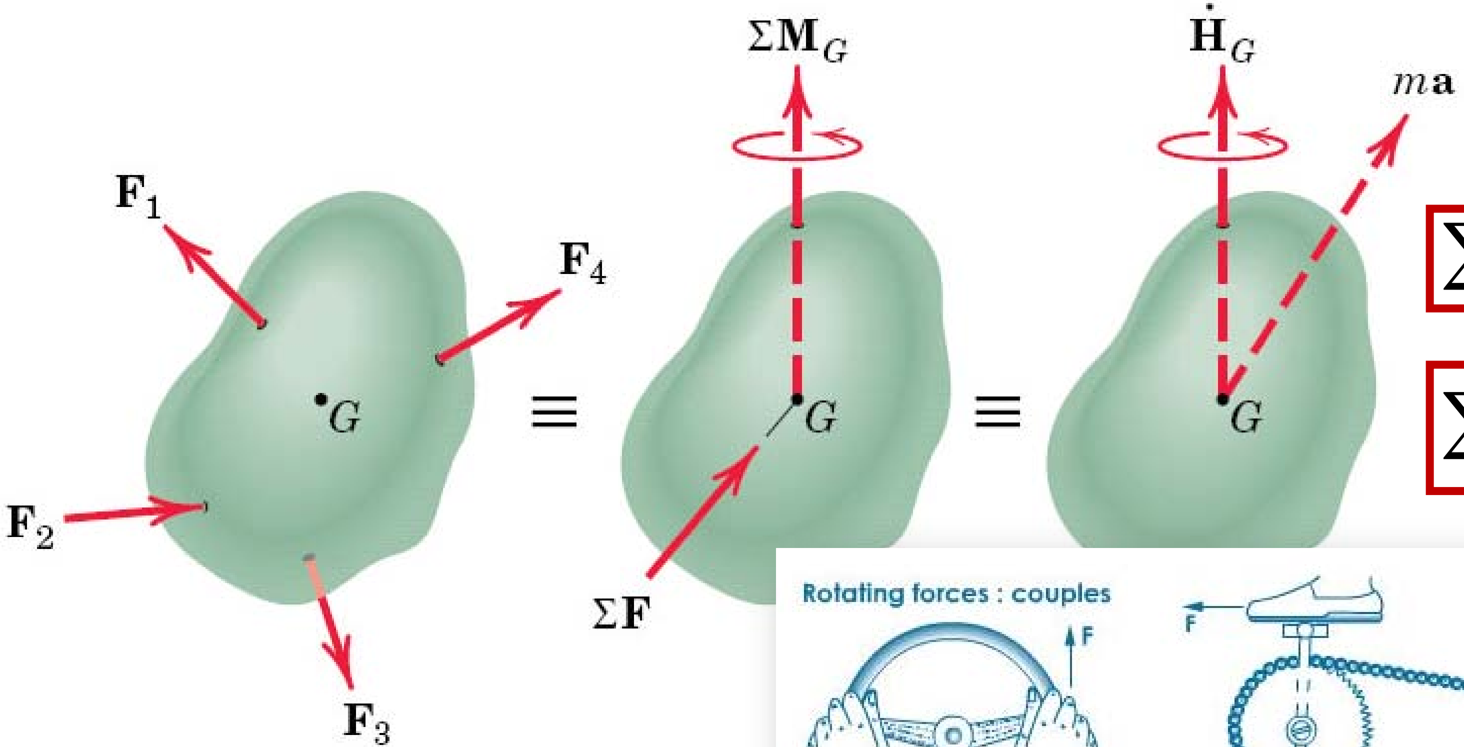
General Equations of Motion

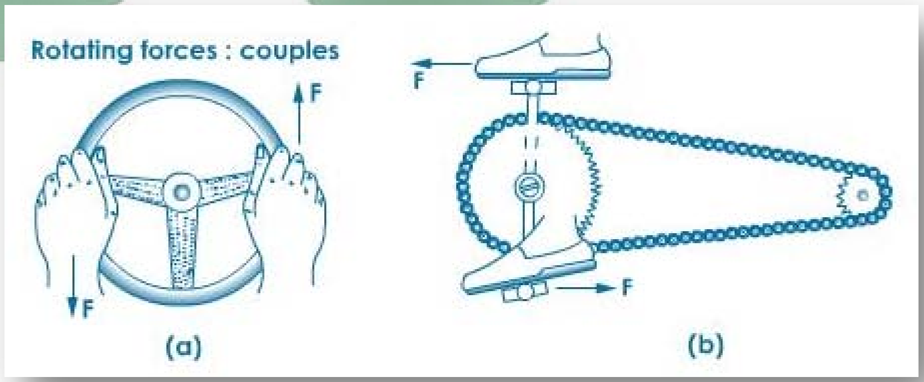
Perhaps the most important concept in dynamics!

free-body diagram ***force-couple diagram*** ***kinetic diagram***

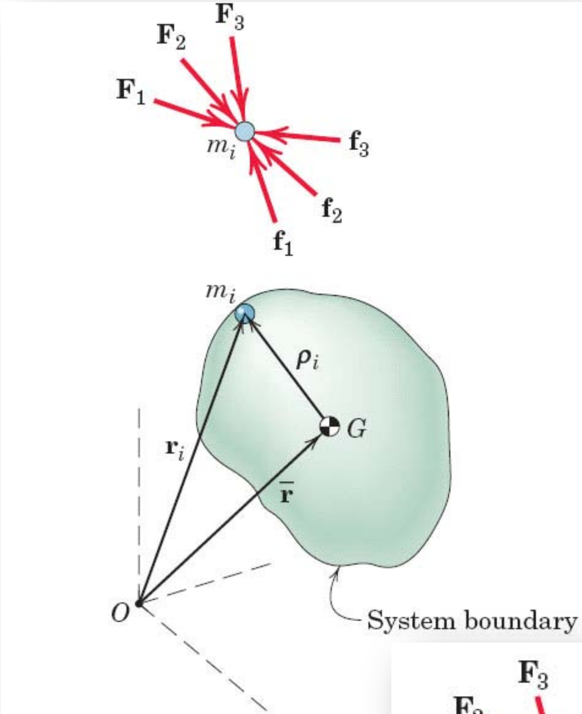


$$\sum \mathbf{F} = m\mathbf{a}$$

$$\sum \mathbf{M}_G = \dot{\mathbf{H}}_G$$

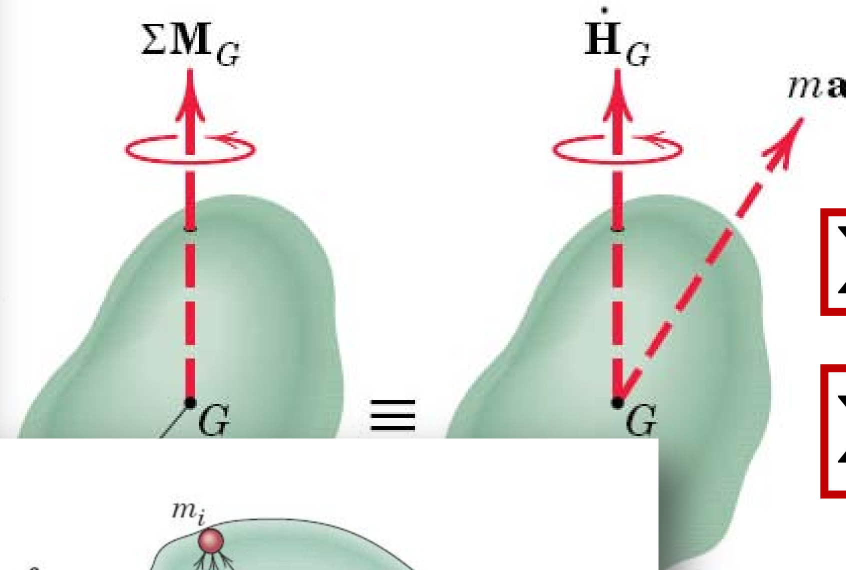


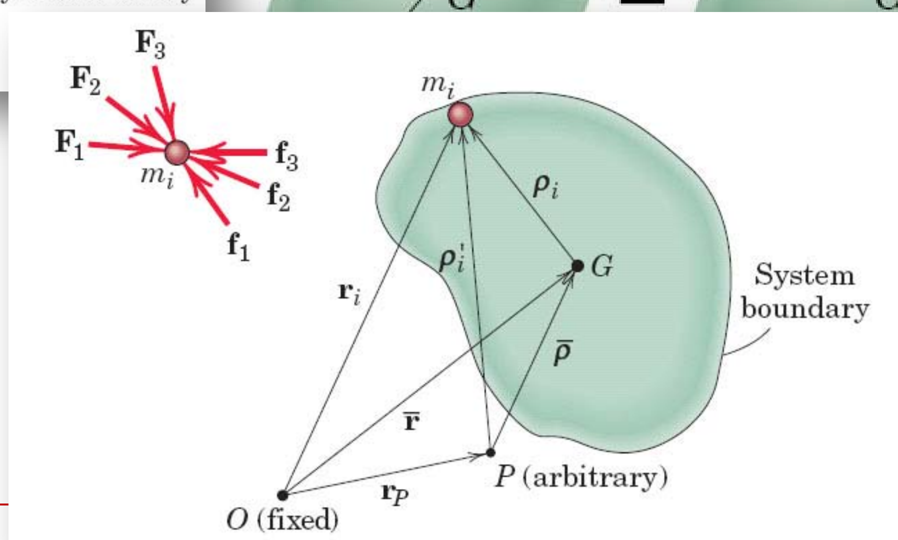
General Equations of Motion: Angular Momentum



force-couple diagram

kinetic diagram





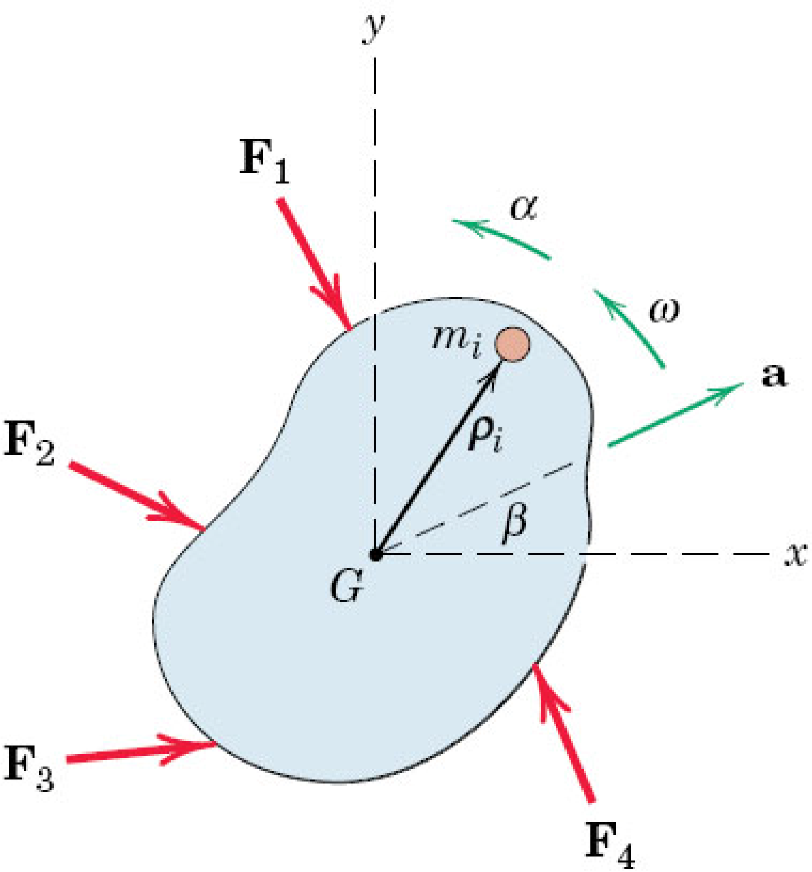
$$\sum \mathbf{F} = m\mathbf{a}$$

$$\sum \mathbf{M}_G = \dot{\mathbf{H}}_G$$

$$\mathbf{H}_G = \sum \boldsymbol{\rho}_i \times m_i \dot{\mathbf{r}}_i$$

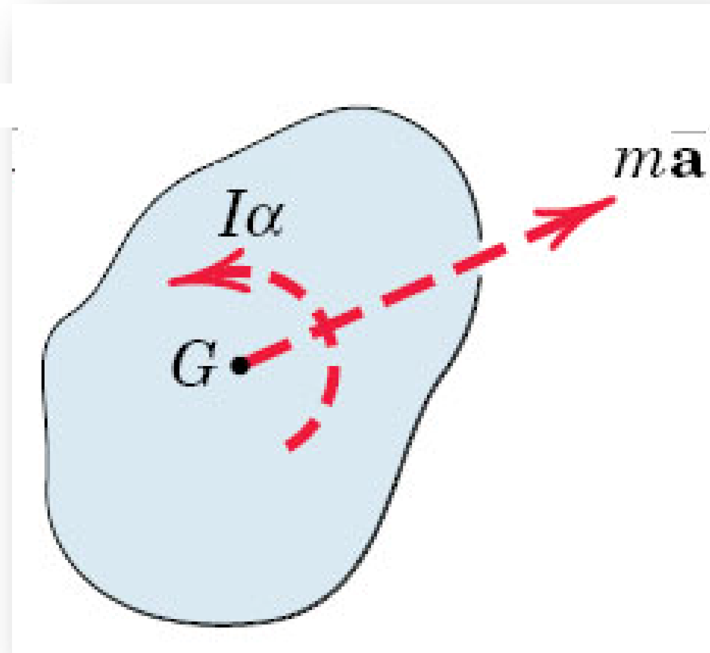
$$\dot{\mathbf{H}}_G = \sum \boldsymbol{\rho}_i \times \mathbf{F}_i$$

Plane-Motion Equations



$$\sum \mathbf{F} = m\mathbf{a}$$

$$\sum \mathbf{M}_G = I_G \boldsymbol{\alpha}$$

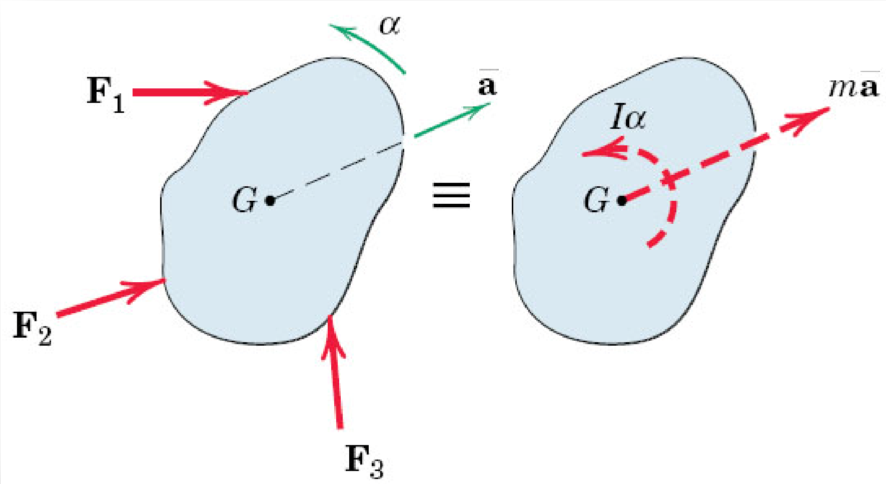


- Rigid body moving in the ***x*-*y* plane**
- Mass center G has an ***acceleration* a**
- Body has an ***angular velocity*** ω and ***angular acceleration*** α

Alternative Moment Equations

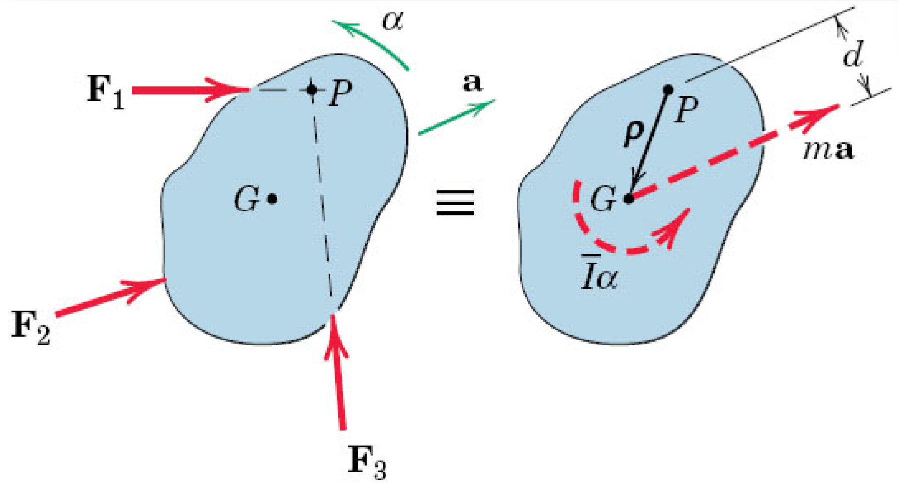
$$\sum \mathbf{F} = m\mathbf{a}$$

Point G is mass center



$$\sum \mathbf{M}_G = I_G \boldsymbol{\alpha}$$

Point P fixed in the body



$$\sum M_P = I_G \alpha + mad$$

$$\sum \mathbf{M}_P = I_P \boldsymbol{\alpha} + \boldsymbol{\rho} \times m\mathbf{a}_P$$

Point O fixed in an inertial reference system

$$\sum \mathbf{M}_O = I_O \boldsymbol{\alpha}$$

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For Next Time…

- Begin Homework #8 due next ***Wednesday (10/19)***
- Read Chapter 6, Articles 6/2 & 6/3