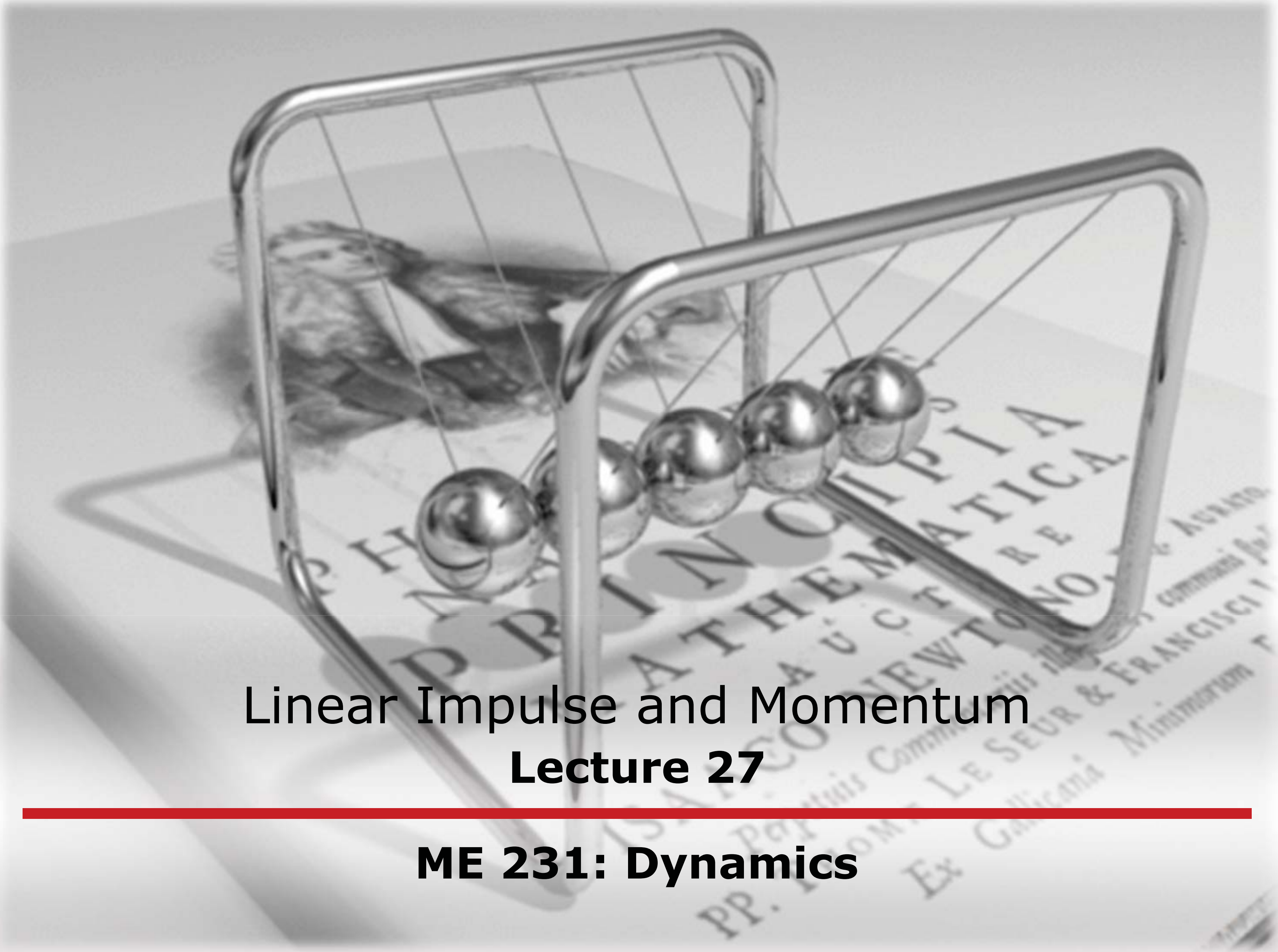
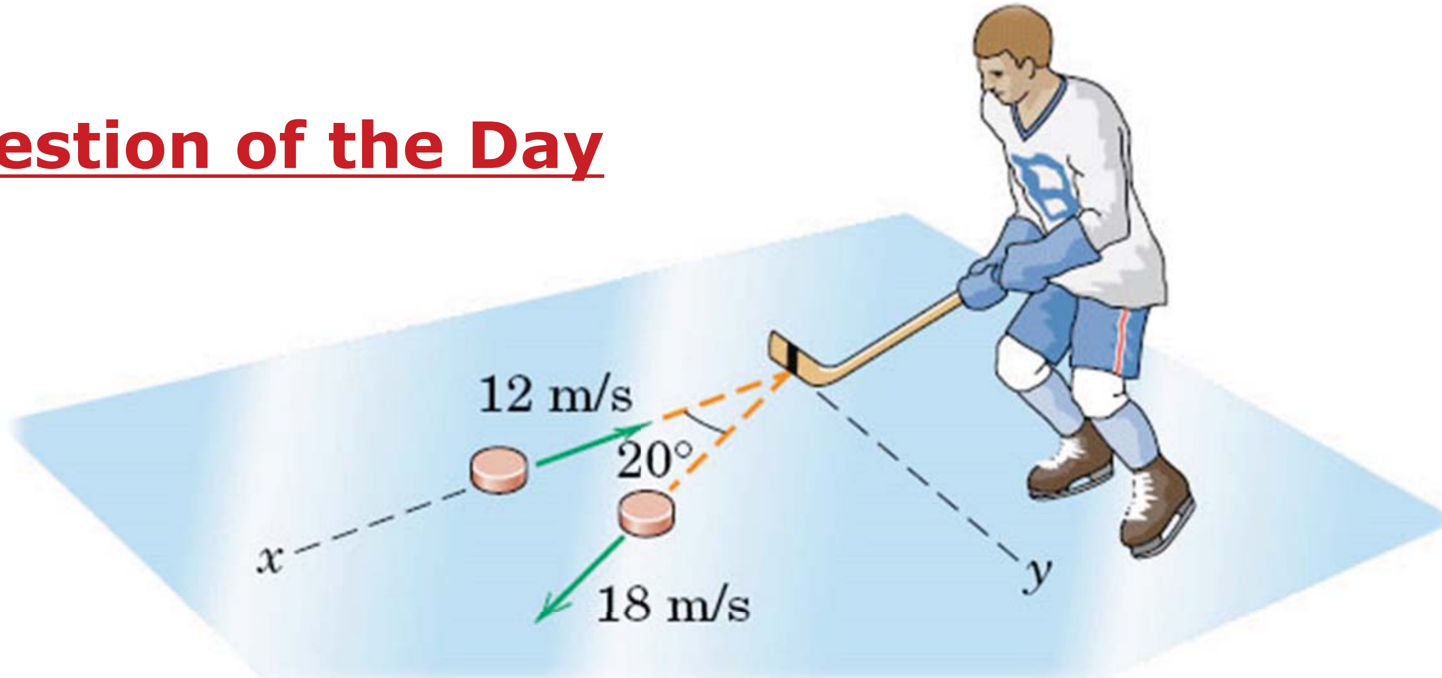


Linear Impulse and Momentum

Lecture 27

ME 231: Dynamics

Question of the Day



An ice-hockey puck with ***mass*** of ***0.20 kg*** has a ***velocity*** of ***12 m/s*** before being struck by the stick. After a ***0.04 s impact***, the puck moves in the new direction shown with a ***velocity*** of ***18 m/s.***

Determine the magnitude of average ***force*** F exerted by the stick on the puck ***during contact***.

Outline for Today

- Question of the day
- From **F**=*m***a** to impulse and momentum
- Linear impulse and momentum
- Linear impulse-momentum principle
- Conservation of linear momentum
- Answer your questions!

Recall: Possible Solutions to Kinetics Problems

- Direct application of ***Newton's 2nd Law***
  - force-mass-acceleration method
  - *Chapters 3 and 7*
- Use of ***impulse*** and ***momentum*** methods
  - *Chapters 5 and 8*
- Use of ***work*** and ***energy*** principles
  - *Chapter 4*

From F=*ma* to impulse and momentum

- ***Integrate*** equations of motion with respect to ***time***
- ***Linear impulse* (F*t)** on m equals change in ***linear momentum* (G)** of m

$$\Sigma \mathbf{F} = m\mathbf{a}$$

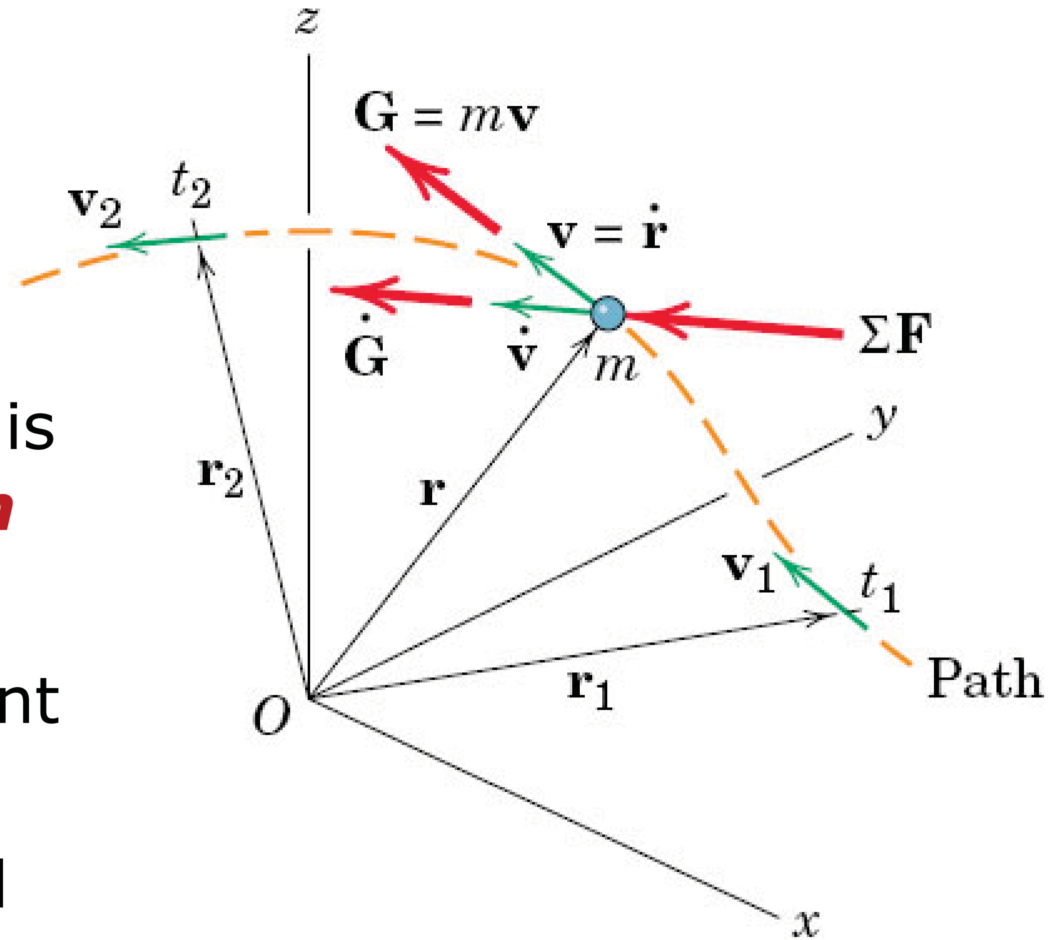
$$\int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \int_{t_1}^{t_2} \frac{d}{dt}(m\mathbf{v})\, dt$$

$$\int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \int_{t_1}^{t_2} \dot{\mathbf{G}}\, dt$$

- Facilitates the ***solution*** of problems where ***forces*** act over ***specified time*** interval or during extremely ***short periods of time*** (e.g., ***impact***)

Linear impulse and momentum

- Particle of ***mass m*** is located by ***position vector*** $\mathbf{r}$
- ***Velocity*** $\mathbf{v}$ is tangent to its path
- ***Resultant*** $\Sigma\mathbf{F}$ of all ***forces*** on ***m*** is in the direction of its ***acceleration*** $\mathbf{a}$
- Valid only when ***mass m*** is ***constant***



$$\Sigma\mathbf{F} = m\dot{\mathbf{v}} = \frac{d}{dt}(m\mathbf{v})$$

$$\boxed{\Sigma\mathbf{F} = \dot{\mathbf{G}}} \qquad \boxed{\mathbf{G} = m\mathbf{v}}$$

$$\sum F_x = \dot{G}_x$$
$$\sum F_y = \dot{G}_y$$
$$\sum F_z = \dot{G}_z$$

Linear Impulse-Momentum Principle

$$\boxed{\Sigma\mathbf{F} = \dot{\mathbf{G}}} \qquad \int_{t_1}^{t_2} \Sigma\mathbf{F}\, dt = \int_{t_1}^{t_2} \dot{\mathbf{G}}\, dt$$

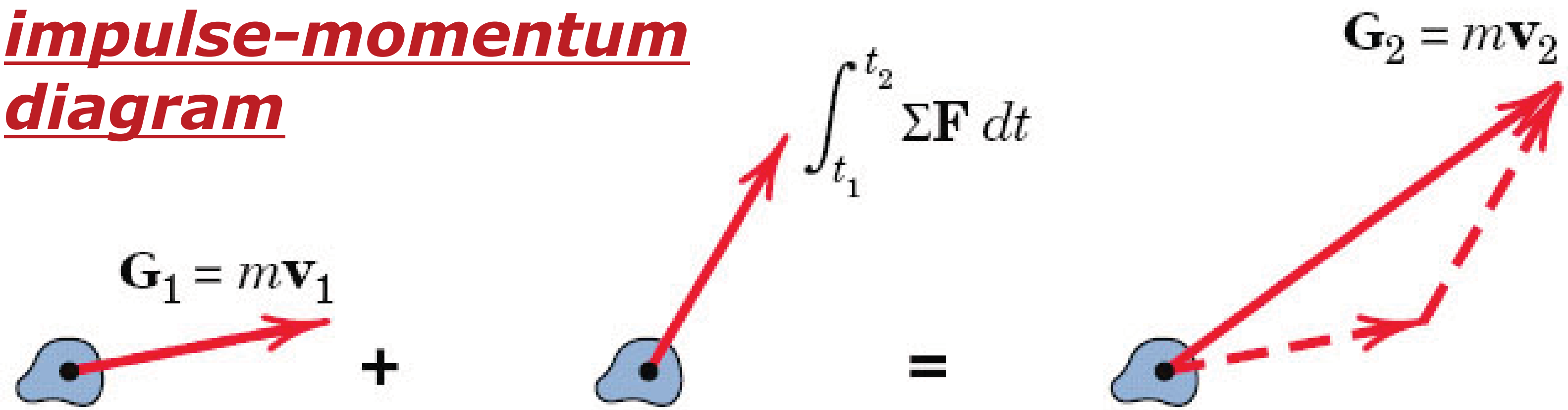
$$\boxed{\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma\mathbf{F}\, dt = \mathbf{G}_2}$$

$$m(v_1)_x + \int_{t_1}^{t_2} \Sigma F_x\, dt = m(v_2)_x$$

$$m(v_1)_y + \int_{t_1}^{t_2} \Sigma F_y\, dt = m(v_2)_y$$

$$m(v_1)_z + \int_{t_1}^{t_2} \Sigma F_z\, dt = m(v_2)_z$$

impulse-momentum diagram



- ***Integrate*** to describe the effect of the ***resultant force*** $\Sigma\mathbf{F}$ on ***linear momentum*** over a finite period of ***time***

Conservation of Linear Momentum

$$\Sigma \mathbf{F} = \dot{\mathbf{G}} \qquad \int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \int_{t_1}^{t_2} \dot{\mathbf{G}}\, dt$$

$$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \mathbf{G}_2$$

$$\Delta \mathbf{G} = \mathbf{0}$$

or

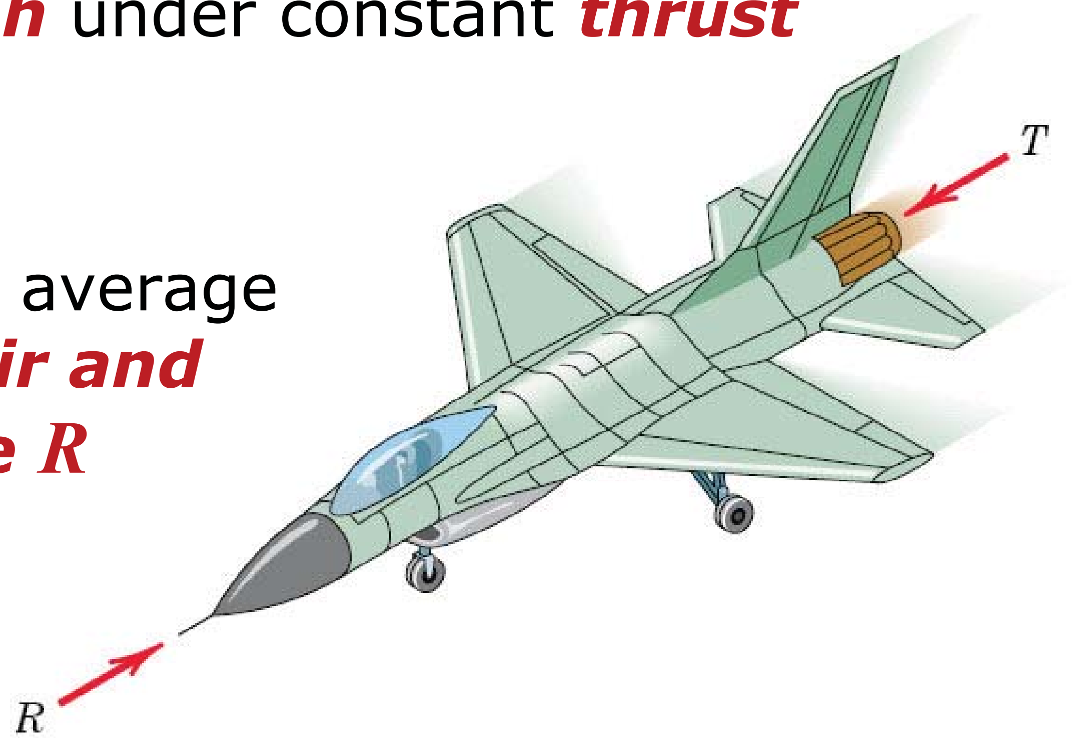
$$\mathbf{G}_1 = \mathbf{G}_2$$

- If the ***resultant force*** $\Sigma \mathbf{F}$ is zero, then ***linear momentum*** remains ***constant***, or is said to be ***conserved***
- Linear momentum may be ***conserved*** in ***one coordinate*** (e.g., x), but ***not necessarily*** in ***others*** (e.g., y or z)

Linear Impulse-Momentum: Exercise

A jet fighter with a ***mass*** of ***6450 kg*** requires ***10 seconds*** from ***rest*** to reach its takeoff ***speed*** of ***250 km/h*** under constant ***thrust*** ***T = 48 kN***.

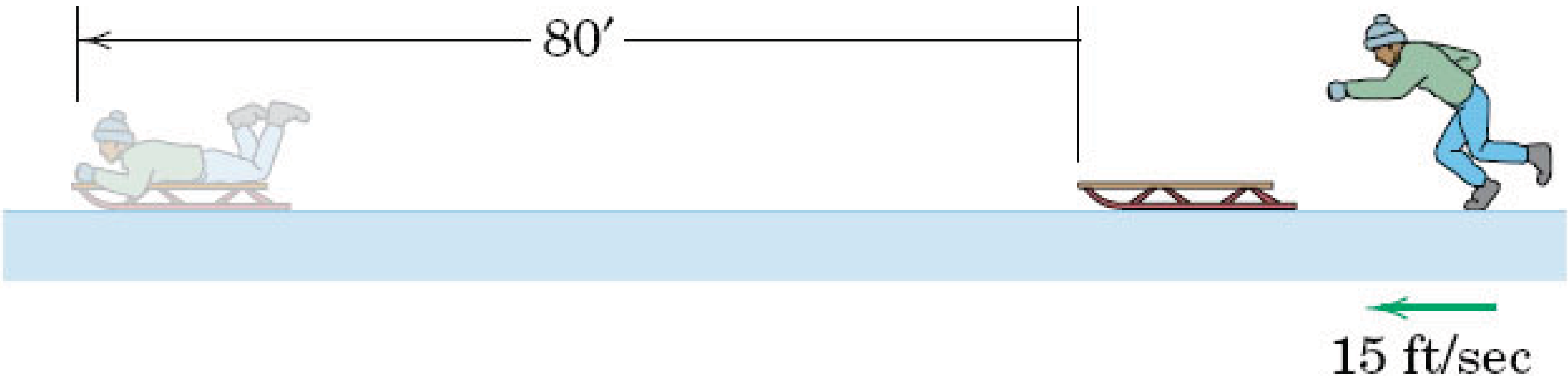
Determine the time average of the ***combined air and ground resistance R*** during takeoff.



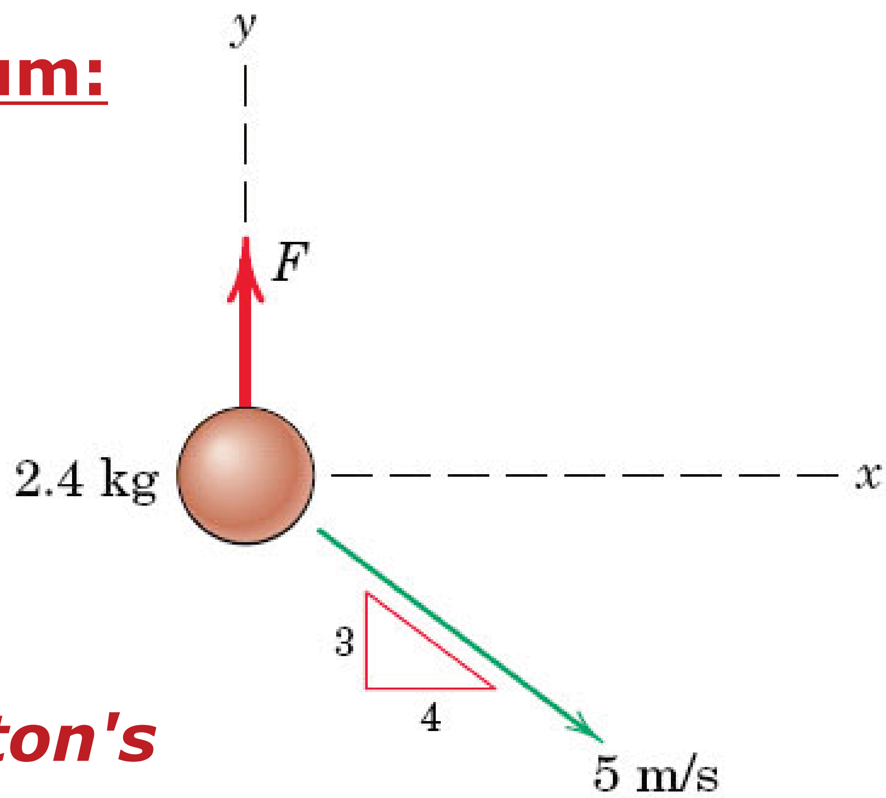
Linear Impulse-Momentum: Another Exercise

A ***100-lb boy*** runs with a ***velocity*** of ***15 ft/s*** and jumps on his ***20-lb sled***. The sled and boy coast ***80 ft*** on level snow before coming to ***rest***.

Determine the ***coefficient of kinetic friction*** between the snow and sled.



Linear Impulse-Momentum: Yet Another Exercise



A ***2.4-kg*** particle moves in the ***x-y plane*** and has the ***velocity*** shown at time ***t = 0***. A ***force $F = 2+3t^2/4$ Newton's*** is applied in the ***y-direction*** at ***t = 0***.

Determine the ***velocity*** of the particle ***4 seconds*** after ***F*** is applied and specify the ***angle θ*** measured counter clockwise from the ***x-axis*** to the ***direction*** of the ***velocity***.

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For Next Time…

- Continue Homework #9 due on ***Thursday (11/1)***
- Read Chapter 5, Section 5.3