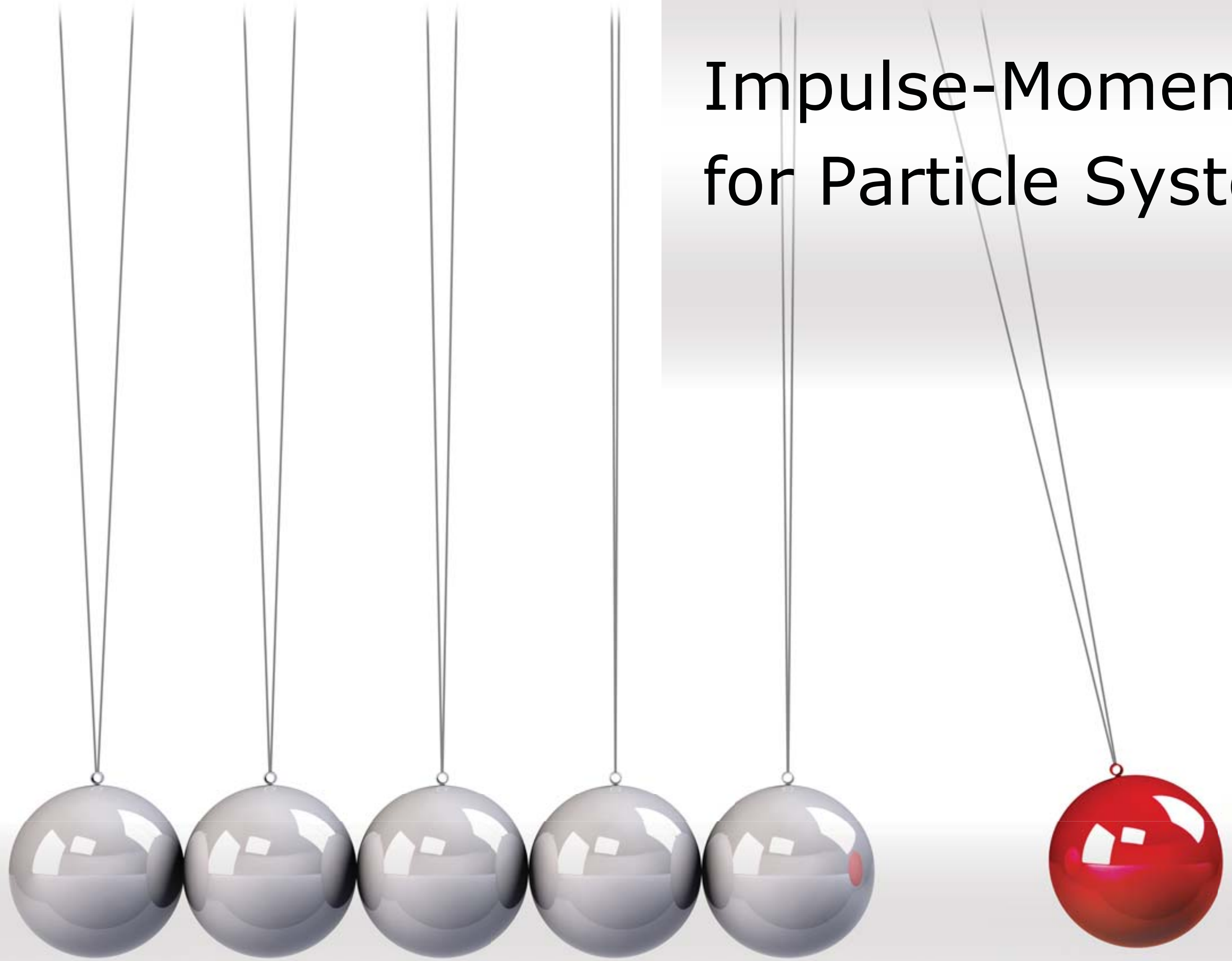
Impulse-Momentum for Particle Systems

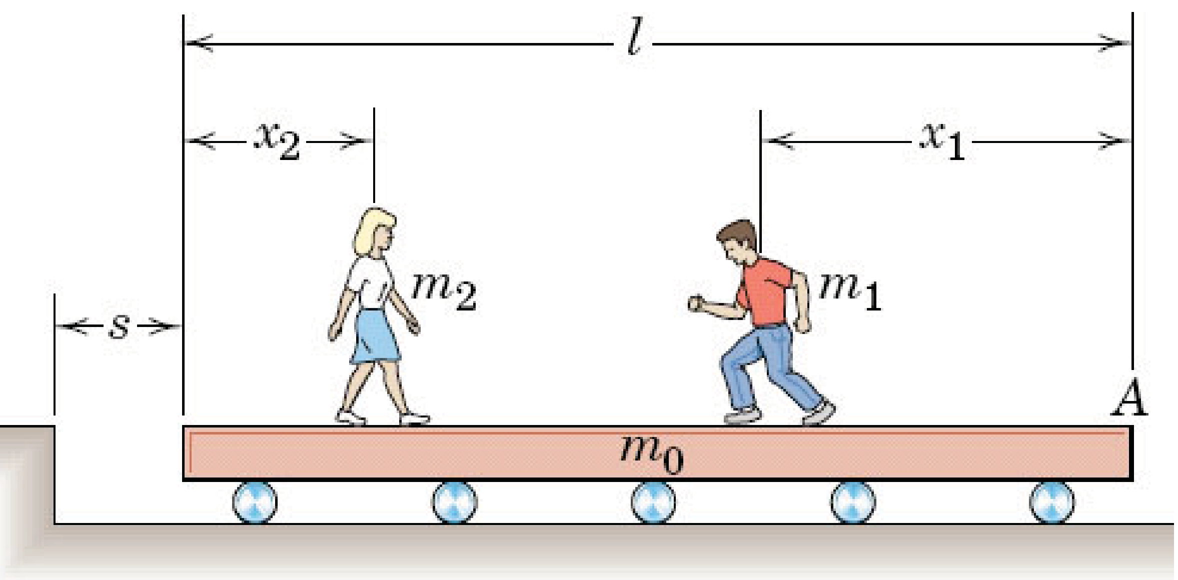


Lecture 29

ME 231: Dynamics

Question of the Day

A man of ***mass*** m_1 and woman of ***mass*** m_2 are at opposite ends and begin to approach each other on a platform of ***mass*** m_0 which moves with negligible friction and initially at rest with $s = 0$.

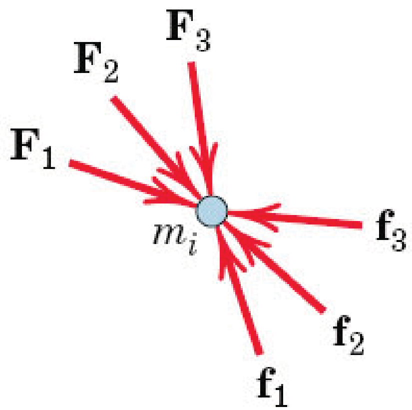


Determine an expression for the ***displacement*** s of the platform when the two meet ***in terms*** of x_1 relative to the platform.

Outline for Today

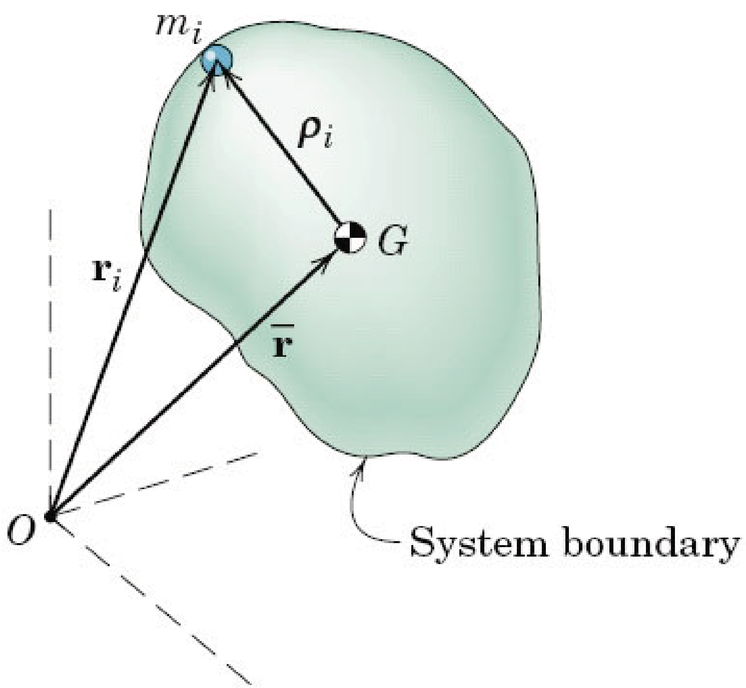
- Question of the day
- Linear momentum for particle systems
- Angular momentum for particle systems
- Conservation of momentum for particle systems
- Answer your questions!

Recall: Newton's 2nd Law for Particle Systems



$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots + \mathbf{f}_1 + \mathbf{f}_2 + \mathbf{f}_3 + \cdots = m_i \ddot{\mathbf{r}}_i$$

$$\sum \mathbf{F} + \sum \mathbf{f} = \sum m_i \ddot{\mathbf{r}}_i$$



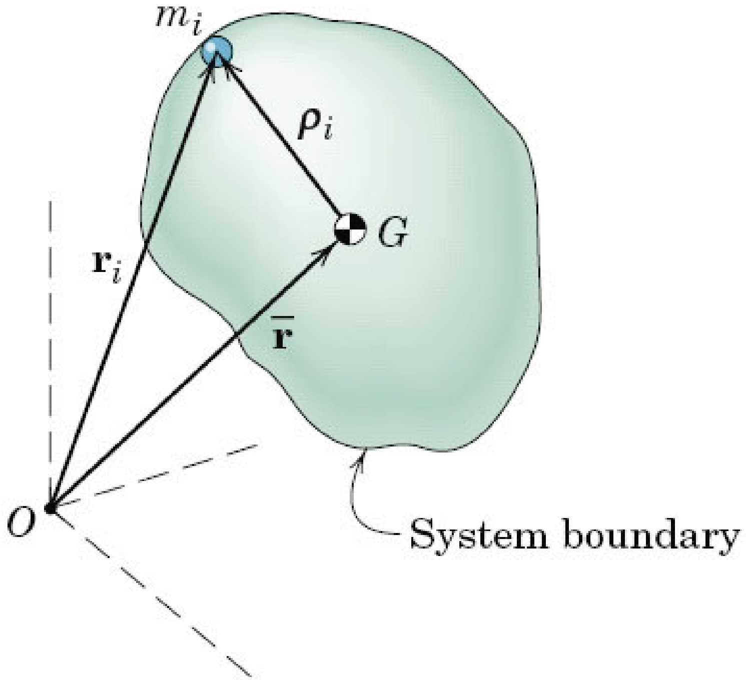
$$\boxed{\sum \mathbf{F} = m\ddot{\bar{\mathbf{r}}}}$$ or $$\boxed{\sum \mathbf{F} = m\bar{\mathbf{a}}}$$

$$\sum F_x = m\bar{a}_x$$

$$\sum F_y = m\bar{a}_y$$

$$\sum F_z = m\bar{a}_z$$

Linear Momentum for Particle Systems



$$\mathbf{G}_i = m_i \mathbf{v}_i$$

$$\mathbf{G} = \sum m_i \mathbf{v}_i$$

$$\mathbf{G} = \sum m_i \left(\bar{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i \right)$$

$$\mathbf{G} = \sum m_i \bar{\mathbf{v}} + \frac{d}{dt} \sum m_i \boldsymbol{\rho}_i$$

$$m\bar{\boldsymbol{\rho}} = \mathbf{0}$$

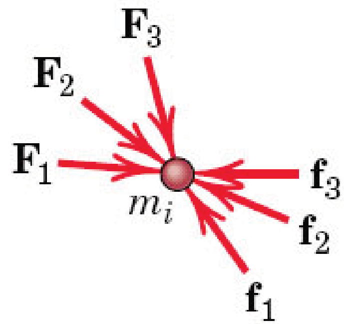
$$\mathbf{G} = m\bar{\mathbf{v}}$$

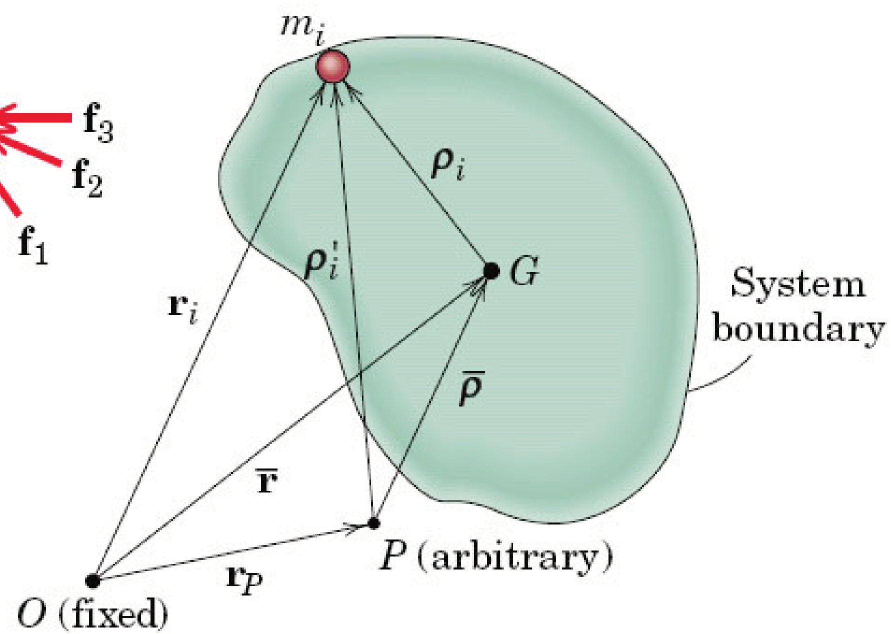
$$\dot{\mathbf{G}} = m\dot{\bar{\mathbf{v}}} = m\bar{\mathbf{a}}$$

$$\sum \mathbf{F} = \dot{\mathbf{G}}$$

Angular Momentum for Particle Systems

About a Fixed Point O





$$(\mathbf{H}_O)_i = \mathbf{r}_i \times m_i \mathbf{v}_i$$

$$\mathbf{H}_O = \sum(\mathbf{r}_i \times m_i \mathbf{v}_i)$$

$$\dot{\mathbf{H}}_O = \sum(\dot{\mathbf{r}}_i \times m_i \mathbf{v}_i) + \sum(\mathbf{r}_i \times m_i \dot{\mathbf{v}}_i)$$

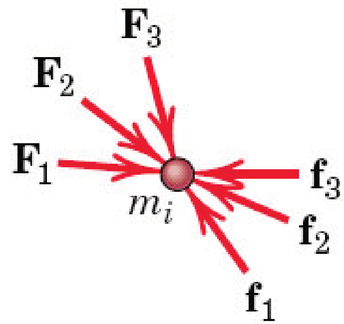
0

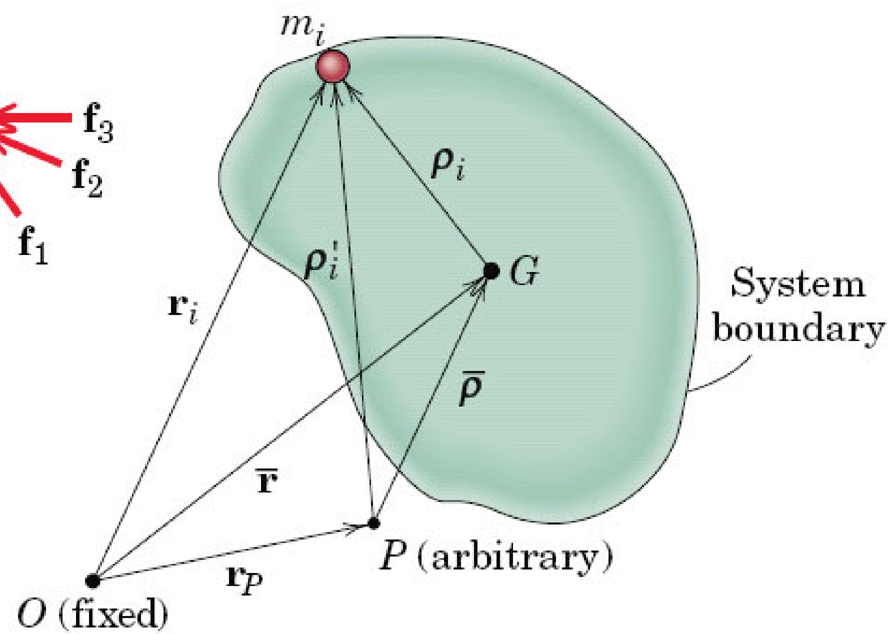
$$\dot{\mathbf{H}}_O = \sum(\mathbf{r}_i \times \mathbf{F}_i)$$

$$\sum \mathbf{M}_O = \dot{\mathbf{H}}_O$$

Angular Momentum for Particle Systems

About the Mass Center G





$$\left(\mathbf{H}_G\right)_i = \boldsymbol{\rho}_i \times m_i \dot{\mathbf{r}}_i$$

$$\mathbf{H}_G = \sum\left(\boldsymbol{\rho}_i \times m_i \dot{\mathbf{r}}_i\right)$$

$$\mathbf{H}_G = \sum\left[\boldsymbol{\rho}_i \times m_i\left(\dot{\bar{\mathbf{r}}} + \dot{\boldsymbol{\rho}}_i\right)\right]$$

$$\mathbf{H}_G = \sum\left(\boldsymbol{\rho}_i \times m_i \dot{\bar{\mathbf{r}}}\right) + \sum\left(\boldsymbol{\rho}_i \times m_i \dot{\boldsymbol{\rho}}_i\right)$$

$$\mathbf{H}_G = \left(-\dot{\bar{\mathbf{r}}} \times \cancelto{\mathbf{0}}{\sum m_i \boldsymbol{\rho}_i}\right) + \sum\left(\boldsymbol{\rho}_i \times m_i \dot{\boldsymbol{\rho}}_i\right)$$

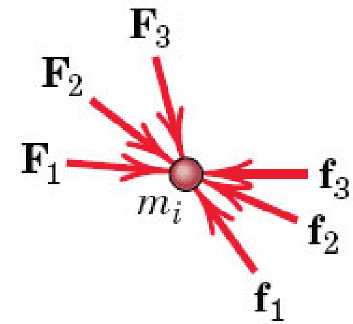
$$\mathbf{H}_G = \sum\left(\boldsymbol{\rho}_i \times m_i \dot{\boldsymbol{\rho}}_i\right)$$

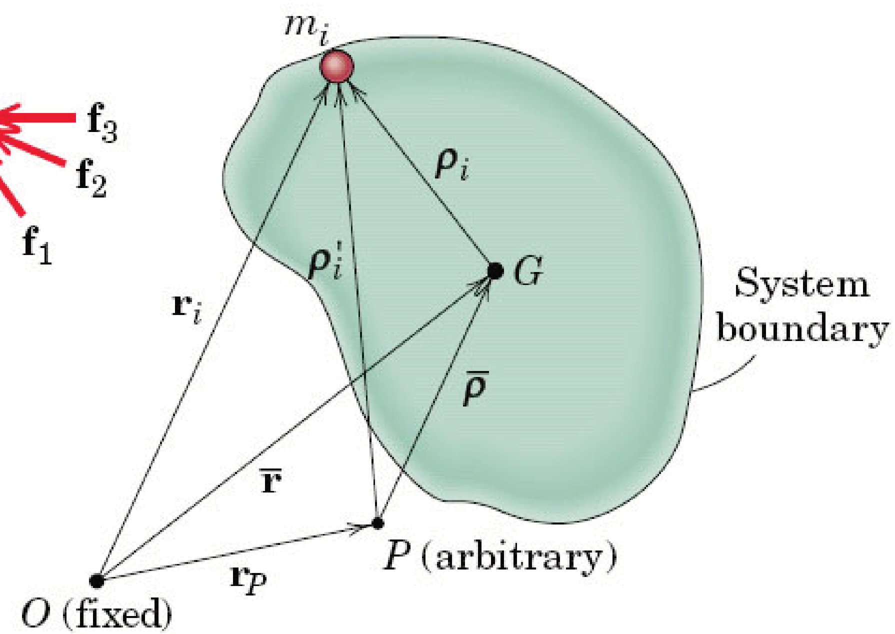
$$\dot{\mathbf{H}}_G = \sum\left(\boldsymbol{\rho}_i \times \mathbf{F}_i\right)$$

$$\boxed{\sum \mathbf{M}_G = \dot{\mathbf{H}}_G}$$

Angular Momentum for Particle Systems

About an Arbitrary Point P





$$(\mathbf{H}_P)_i = \boldsymbol{\rho}_i' \times m_i \dot{\mathbf{r}}_i$$

$$\mathbf{H}_P = \sum (\boldsymbol{\rho}_i' \times m_i \dot{\mathbf{r}}_i)$$

$$\mathbf{H}_P = \sum [(\bar{\boldsymbol{\rho}} + \boldsymbol{\rho}_i) \times m_i \dot{\mathbf{r}}_i]$$

$$\mathbf{H}_P = \sum (\bar{\boldsymbol{\rho}} \times m_i \dot{\mathbf{r}}_i) + \sum (\boldsymbol{\rho}_i \times m_i \dot{\mathbf{r}}_i)$$

$$\mathbf{H}_P = \bar{\boldsymbol{\rho}} \times \sum m_i \mathbf{v}_i + \mathbf{H}_G$$

$$\mathbf{H}_P = \mathbf{H}_G + \bar{\boldsymbol{\rho}} \times m \bar{\mathbf{v}}$$

$$\sum \mathbf{M}_P = \dot{\mathbf{H}}_G + \bar{\boldsymbol{\rho}} \times m \bar{\mathbf{a}}$$

Conservation of Momentum

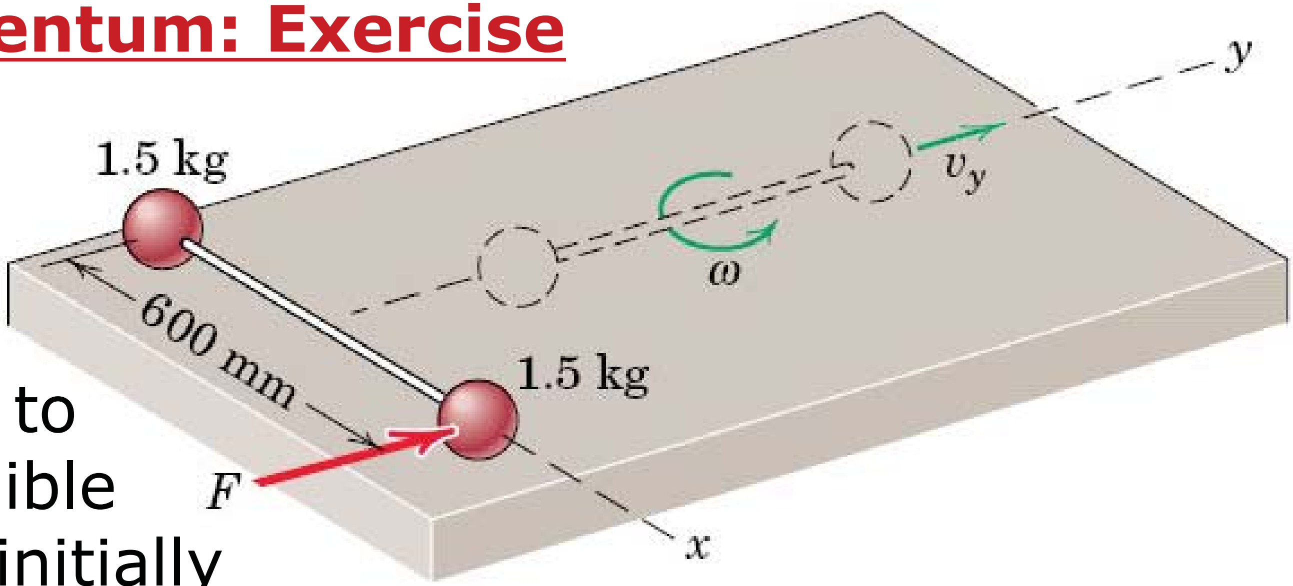
$$\mathbf{G}_1 = \mathbf{G}_2$$

$$\left(\mathbf{H}_O\right)_1 = \left(\mathbf{H}_O\right)_2$$

$$\left(\mathbf{H}_G\right)_1 = \left(\mathbf{H}_G\right)_2$$

- If the ***resultant external force*** $\Sigma\mathbf{F}$ is zero, then ***linear momentum*** is ***conserved***
- If the ***resultant moment about*** a fixed ***point*** O or ***mass center*** G is zero, then ***angular momentum*** is ***conserved***

Impulse-Momentum: Exercise

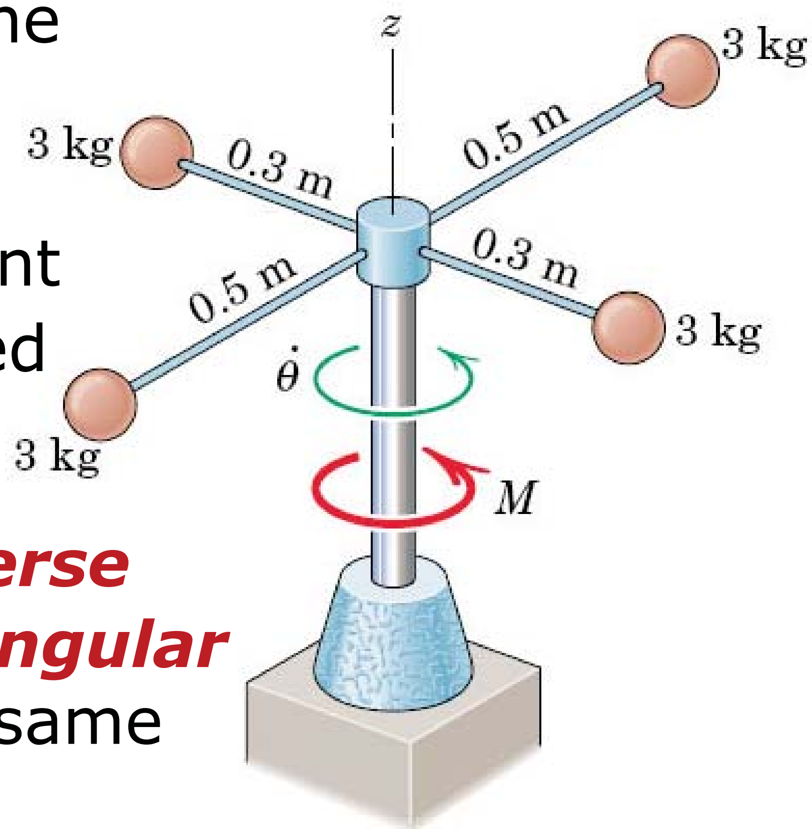


Two spheres are connected to a rod of negligible mass and are initially at rest. A ***force F*** is applied to ***one sphere*** in the ***y-direction*** and imparts an impulse of ***10 Ns*** during a negligibly short time.

Determine the ***velocity*** of each sphere as they pass the ***dashed position***.

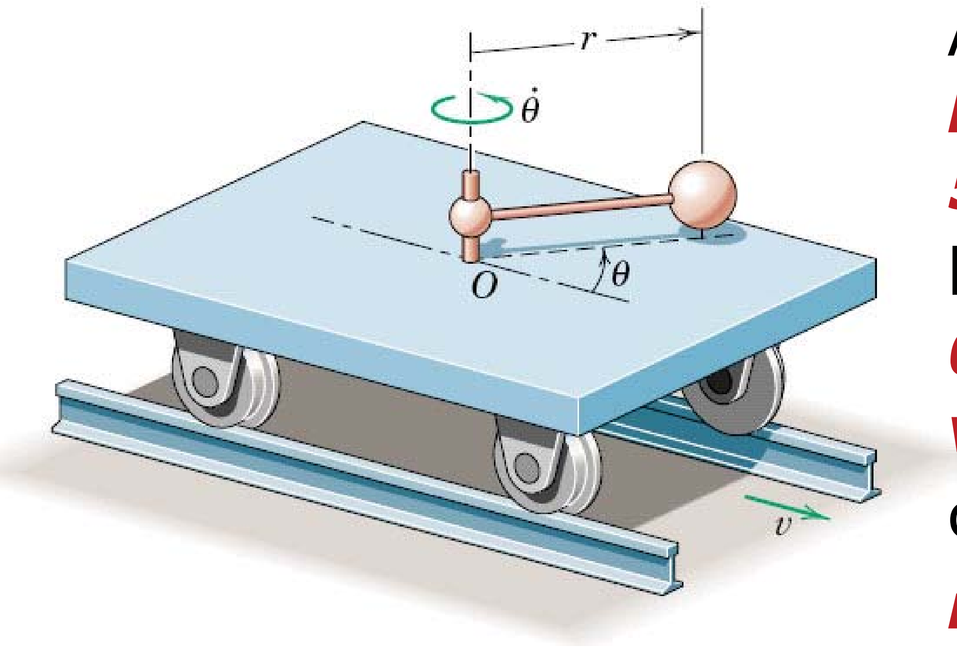
Impulse-Momentum: Another Exercise

Four ***3-kg*** balls are mounted to a frame freely rotating about the vertical ***z-axis*** at a rate of ***20 rad/s*** clockwise when viewed from above. A constant ***torque*** M ***= 30 Nm*** is applied to reverse the rotation.



Determine the ***time*** t to ***reverse the rotation*** and reach an ***angular velocity*** of ***20 rad/s*** in the same sense as M.

Impulse-Momentum: Yet Another Exercise



A small ***car*** with mass of ***20 kg*** rolls freely and carries a ***5-kg sphere*** mounted on a light rotating ***rod*** with ***r = 0.4 m*** and ***angular velocity*** of ***4 rad/s***. The car has a ***velocity v = 0.6 m/s*** when ***$\theta = 0^o$***.

Determine ***v*** when ***$\theta = 60^o$***.

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For Next Time…

- Continue Homework #10 due on ***Thursday (11/8)***
- Read Chapter 8, Section 8.2
- Read Chapter 5, Section 5.1