

General Plane Motion



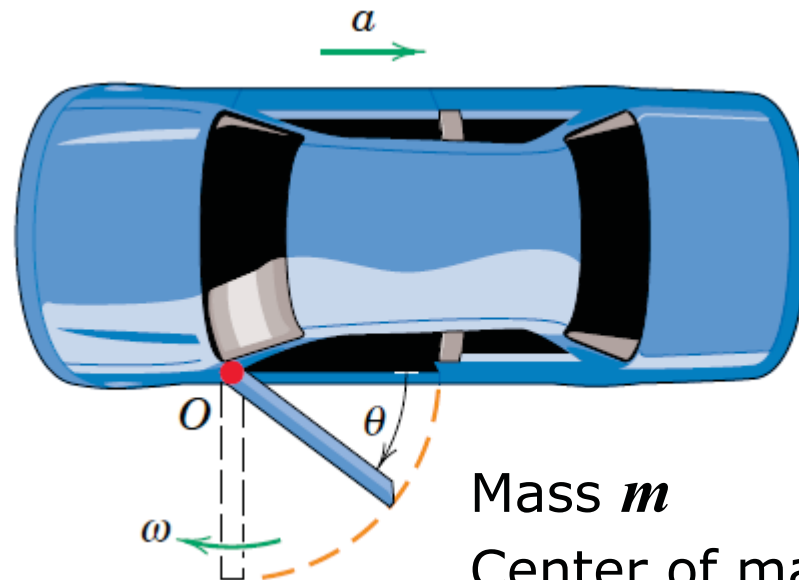
Lecture 25

ME 231: Dynamics

Question of the Day

The door of a moving car is inadvertently left slightly open. The brakes are applied to give the car a constant rearward **acceleration**.

Determine expressions for the **angular velocity** of the door and components of the hinge **reaction forces**.



Mass m

Center of mass r

Radius of gyration k_O

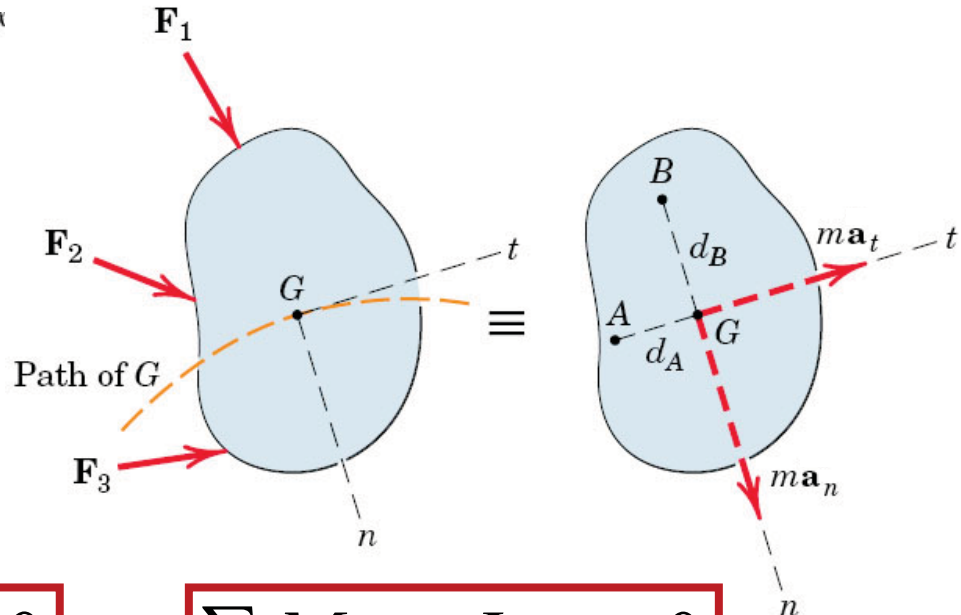
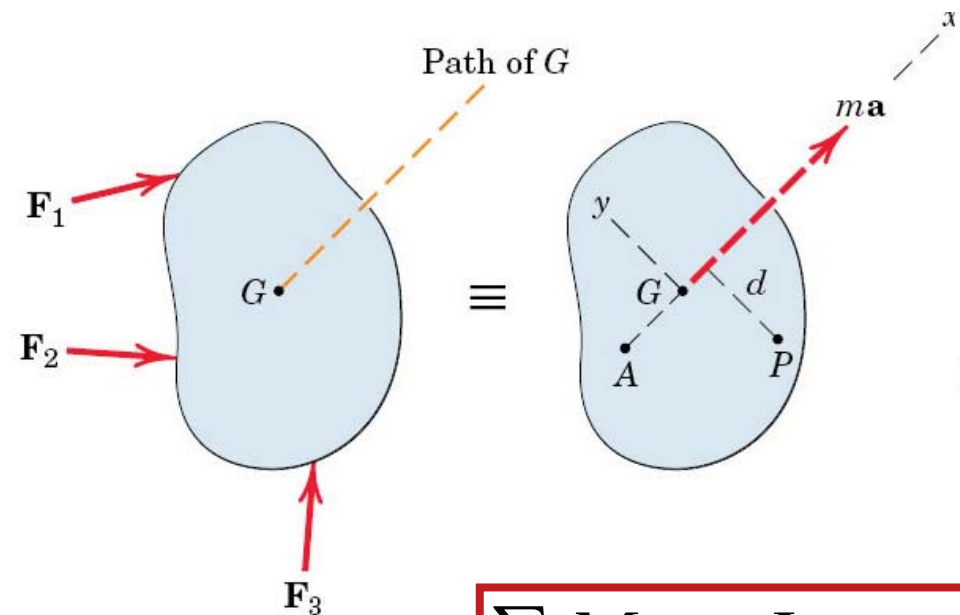
Outline for Today

- Question of the day
- Combined translation and rotation
- Answer your questions!

Recall: Rigid-Body Translation

rectilinear

curvilinear



$$\sum \mathbf{F} = m\mathbf{a}$$

$$\alpha = 0$$

$$\omega = 0$$

$$\sum M_G = I_G \alpha = 0$$

$$\sum M_P = mad$$

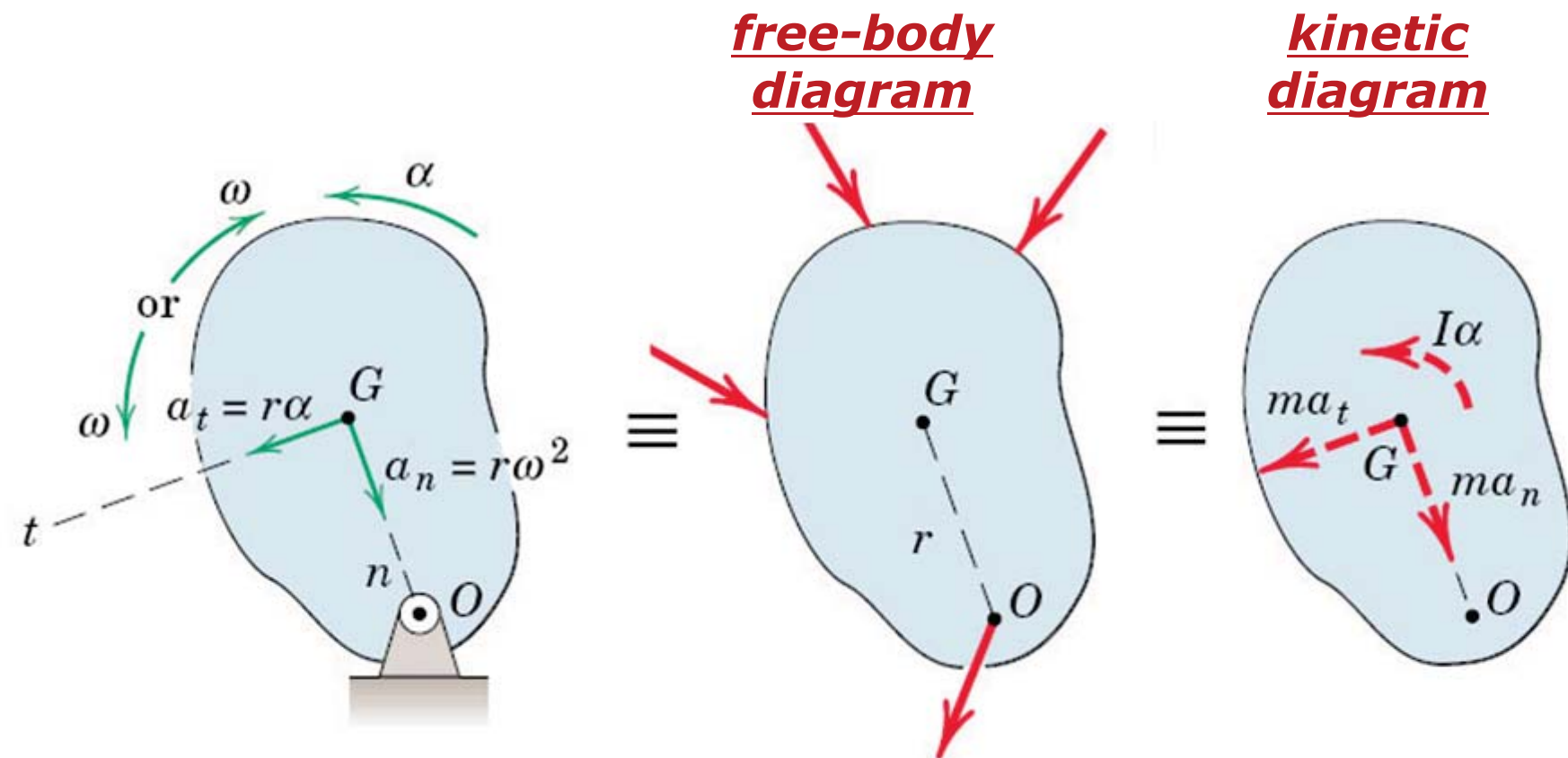
$$\sum M_A = 0$$

$$\sum M_G = I_G \alpha = 0$$

$$\sum M_A = ma_n d_A$$

$$\sum M_B = ma_t d_B$$

Recall: Fixed-Axis Rotation



- Mass center's circular motion easily expressed in n - t coordinates
- Plane-motion equations:

$$\sum \mathbf{F} = m\mathbf{a}$$

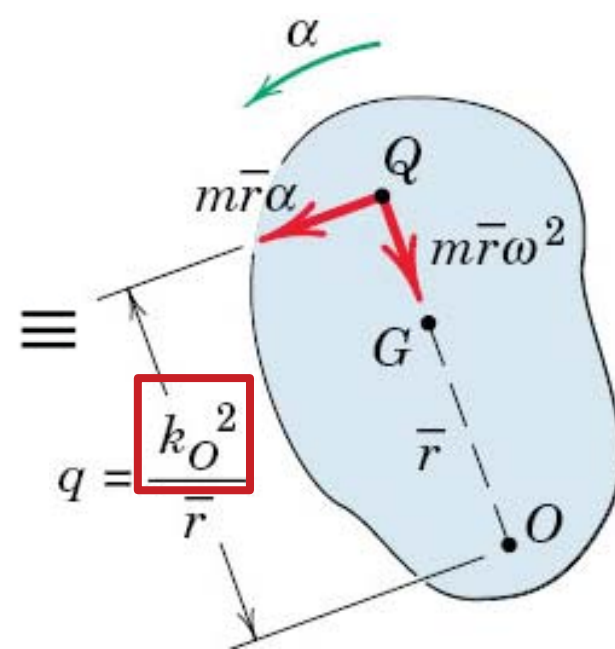
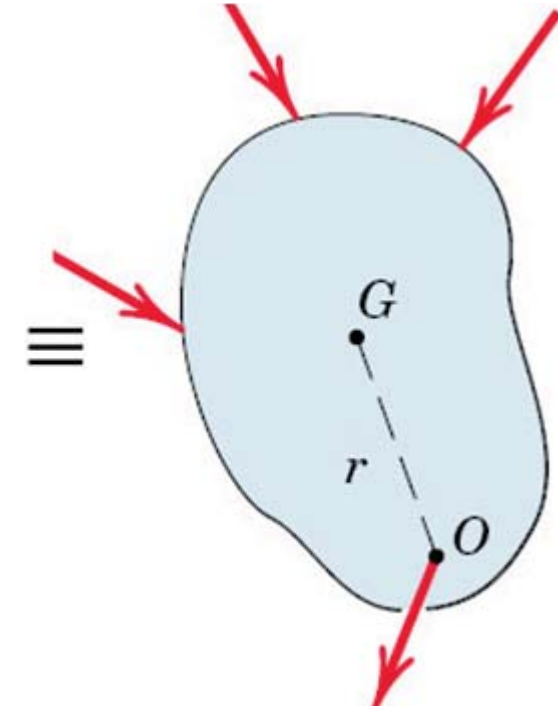
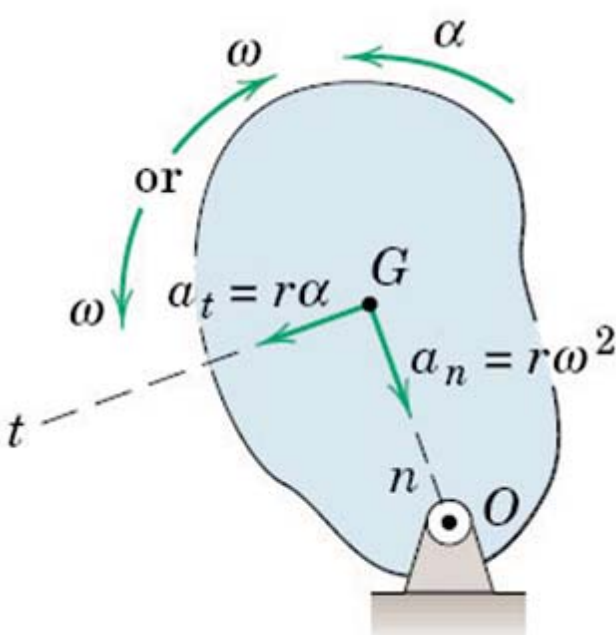
$$\sum \mathbf{M}_G = I_G\boldsymbol{\alpha}$$

$$\sum \mathbf{M}_O = I_O\boldsymbol{\alpha}$$

Recall: Center of Percussion

free-body diagram

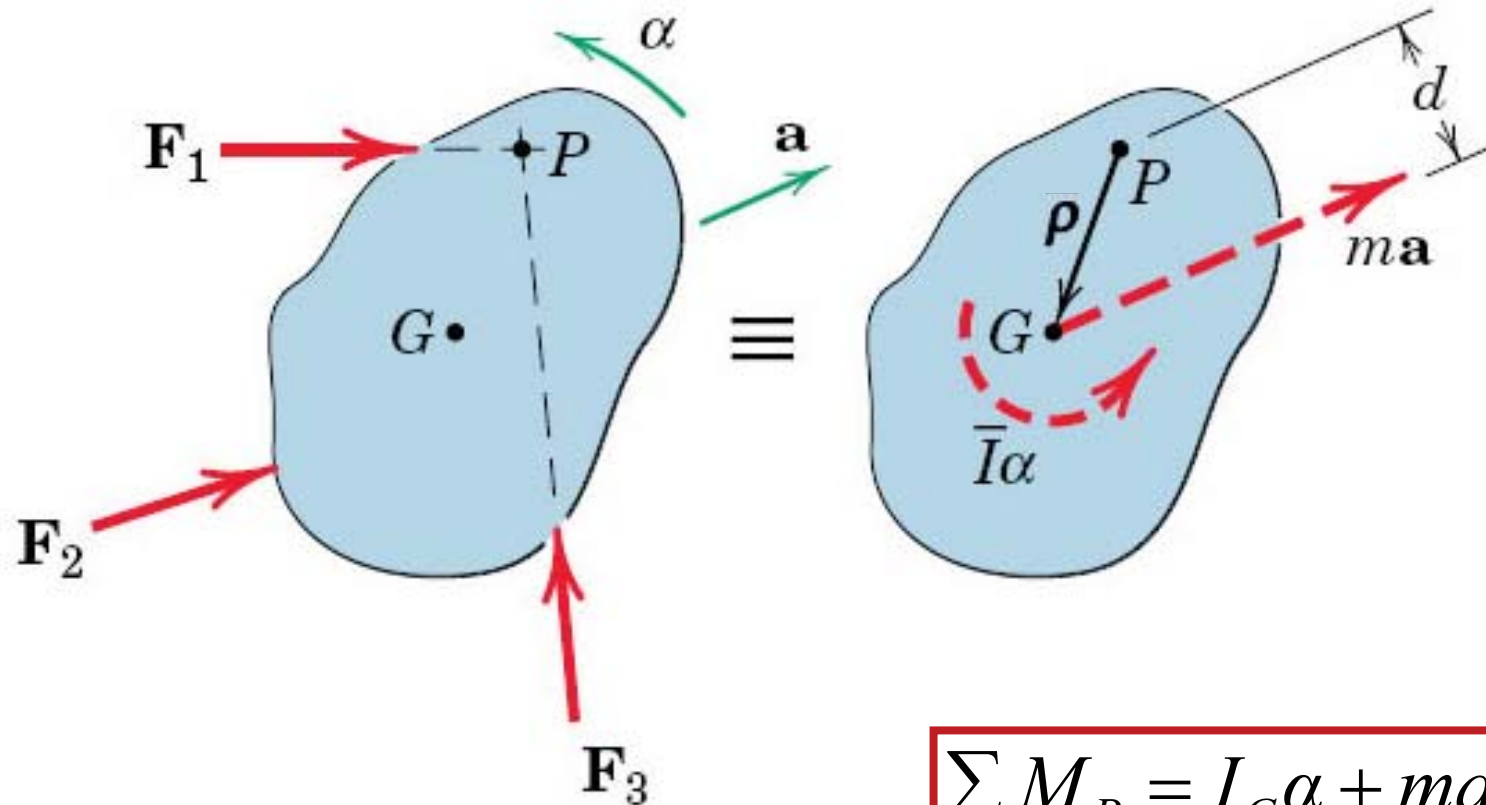
kinetic diagram



$$mr\alpha q = I_G\alpha + mr\alpha(r) \quad \boxed{\sum M_Q = 0}$$

combine ma_t by moving ma_t to point Q (center of percussion)
 $I_G\alpha$

Combined Translation and Rotation



$$\sum \mathbf{F} = m\mathbf{a}$$

$$\sum \mathbf{M}_G = I_G \boldsymbol{\alpha}$$

$$\sum M_P = I_G \alpha + mad$$

$$\sum \mathbf{M}_P = I_P \boldsymbol{\alpha} + \boldsymbol{\rho} \times m\mathbf{a}_P$$

General Plane Motion: Exercise

In a study of head injury during a crash where lap belts are used, the segmented human model is analyzed. The **hip joint O** remains fixed relative to the car, and the **torso** is treated as a rigid body of **mass m** pivoted at **O** .

$$m = 50 \text{ kg}$$

$$\bar{r} = 450 \text{ mm}$$

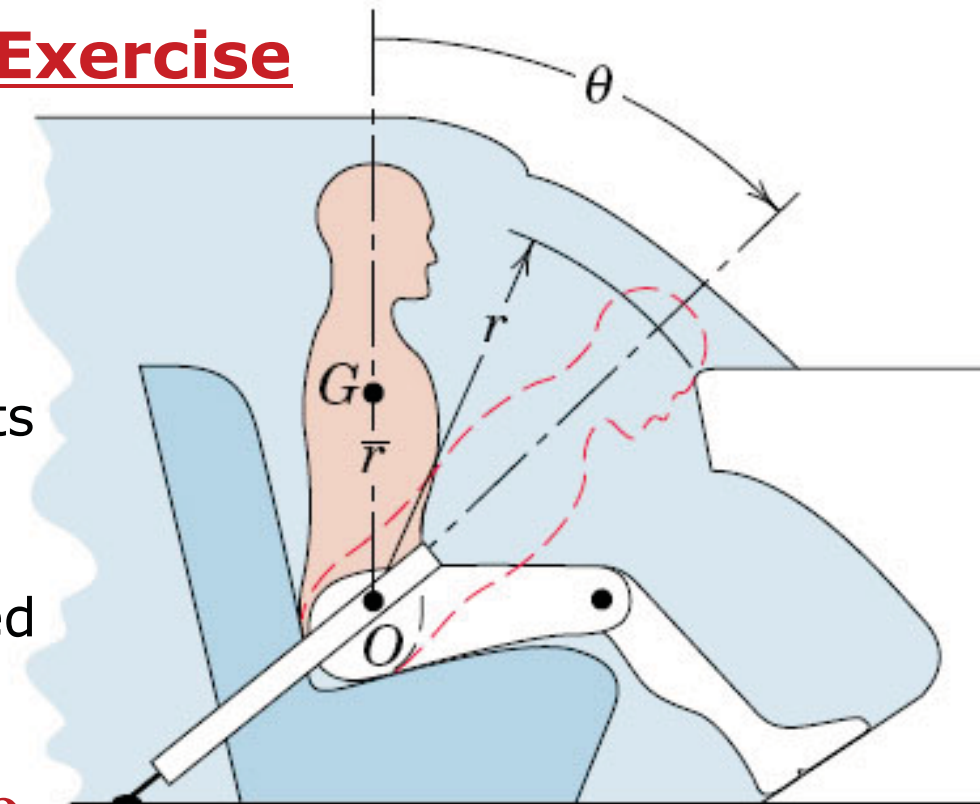
$$r = 800 \text{ mm}$$

$$k_O = 550 \text{ mm}$$

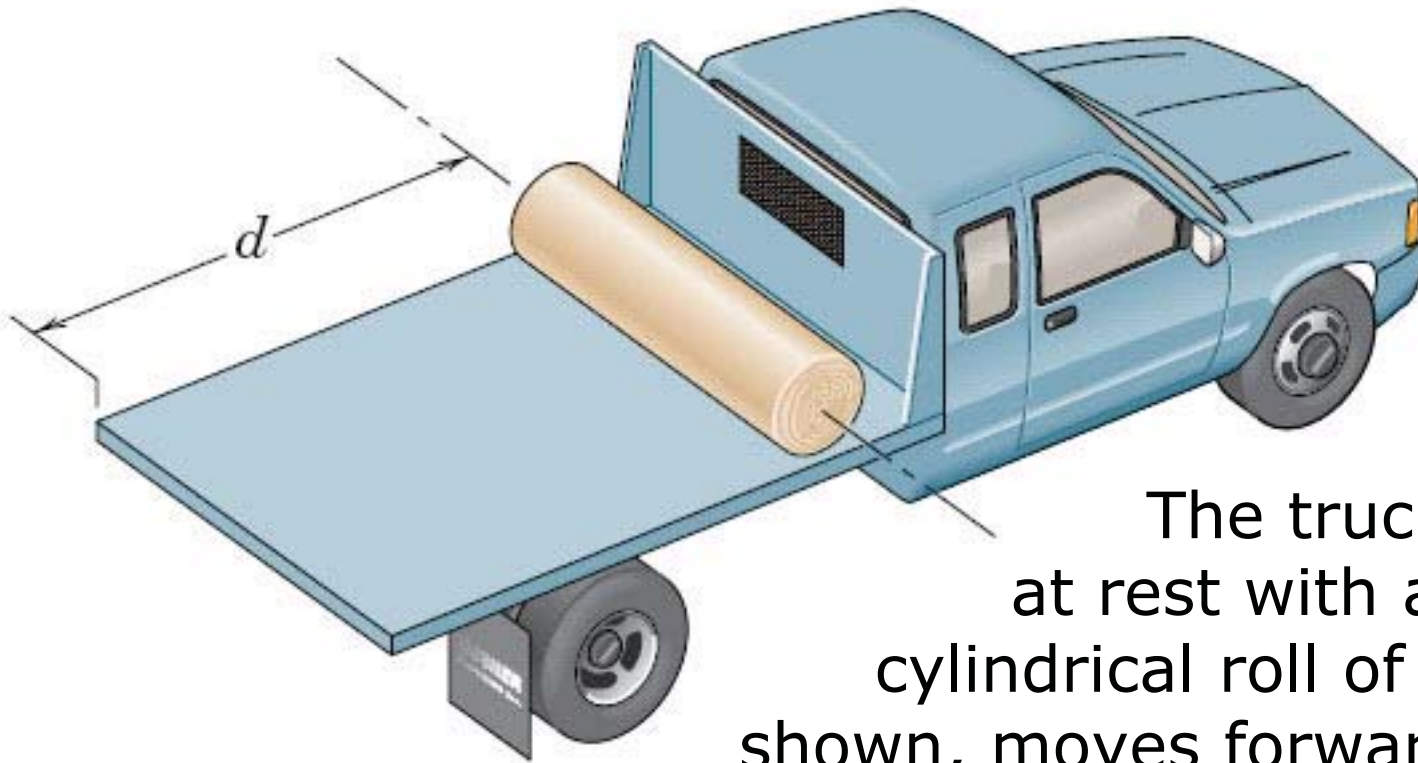
$$\theta = 45^\circ$$

$$a = 10g$$

Determine the **velocity v** relative to the car with which the model's head strikes the instrument panel when the car is stopped with a constant **deceleration a** .



General Plane Motion: Another Exercise



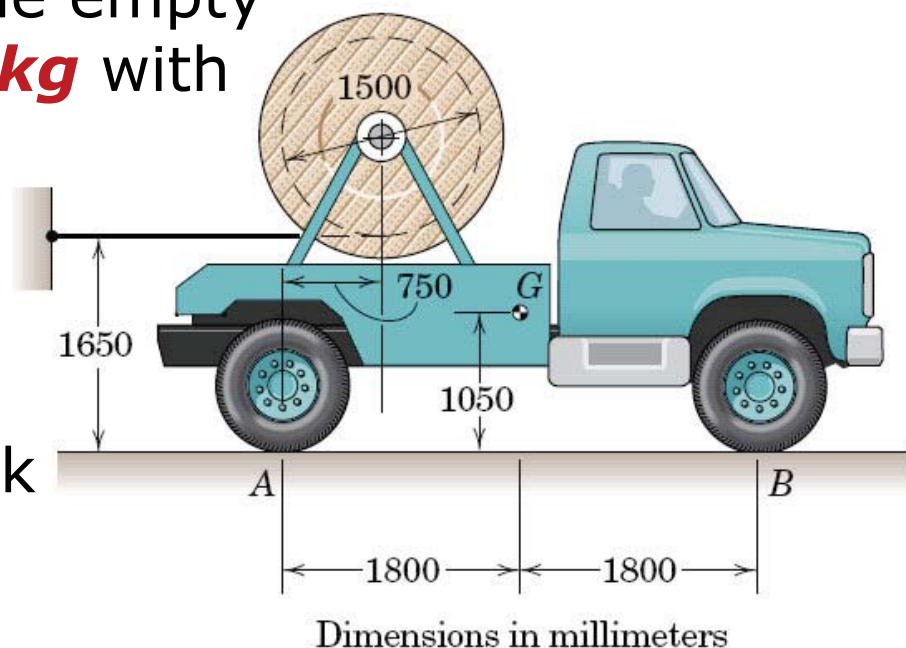
The truck, initially at rest with a solid cylindrical roll of paper shown, moves forward with a constant **acceleration a** .

Determine the **distance s** which the truck goes before the paper rolls (without slipping) off the bed of the truck.

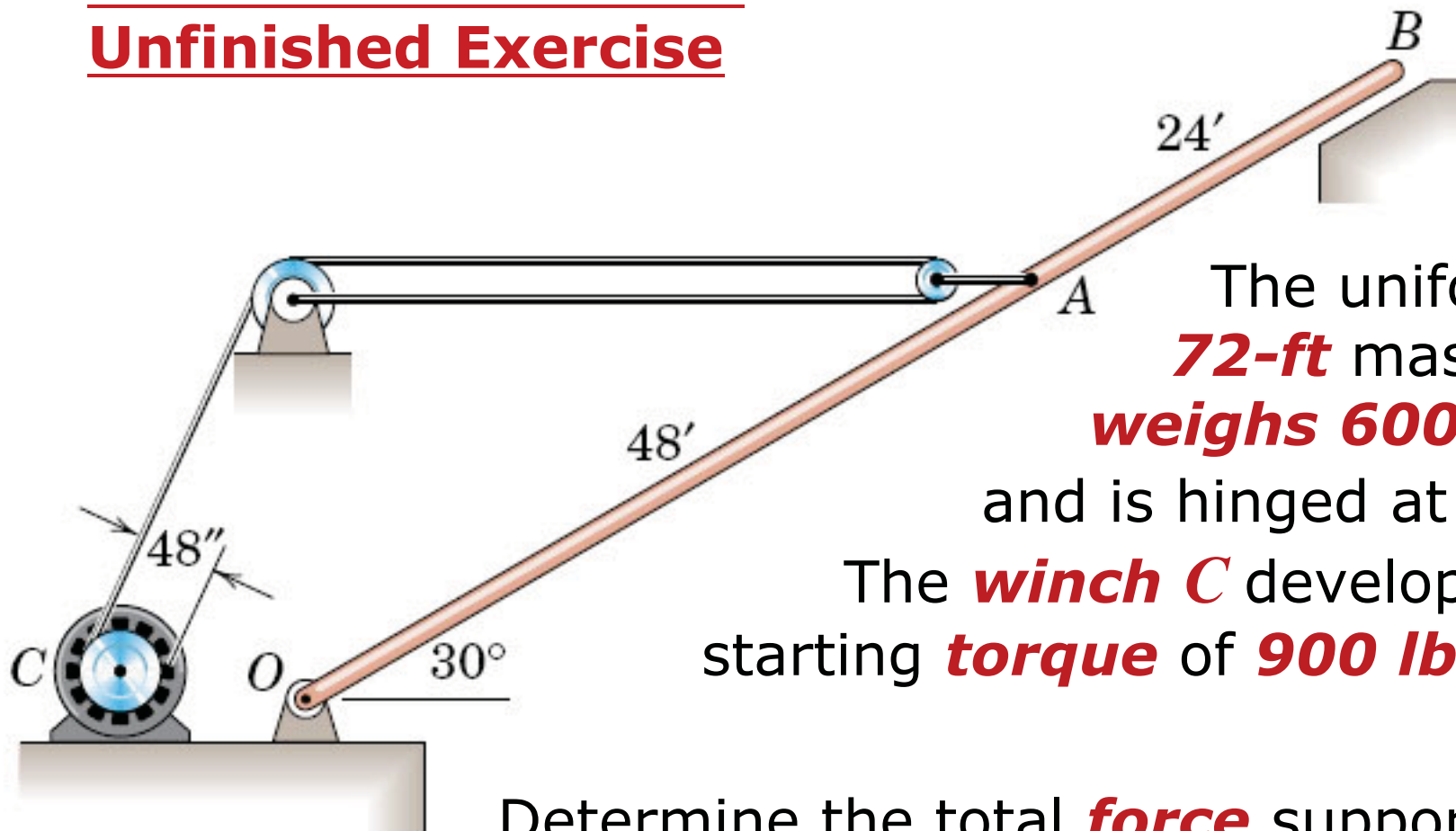
General Plane Motion: Yet Another Exercise

A truck has a **mass** of **2030 kg** and carries a **1500-mm-diameter spool** of cable with a **mass** of **0.75 kg per meter of length**. There are **150 turns** on the full spool. The empty spool has a **mass** of **140 kg** with **radius of gyration** of **530 mm**.

Determine the **tension T** in the cable when the truck starts from rest with an **acceleration** of **$0.2g$** .



Fixed-Axis Rotation: Unfinished Exercise



The uniform **72-ft** mast **weighs 600 lb** and is hinged at **O**.
The **winch C** develops a starting **torque** of **900 lb-ft**.

Determine the total **force** supported by the **pin** at **O** as the mast begins to lift off its support at **B**. Also find the corresponding **angular acceleration** α of the mast.

For Next Time...

- Begin Homework #9 due on ***Wednesday (10/31)***
- Read Chapter 7, Section 7.4