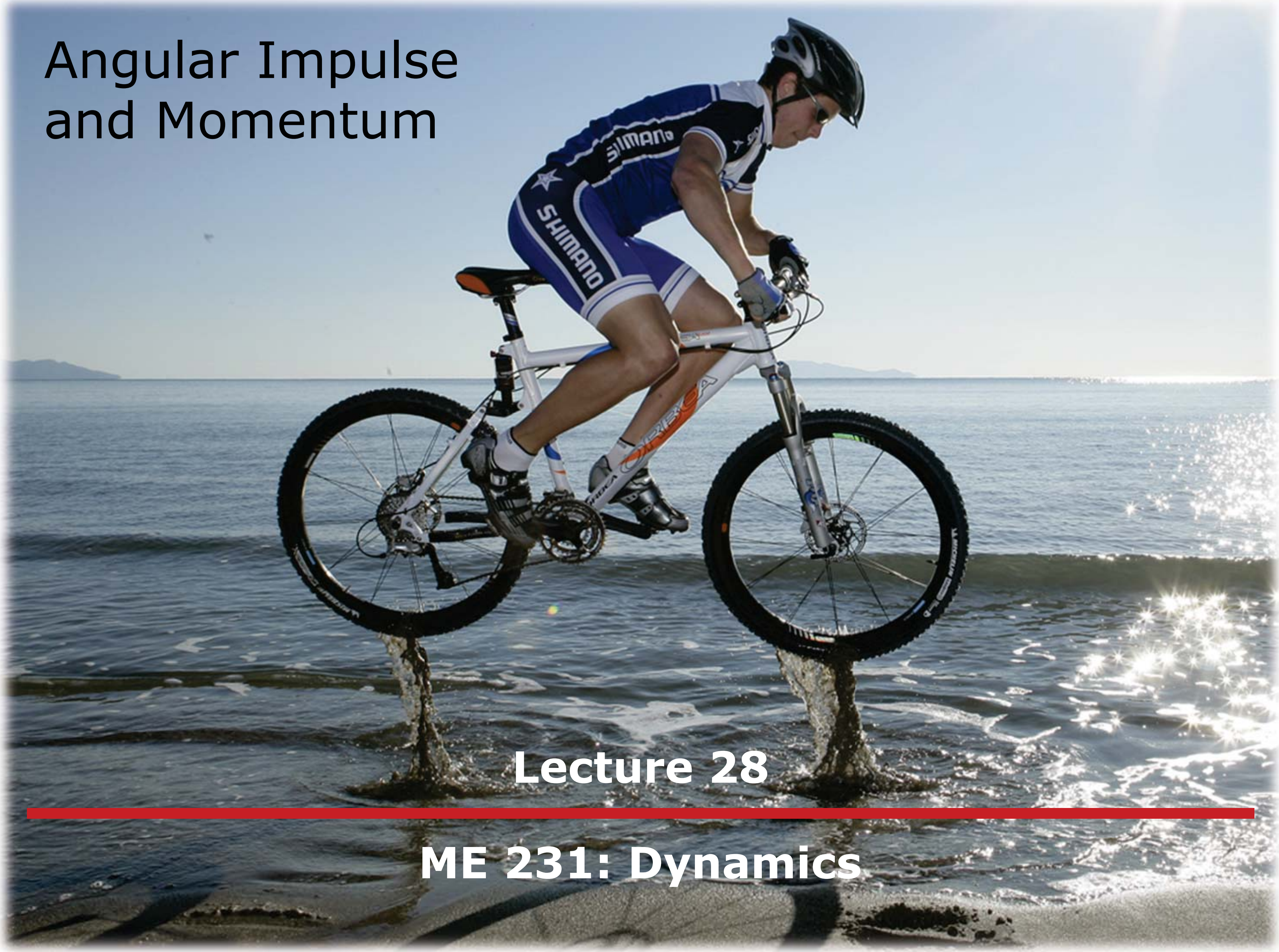
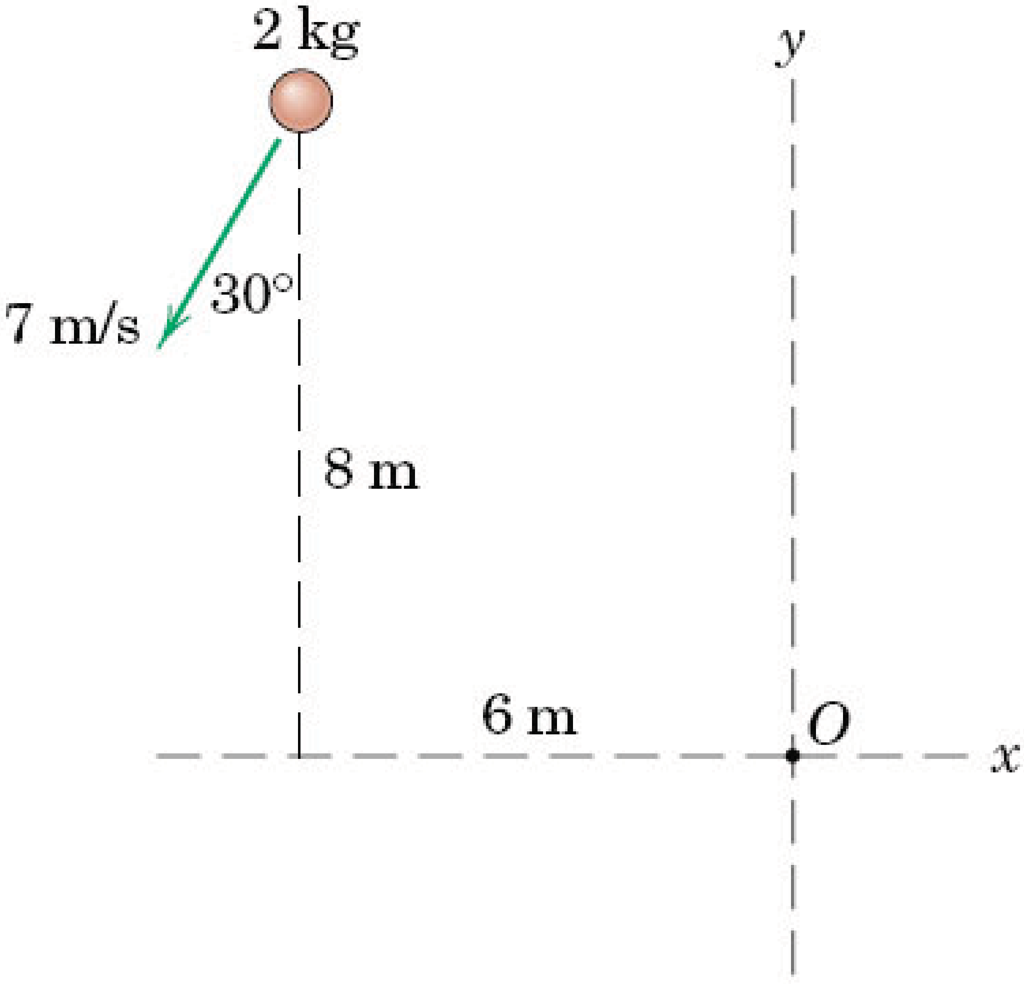
Angular Impulse and Momentum



Lecture 28

ME 231: Dynamics

Question of the Day



Determine the magnitude H_O of the ***angular momentum*** of the ***2-kg*** sphere about ***point O***.

Hint: H_O is the moment of linear momentum about point O.

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$$

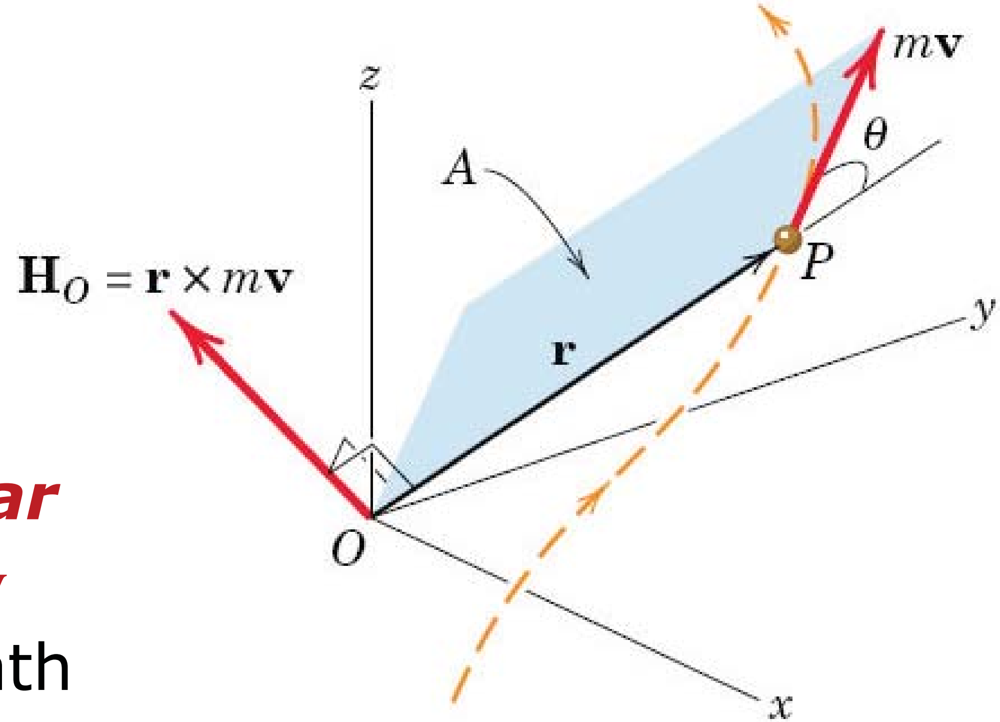
Outline for Today

- Question of the day
- Angular momentum
- Rate of change of angular momentum
- Angular impulse-momentum principle
- Plane-motion applications
- Conservation of angular momentum
- Answer your questions!

- Exam 2 Survey

Angular Momentum

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$$

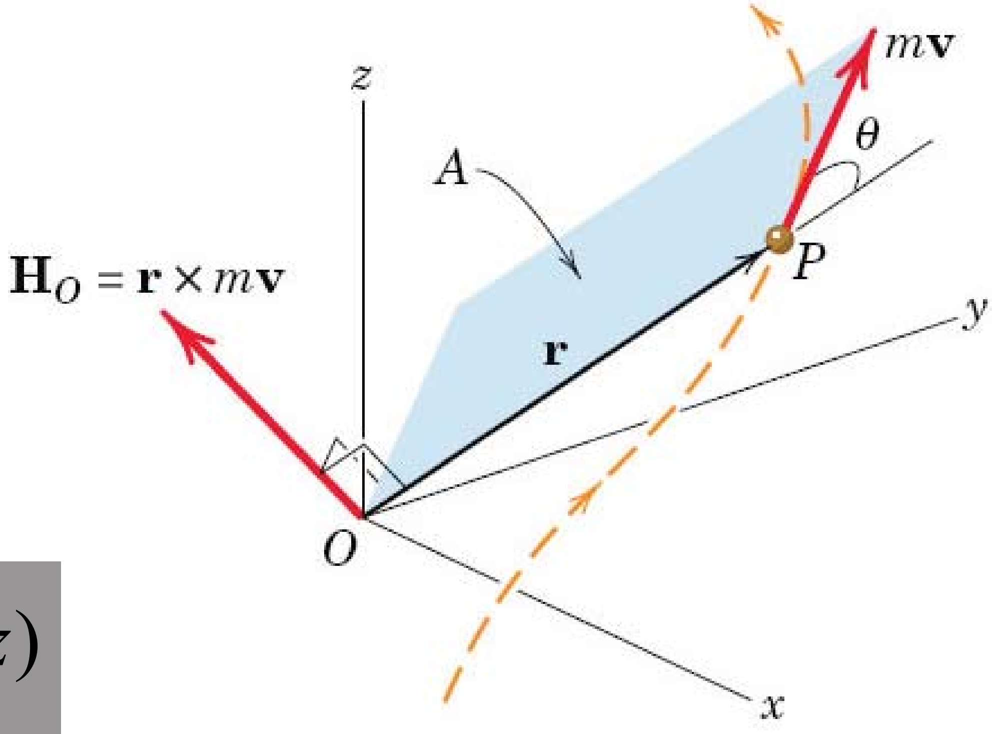


- Particle of ***mass*** m is located by ***position vector*** $\mathbf{r}$
- ***Velocity*** $\mathbf{v}$ and ***linear momentum*** $\mathbf{G} = m\mathbf{v}$ are tangent to its path
- The ***moment*** of the ***linear momentum*** vector $m\mathbf{v}$ about ***point O*** is the ***angular momentum $\mathbf{H}_O$ of P about O***
- ***Perpendicular*** to ***plane A*** defined by $\mathbf{r}$ and $\mathbf{v}$

Angular Momentum

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$$

$$\mathbf{H}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ mv_x & mv_y & mv_z \end{vmatrix}$$



$$(\mathbf{H}_O)_x = m(v_z y - v_y z)$$

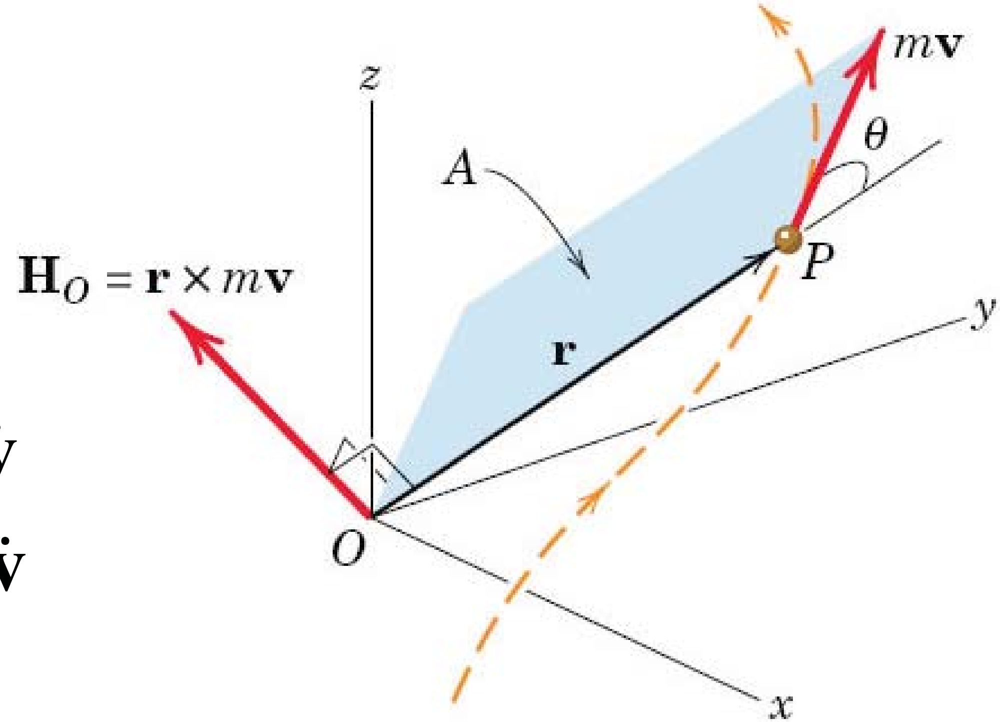
$$(\mathbf{H}_O)_y = m(v_x z - v_z x)$$

$$(\mathbf{H}_O)_z = m(v_y x - v_x y)$$

Rate of Change of Angular Momentum

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$$

$$\sum \mathbf{M}_O = \dot{\mathbf{H}}_O$$



- ***Differentiate*** $\mathbf{H}_O$ with respect to ***time***

$$\dot{\mathbf{H}}_O = \dot{\mathbf{r}} \times m\mathbf{v} + \mathbf{r} \times m\dot{\mathbf{v}}$$

$$\dot{\mathbf{H}}_O = \cancelto{0}{\mathbf{v} \times m\mathbf{v}} + \mathbf{r} \times m\dot{\mathbf{v}}$$

$$\dot{\mathbf{H}}_O = \mathbf{r} \times m\dot{\mathbf{v}}$$

- ***Resultant*** $\Sigma \mathbf{M}_O$ of all ***moments of forces*** on ***m*** is the vector cross product of $\mathbf{r}$ and $\Sigma\mathbf{F}$

$$\Sigma \mathbf{M}_O = \mathbf{r} \times \Sigma\mathbf{F} = \mathbf{r} \times m\dot{\mathbf{v}}$$

$$(\Sigma \mathbf{M}_O)_x = (\dot{\mathbf{H}}_O)_x$$

$$(\Sigma \mathbf{M}_O)_y = (\dot{\mathbf{H}}_O)_y$$

$$(\Sigma \mathbf{M}_O)_z = (\dot{\mathbf{H}}_O)_z$$

Angular Impulse-Momentum Principle

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$$

$$\sum \mathbf{M}_O = \dot{\mathbf{H}}_O$$

$$\int_{t_1}^{t_2} \Sigma \mathbf{M}_O \, dt = \int_{t_1}^{t_2} \dot{\mathbf{H}}_O \, dt$$

$$\left(\mathbf{H}_O\right)_1 + \int_{t_1}^{t_2} \Sigma \mathbf{M}_O \, dt = \left(\mathbf{H}_O\right)_2$$

- ***Integrate*** to describe the effect of the ***angular impulse*** $\Sigma M_O * t$ on ***angular momentum*** H_O ***of*** m ***about*** O over a finite period of ***time***

$$m\left(v_z y - v_y z\right)_1 + \int_{t_1}^{t_2} \Sigma\left(\mathbf{M}_O\right)_x \, dt = m\left(v_z y - v_y z\right)_2$$

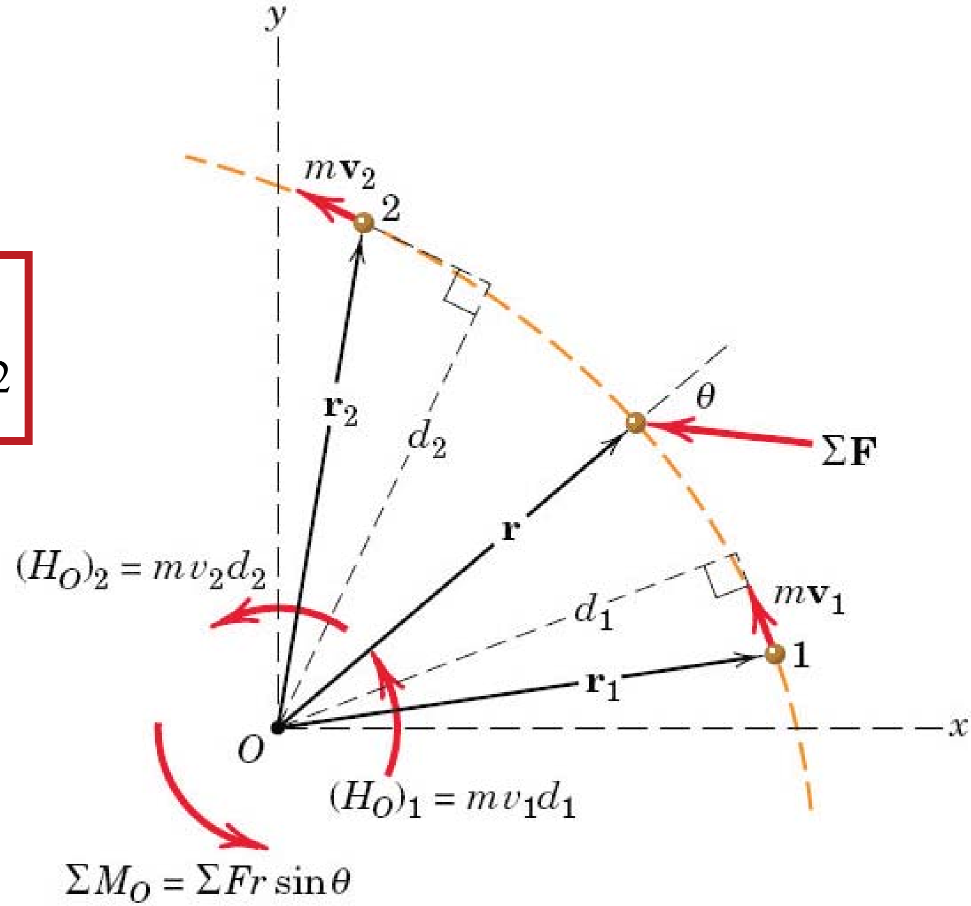
$$m\left(v_x z - v_z x\right)_1 + \int_{t_1}^{t_2} \Sigma\left(\mathbf{M}_O\right)_y \, dt = m\left(v_x z - v_z x\right)_2$$

$$m\left(v_y x - v_x y\right)_1 + \int_{t_1}^{t_2} \Sigma\left(\mathbf{M}_O\right)_z \, dt = m\left(v_y x - v_x y\right)_2$$

Plane-Motion Applications

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$$

$$\sum \mathbf{M}_O = \dot{\mathbf{H}}_O$$



$$(\mathbf{H}_O)_1 + \int_{t_1}^{t_2} \Sigma\mathbf{M}_O \, dt = (\mathbf{H}_O)_2$$

- ***Moments*** taken about a ***single axis normal*** to the ***plane of motion***
- ***Angular momentum*** may change ***magnitude*** and ***sense***, but the ***direction*** is ***constant***

$$mv_1d_1 + \int_{t_1}^{t_2} \Sigma Fr\sin\theta \, dt = mv_2d_2$$

Conservation of Angular Momentum

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$$

$$\sum \mathbf{M}_O = \dot{\mathbf{H}}_O$$

$$\left(\mathbf{H}_O\right)_1 + \int_{t_1}^{t_2} \Sigma \mathbf{M}_O \, dt = \left(\mathbf{H}_O\right)_2$$

$$\Delta \mathbf{H}_O = \mathbf{0}$$

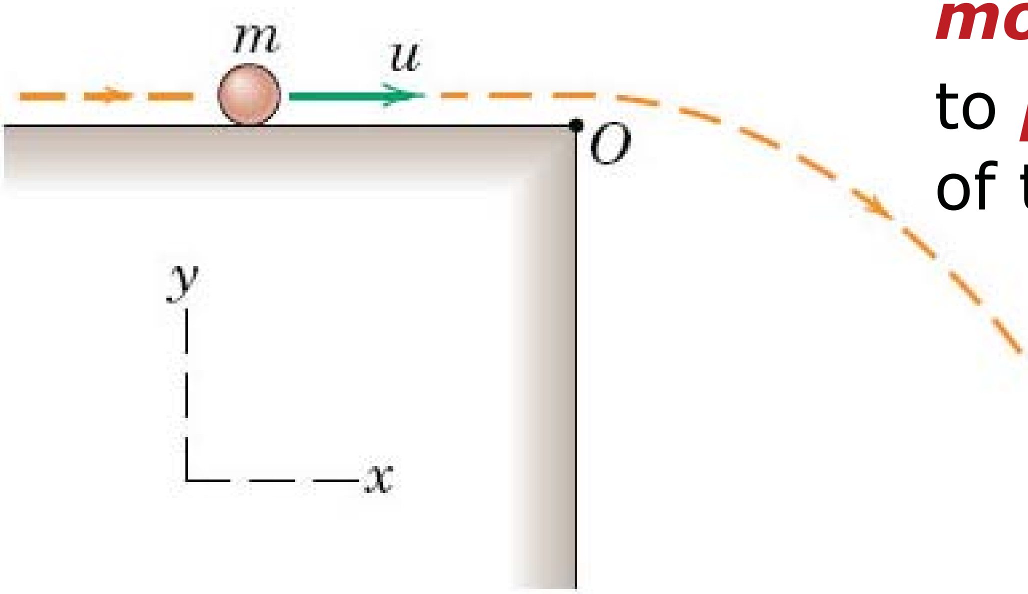
or

$$\left(\mathbf{H}_O\right)_1 = \left(\mathbf{H}_O\right)_2$$

- If the ***resultant moment about*** a fixed ***point*** O is zero, then ***angular momentum*** remains ***constant***, or is said to be ***conserved***
- ***Angular momentum*** may be ***conserved*** in ***one coordinate*** (e.g., x), but ***not necessarily*** in ***others*** (e.g., y or z)

Angular Impulse-Momentum: Exercise

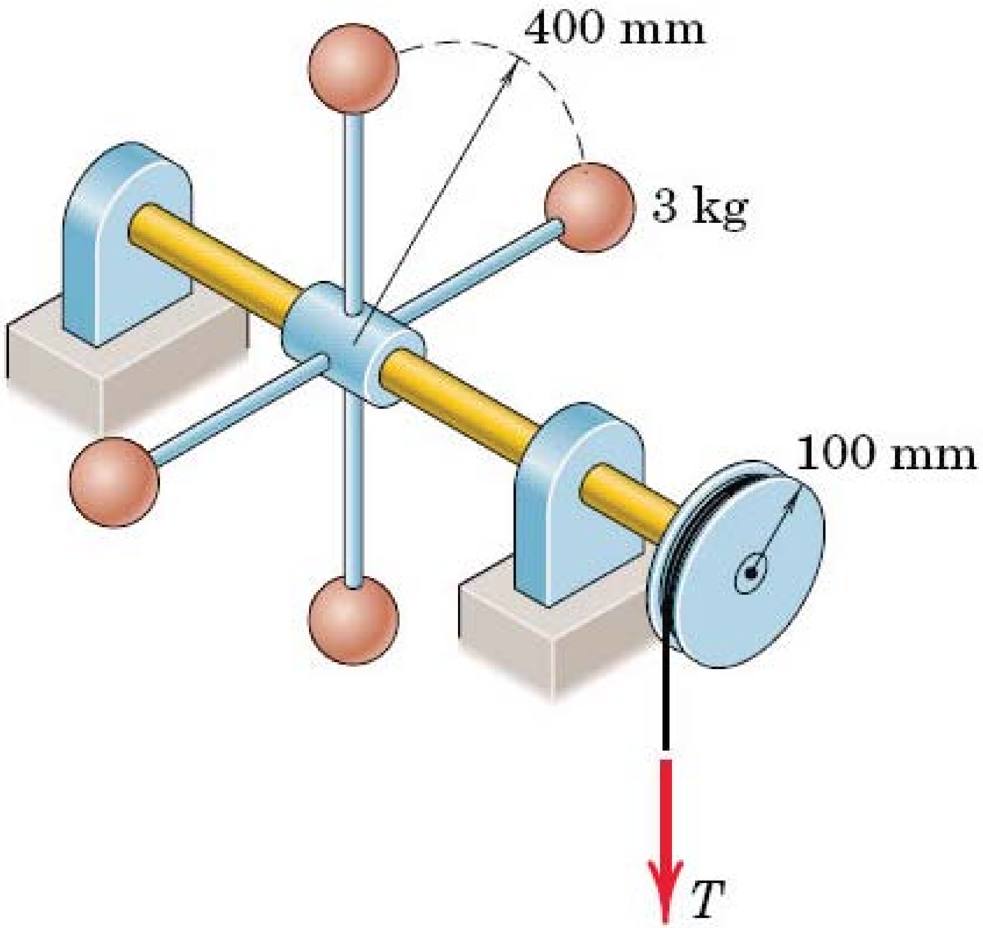
The particle of ***mass*** m is launched from ***point*** O with a horizontal ***velocity*** $\mathbf{u}$ at time $t = 0$.



Determine its ***angular momentum*** $\mathbf{H}_O$ relative to ***point*** O as a function of time.

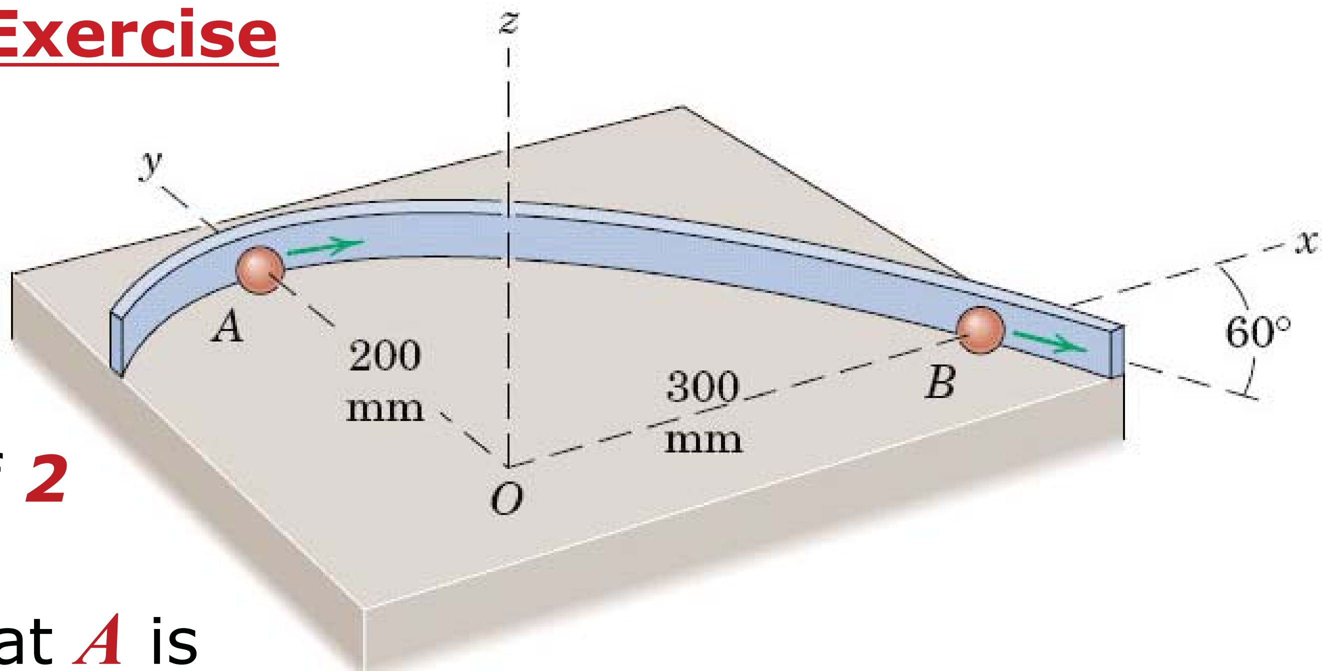
Angular Impulse-Momentum: Another Exercise

The assembly starts from rest and reaches an ***angular speed*** of ***150 rev/min*** under the action of a ***20-N force T*** applied to the string for ***t seconds***. Neglect friction and all masses except those of the four ***3-kg*** spheres.



Determine ***t***.

Angular Impulse-Momentum: Yet Another Exercise



A ***0.1-kg*** particle with a ***velocity*** of ***2 m/s*** in the ***x-direction*** at A is guided by a curved rail. The ***radius of curvature*** of the rail at B is ***500 mm***.

Determine the ***time rate of change*** of the ***angular momentum*** H_O about the ***z-axis*** through O at both A and B.

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For Next Time…

- Begin Homework #10 due on ***Wednesday (11/7)***
- Read Chapter 5, Section 5.3
- Read Chapter 8, Section 8.2