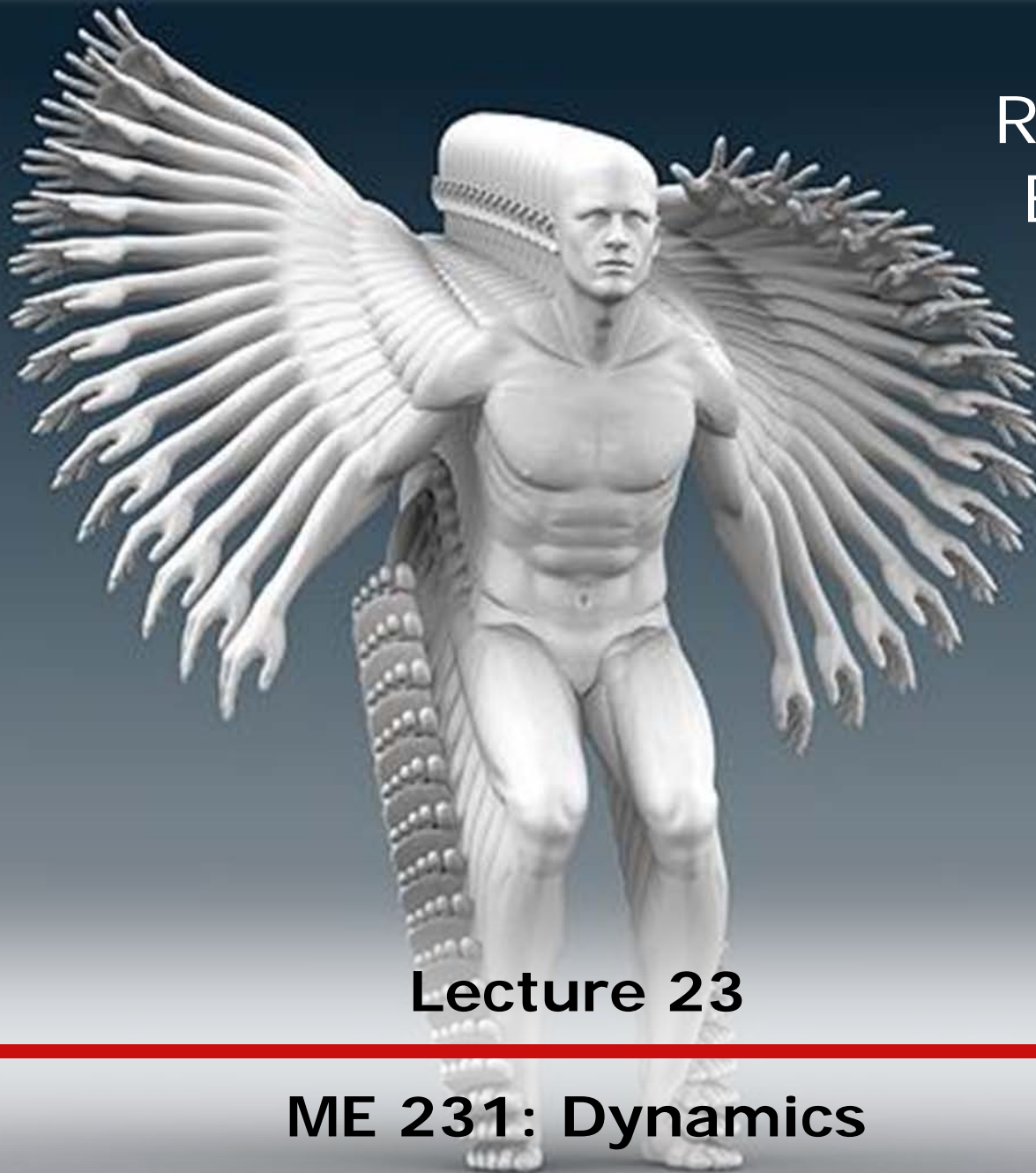


Rigid Body  
Equations  
of Motion



**Lecture 23**

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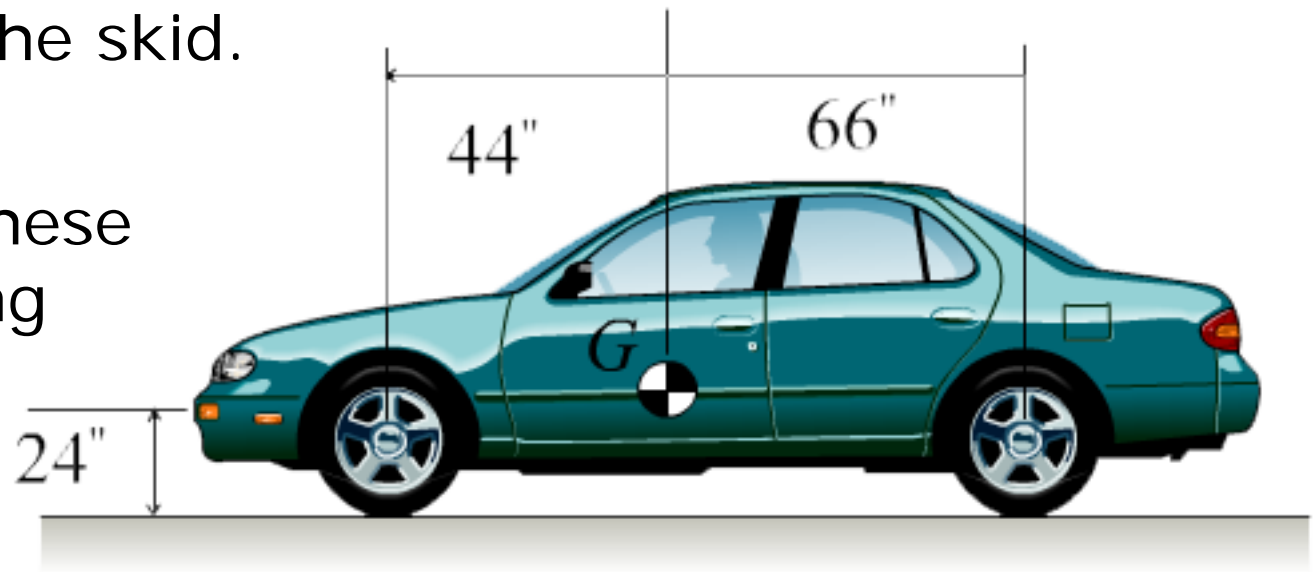
**ME 231: Dynamics**

## Question of the Day

The **3200-lb** front-engine car is traveling forward at a constant velocity when the brakes lock up all four wheels. The coefficient of kinetic friction is 0.8 between the tire and the road.

Determine the **normal force** under each tire just before the skid.

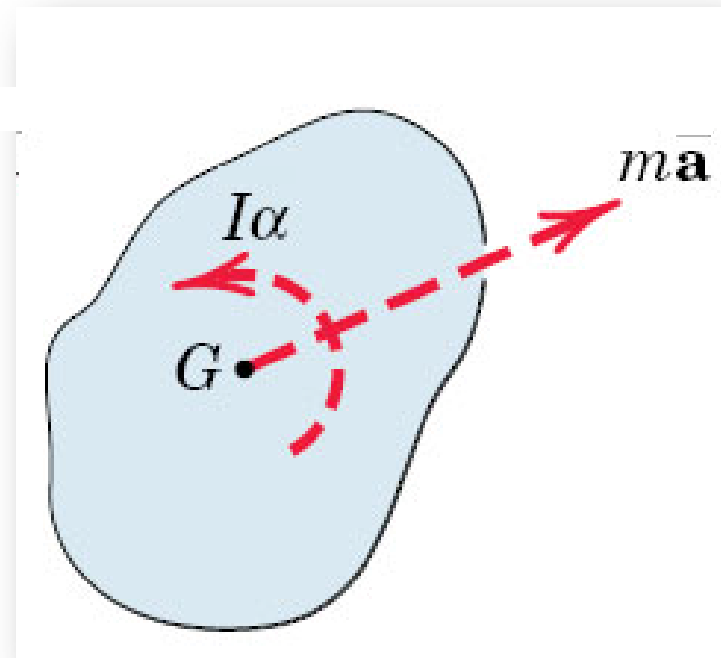
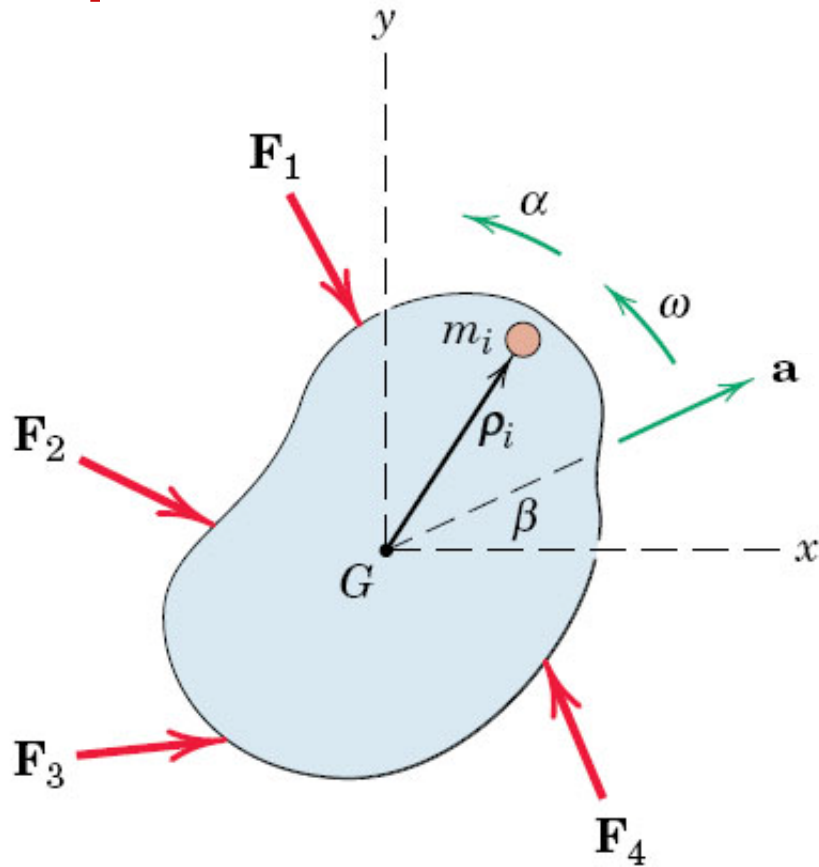
Determine these **forces** during the skid.



## Outline for Today

- Question of the day
- Plane-motion equations (again)
- Unconstrained and constrained motion
- Systems of interconnected bodies
- Step-by-step solution process
- Rigid-body translation
- Answer your questions!

## Recall: Plane-Motion Equations



- Rigid body moving in the  **$x-y$  plane**
- Mass center  $G$  has an **acceleration  $\mathbf{a}$**
- Body has an **angular velocity  $\omega$**  and **angular acceleration  $\alpha$**

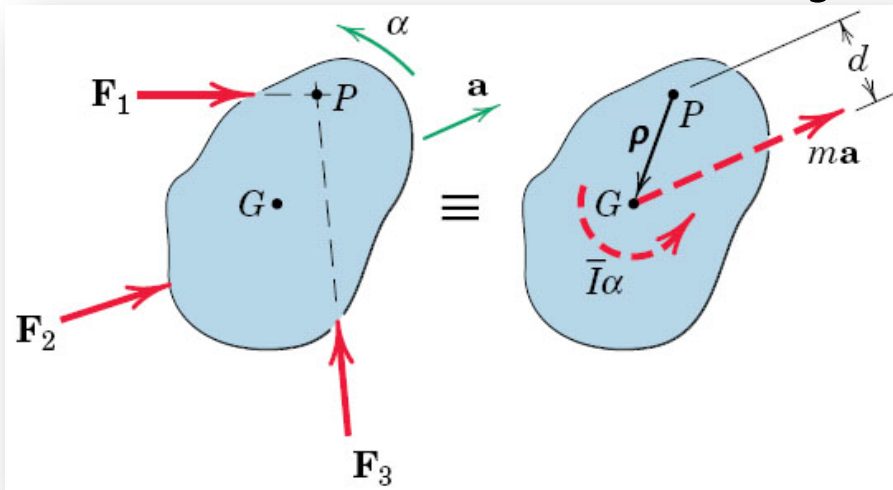
$$\boxed{\sum \mathbf{F} = m\mathbf{a}}$$

$$\boxed{\sum \mathbf{M}_G = I_G \boldsymbol{\alpha}}$$

## Recall: Alternative Moment Equations

$$\Sigma \mathbf{F} = m\mathbf{a}$$

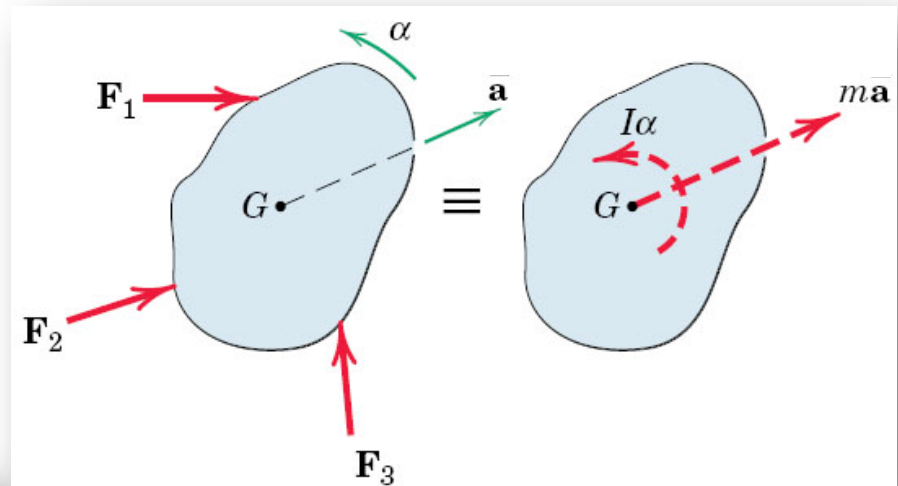
Point  $\mathbf{P}$  fixed in the body



$$\Sigma M_P = I_G \alpha + mad$$

$$\Sigma \mathbf{M}_P = I_P \boldsymbol{\alpha} + \boldsymbol{\rho} \times m\mathbf{a}_P$$

Point  $\mathbf{G}$  is mass center



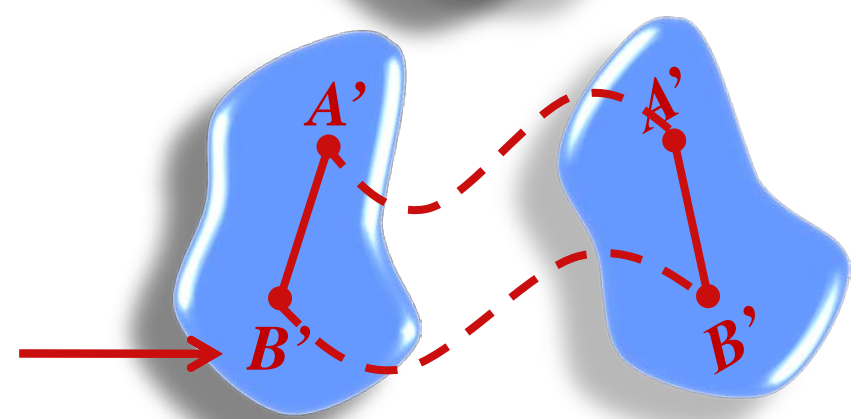
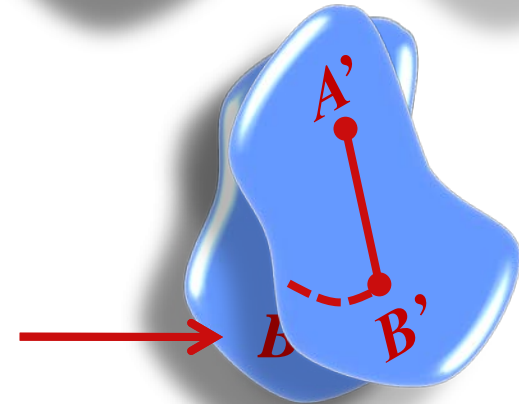
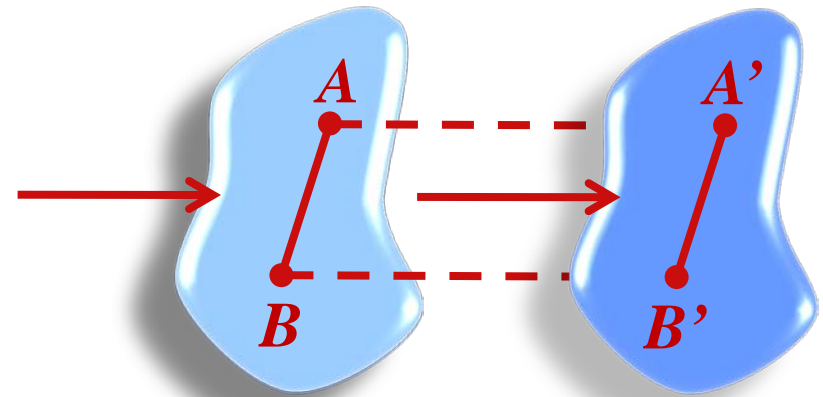
$$\Sigma \mathbf{M}_G = I_G \boldsymbol{\alpha}$$

Point  $\mathbf{O}$  fixed in an  
inertial reference  
system

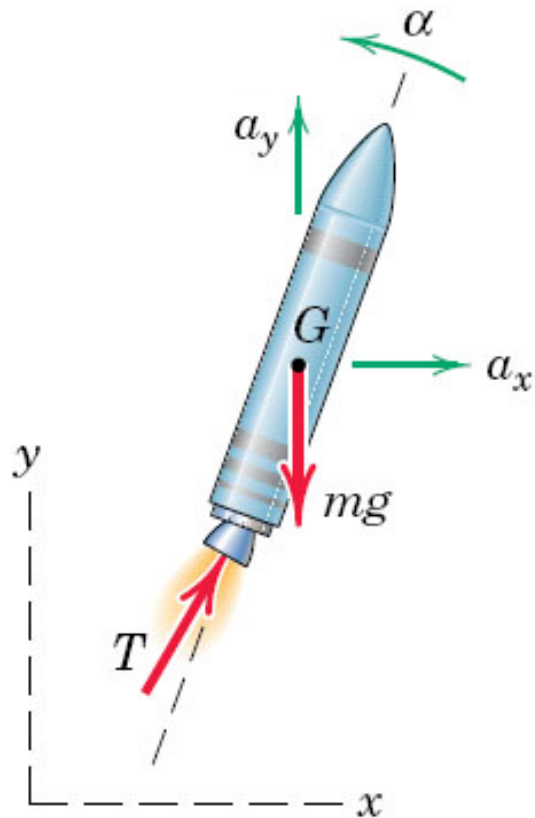
$$\Sigma \mathbf{M}_O = I_O \boldsymbol{\alpha}$$

## Plane Motion Types

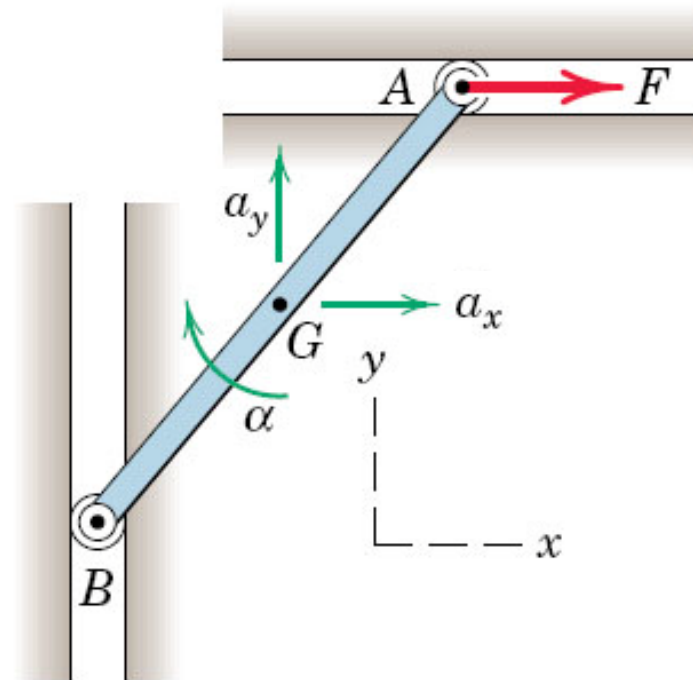
- Translation
- Fixed-axis rotation
- General plane motion



# Unconstrained and Constrained Motion



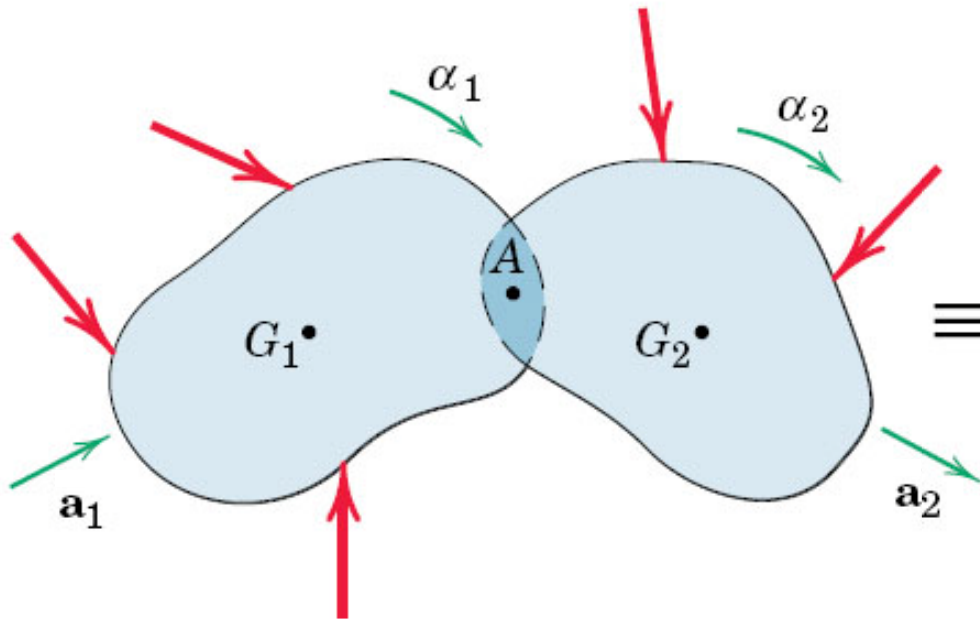
**Unconstrained:**  $a_x$ ,  $a_y$ , and  $\alpha$  may be determined independently from force/moment equations



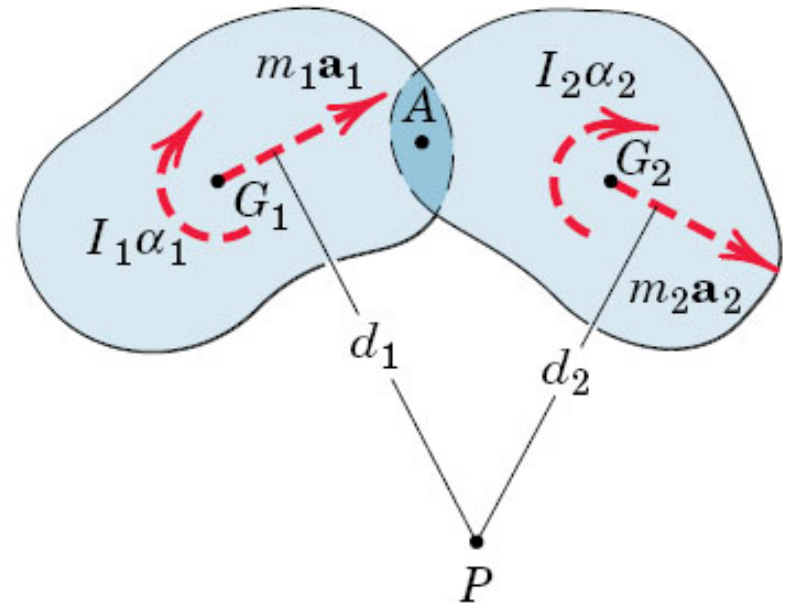
**Constrained:**  $a_x$ ,  $a_y$ , and  $\alpha$  kinematic relationships may be determined and then combined with force/moment equations

# Systems of Interconnected Bodies

free-body diagram



kinetic diagram



$$\Sigma \mathbf{F} = \Sigma m\mathbf{a}$$

$$\Sigma M_P = \Sigma I_G\alpha + \Sigma mad$$



## Outline for Today

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# Step-by-Step Solution Process

## 1. *Kinematics*

- Identify type of *motion*
- Solve for *linear* and *angular accelerations*

## 2. *Diagram*

- Assign *inertial coordinate system*
- Draw complete *free-body diagram*
- Draw *kinetic diagram* to clarify equations

## 3. *Equations of motion*

- Apply *2 linear* and *1 angular equations*
- Maintain *consistent sense*
- Solve for no more than 5 scalar unknowns (*3 scalar equations of motion* and *2 scalar relations* from the *relative-acceleration equation*)

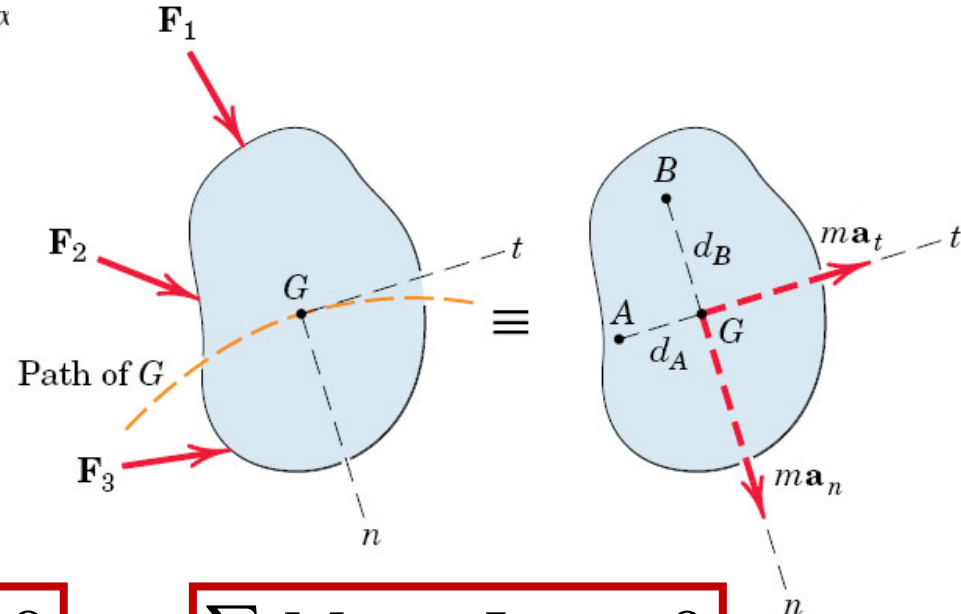
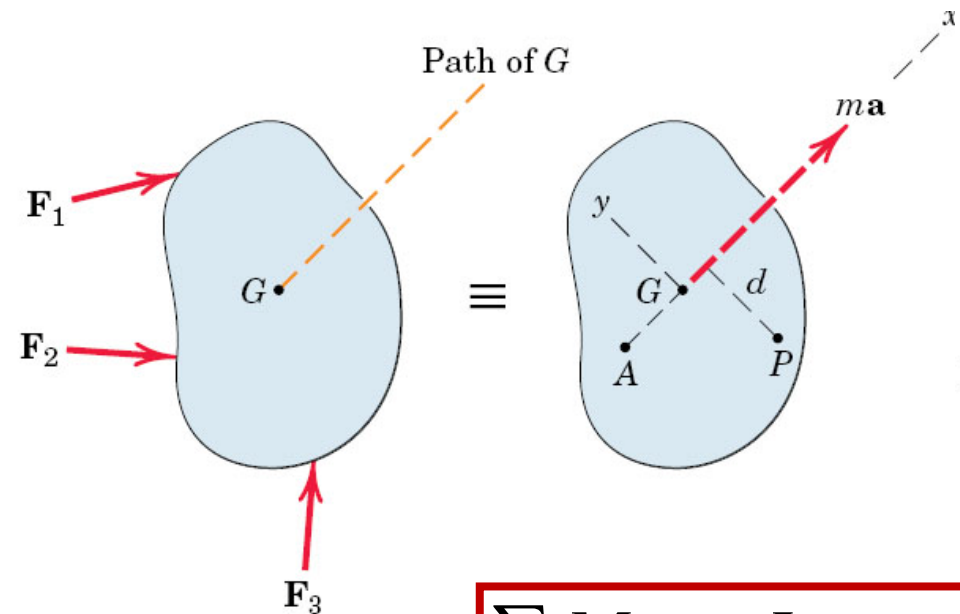
## Outline for Today

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# Rigid-Body Translation

rectilinear

curvilinear



$$\sum \mathbf{F} = m\mathbf{a}$$

$$\alpha = 0$$

$$\omega = 0$$

$$\sum M_G = I_G \alpha = 0$$

$$\sum M_P = mad$$

$$\sum M_A = 0$$

$$\sum M_G = I_G \alpha = 0$$

$$\sum M_A = ma_n d_A$$

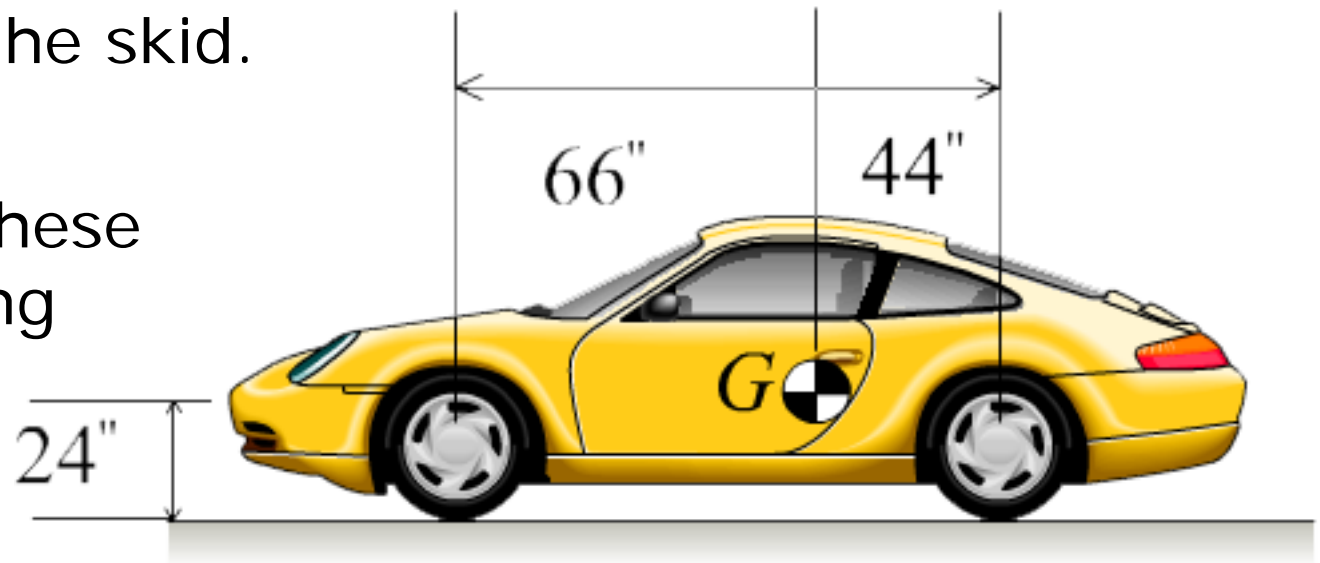
$$\sum M_B = ma_t d_B$$

## Rigid-Body Translation: Exercise

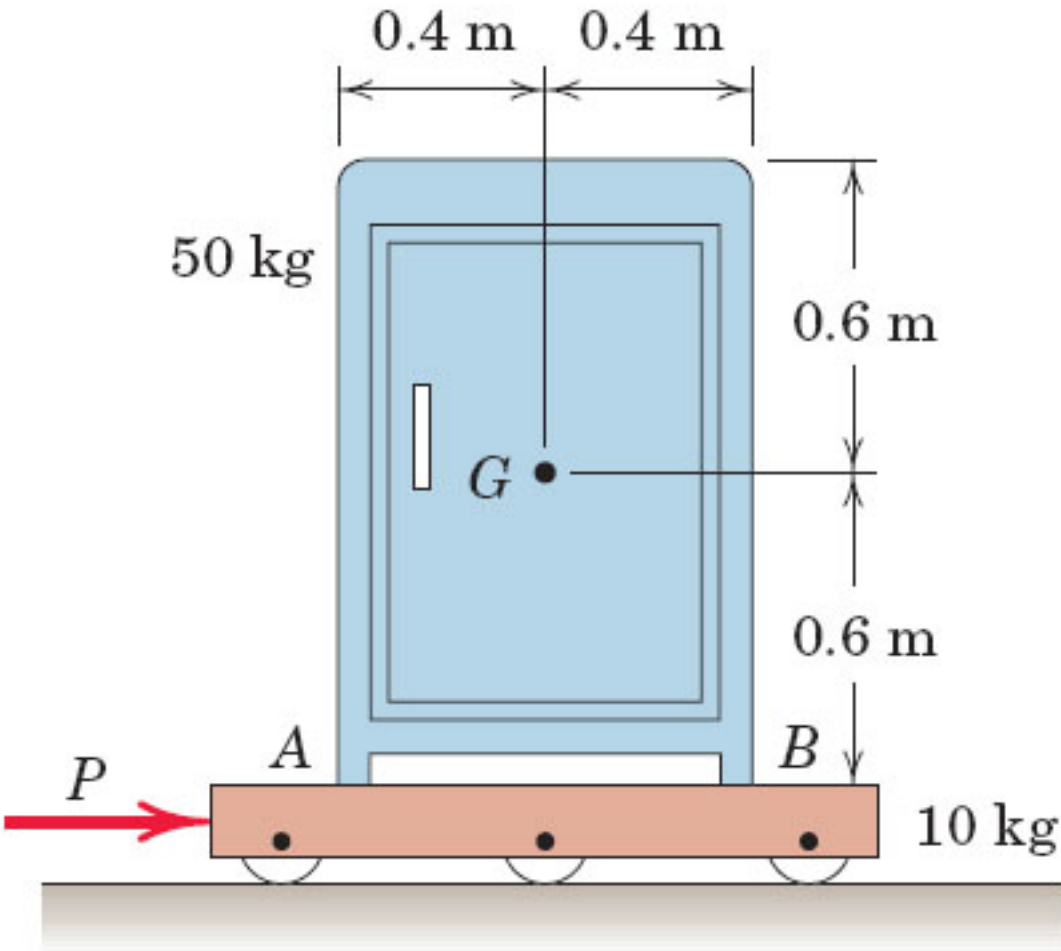
The **3200-lb** rear-engine car is traveling forward at a constant velocity when the brakes lock up all four wheels. The coefficient of kinetic friction is 0.8 between the tire and the road.

Determine the **normal force** under each tire just before the skid.

Determine these **forces** during the skid.



## Rigid-Body Translation: Another Exercise



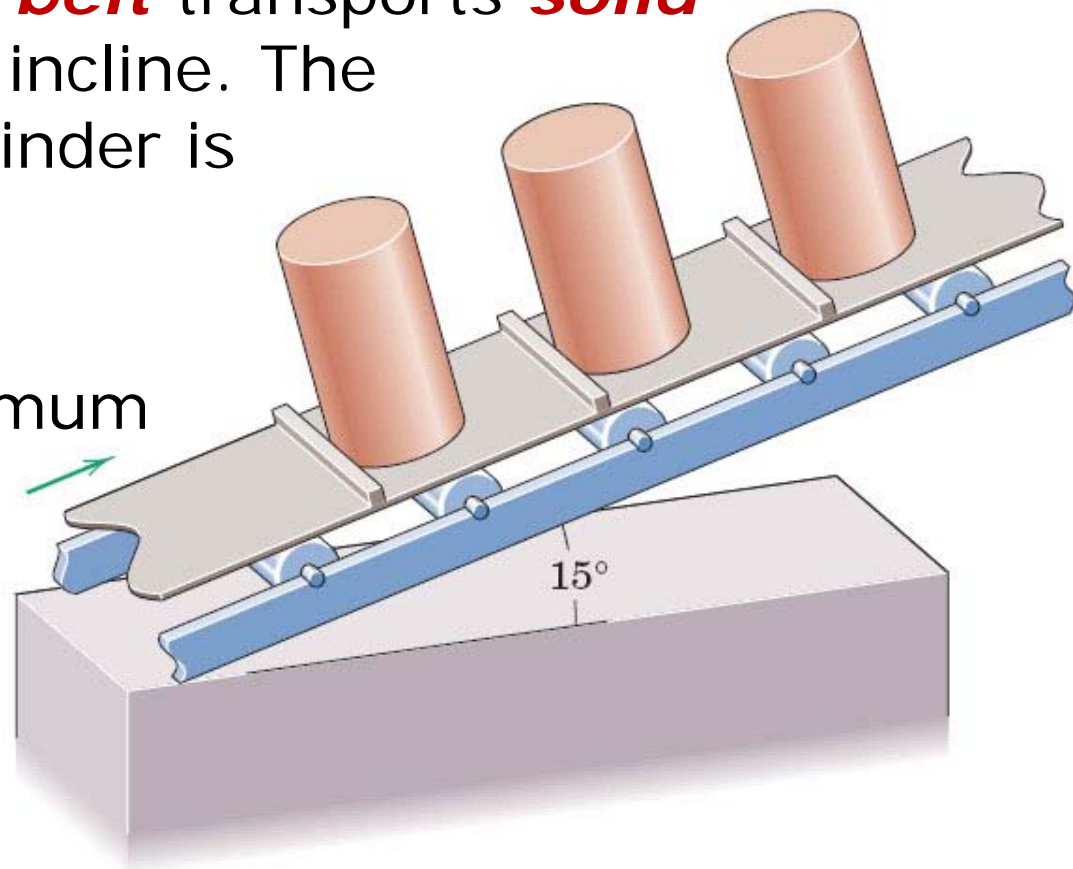
Determine the value of the **force  $P$**  which would cause the cabinet to begin to tip.

What **coefficient of static friction** is necessary to ensure tipping occurs without slipping?

## Rigid-Body Translation: Yet Another Exercise

A cleated **conveyor belt** transports **solid cylinders** up a  $15^\circ$  incline. The diameter of each cylinder is half its height.

Determine the maximum **acceleration** for the **belt** without tipping the **cylinders** as it starts.



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## For Next Time...

- Continue Homework #8 due next ***Wednesday (10/19)***
- Read Chapter 6, Article 6/4