Curvilinear (*Two-Dimensional*) Motion

Lecture 3

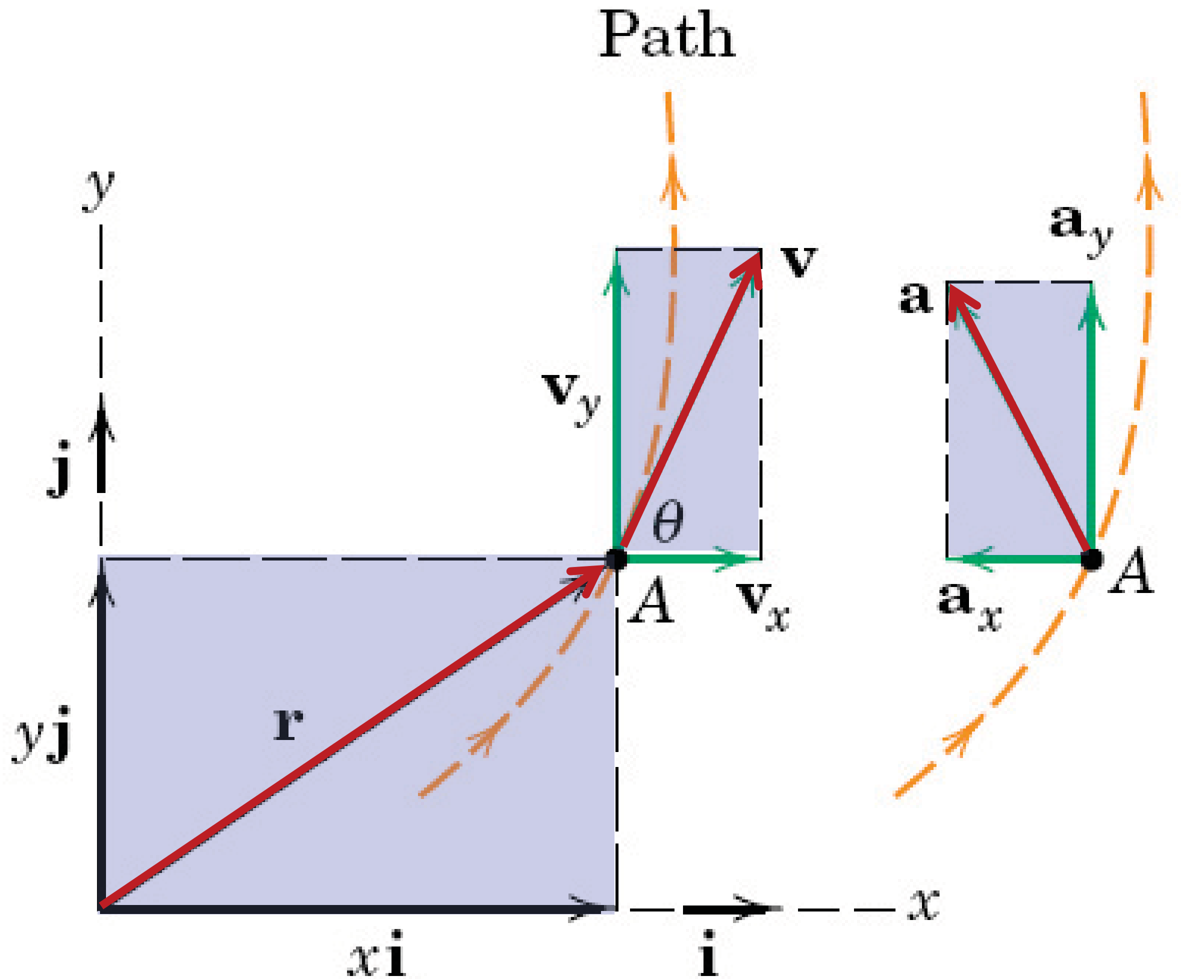
ME 231: Dynamics

Question of the Day

A particle moving along a curved path has a ***position vector*** ($\mathbf{r}$) given by

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

Determine the ***velocity*** and ***acceleration*** of the particle.

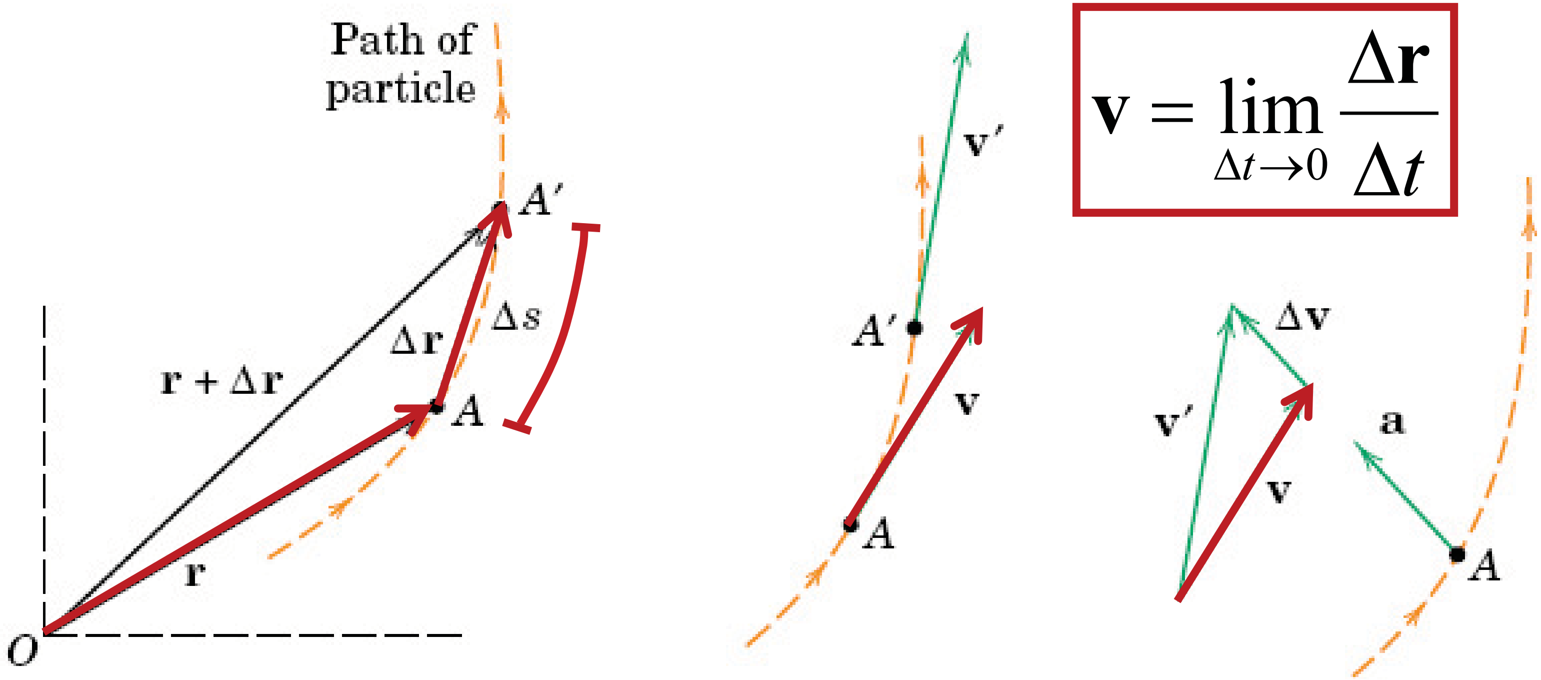


Outline for Today

- Question of the day
- Time derivative of a vector
- Velocity and acceleration
- Visualization of motion
- X-Y vector representation
- Projectile motion
- Answer your questions!

Time Derivative of a Vector

One of the most important concepts in dynamics!

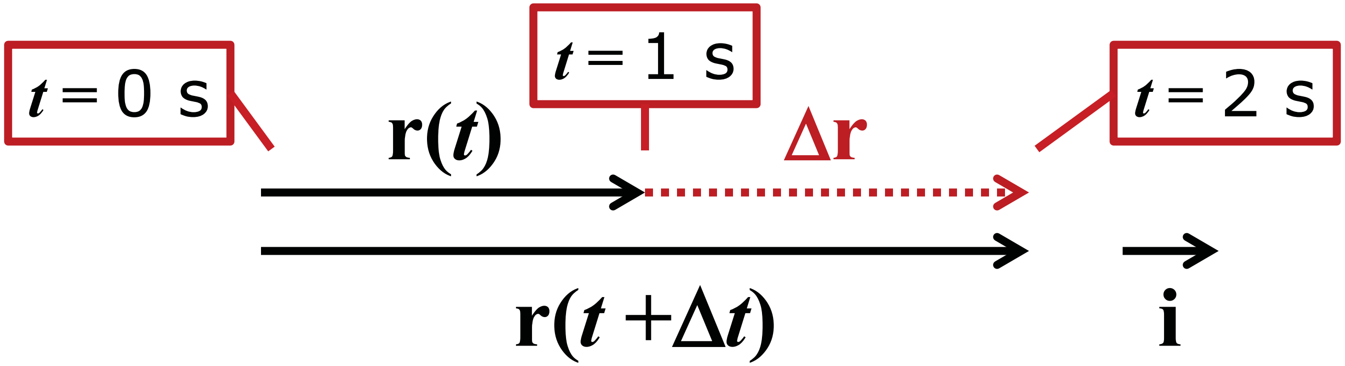


$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t}$$

- Δs is the scalar displacement along the path ($A \rightarrow A'$)
- **<u>Magnitude</u>** and **<u>direction</u>** of $\mathbf{r}$ are known at time t
- $\Delta \mathbf{r}$ is the vector (*not scalar*) change of position at $t + \Delta t$
- $\mathbf{v}$ has direction of $\Delta \mathbf{r}$ (*tangent*) and magnitude $|\Delta \mathbf{r} / \Delta t|$

Time Derivative of a Vector: Exercise

Magnitude changes, but direction constant



$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = 2\,\mathbf{i}$$

$$\mathbf{r}(t) = 2t\,\mathbf{i}$$

$$\Delta\mathbf{r} = (4-2)\,\mathbf{i} = 2\,\mathbf{i}$$

$\mathbf{v}$ has

direction of $\Delta\mathbf{r}$

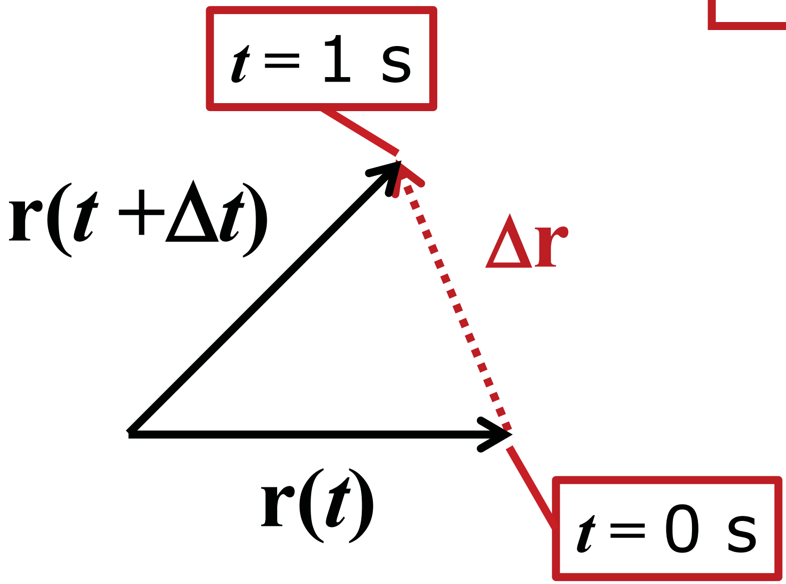
and magnitude $|\Delta\mathbf{r}/\Delta t|$

$$\left|\frac{\Delta\mathbf{r}}{\Delta t}\right| = \left|\frac{(4-2)\,\mathbf{i}}{(2-1)}\right| = \left|\frac{2\,\mathbf{i}}{1}\right| = 2$$

Time Derivative of a Vector: Another Case

Magnitude constant, but direction changes

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t}$$



$\mathbf{v}$ has

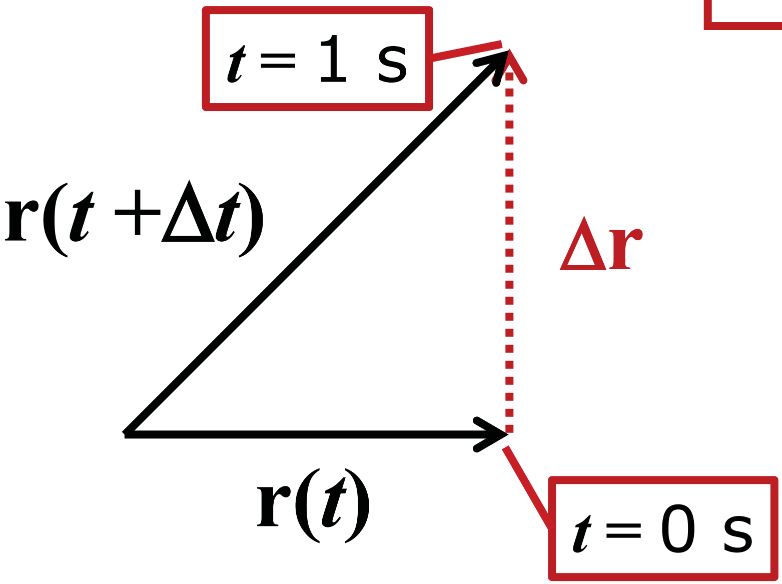
direction of $\Delta \mathbf{r}$

and magnitude $|\Delta \mathbf{r}/\Delta t|$

Time Derivative of a Vector: Another Case

Magnitude changes AND direction changes

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t}$$

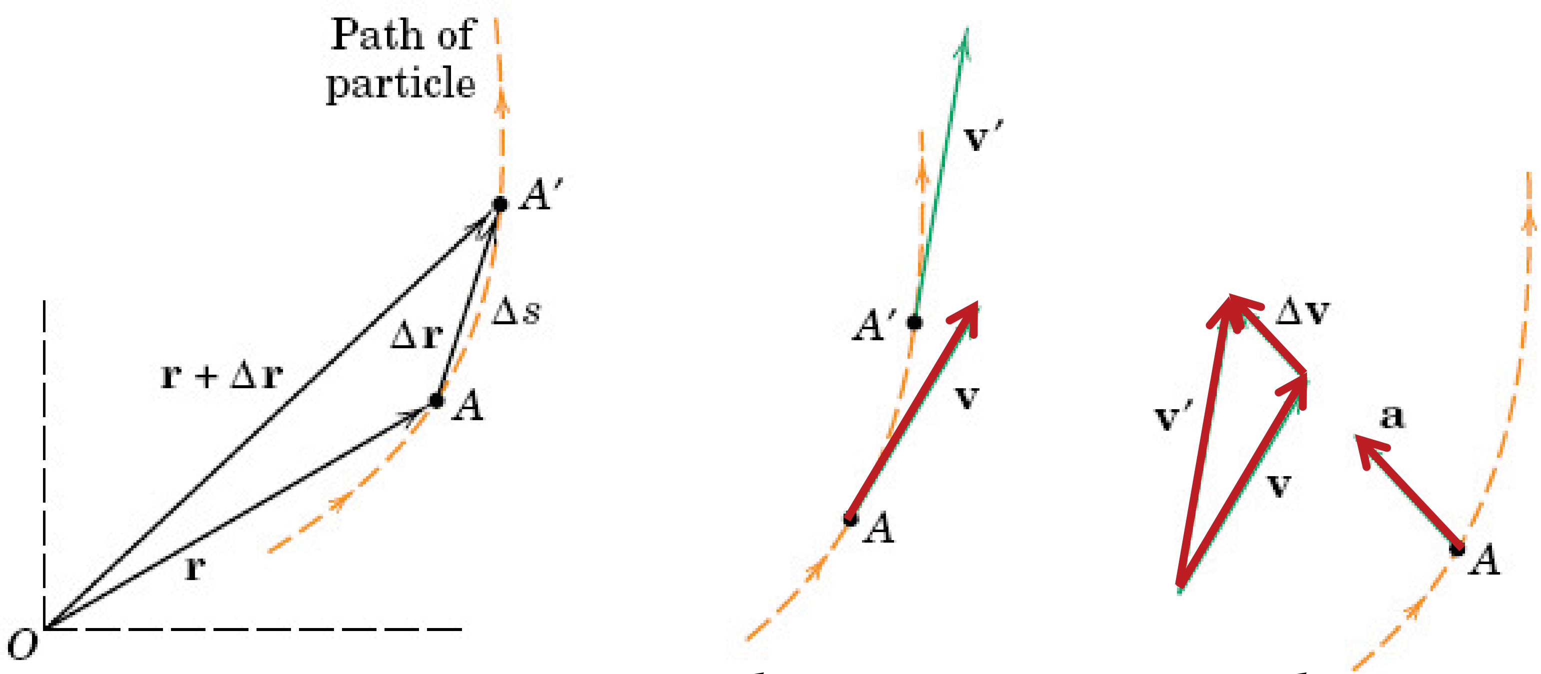


$\mathbf{v}$ has

direction of $\Delta \mathbf{r}$

and magnitude $|\Delta \mathbf{r}/\Delta t|$

Velocity and Acceleration



At A, $\mathbf{v} = \dfrac{d\mathbf{r}}{dt} = \dot{\mathbf{r}}$ $\qquad$ $\mathbf{a} = \dfrac{d\mathbf{v}}{dt} = \dot{\mathbf{v}}$

At A', $\mathbf{v}' = \dfrac{d\left(\mathbf{r} + \Delta\mathbf{r}\right)}{dt}$ $\qquad$ $\Delta\mathbf{v} = \mathbf{v}' - \mathbf{v}$

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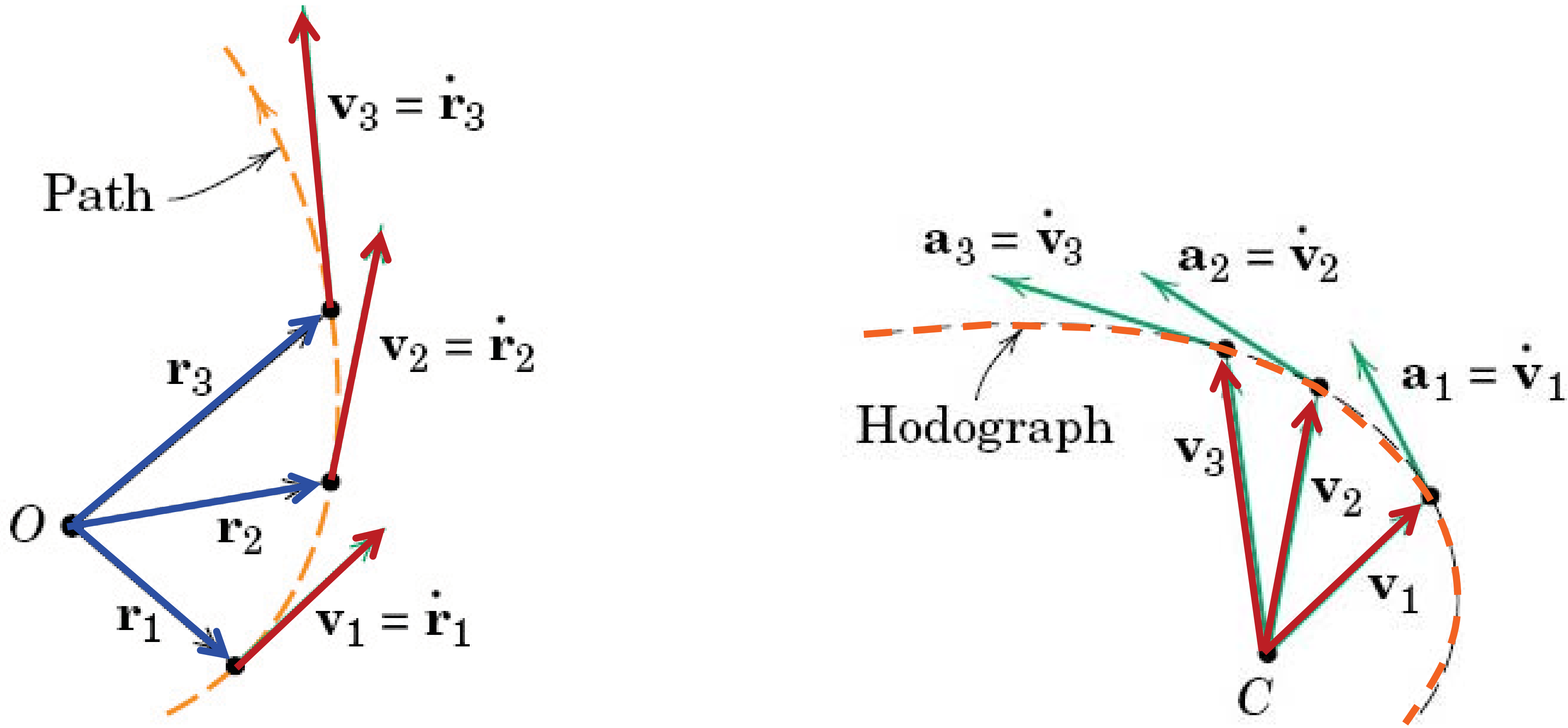
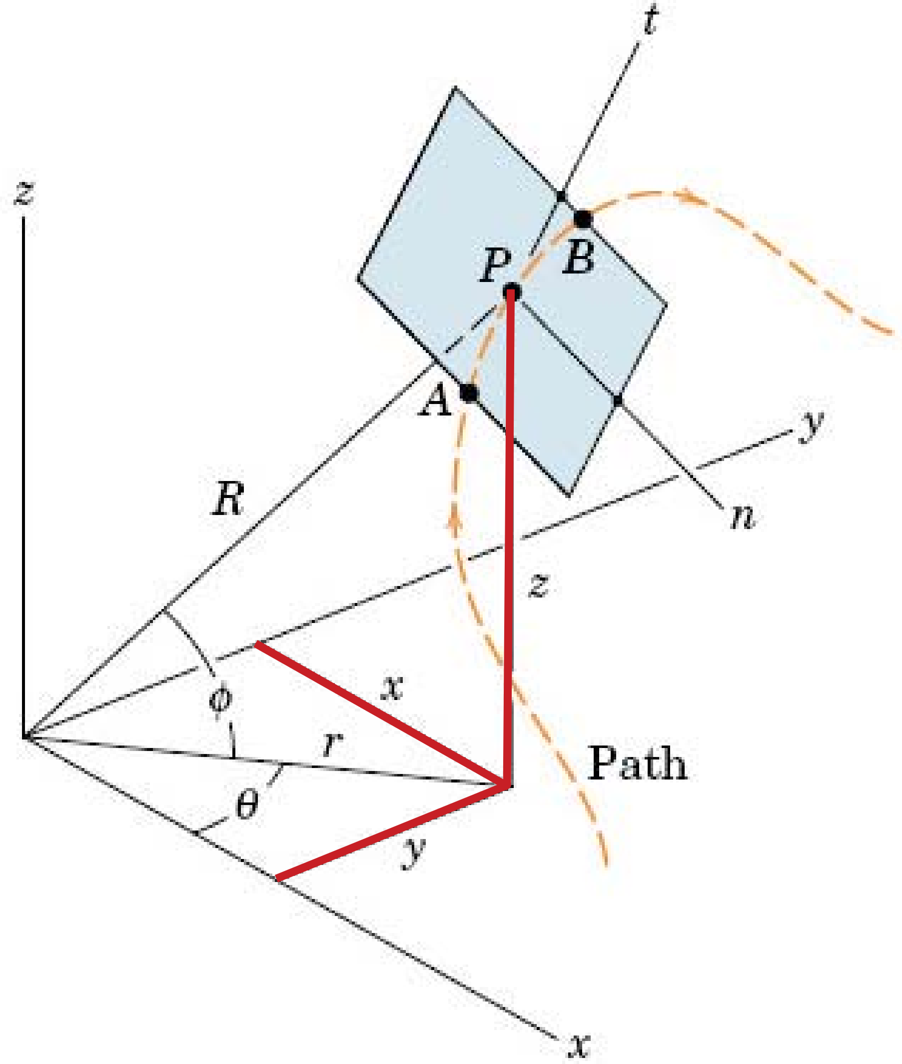
Visualization of Motion



Hodograph is a diagram that gives a vectorial visual representation of the movement of a body.

Recall: Possible Coordinate Systems

- Rectangular (x, y, z)
- Polar (r, θ, z)
- Spherical (R, θ, ϕ)
- Normal and Tangential (n, t)

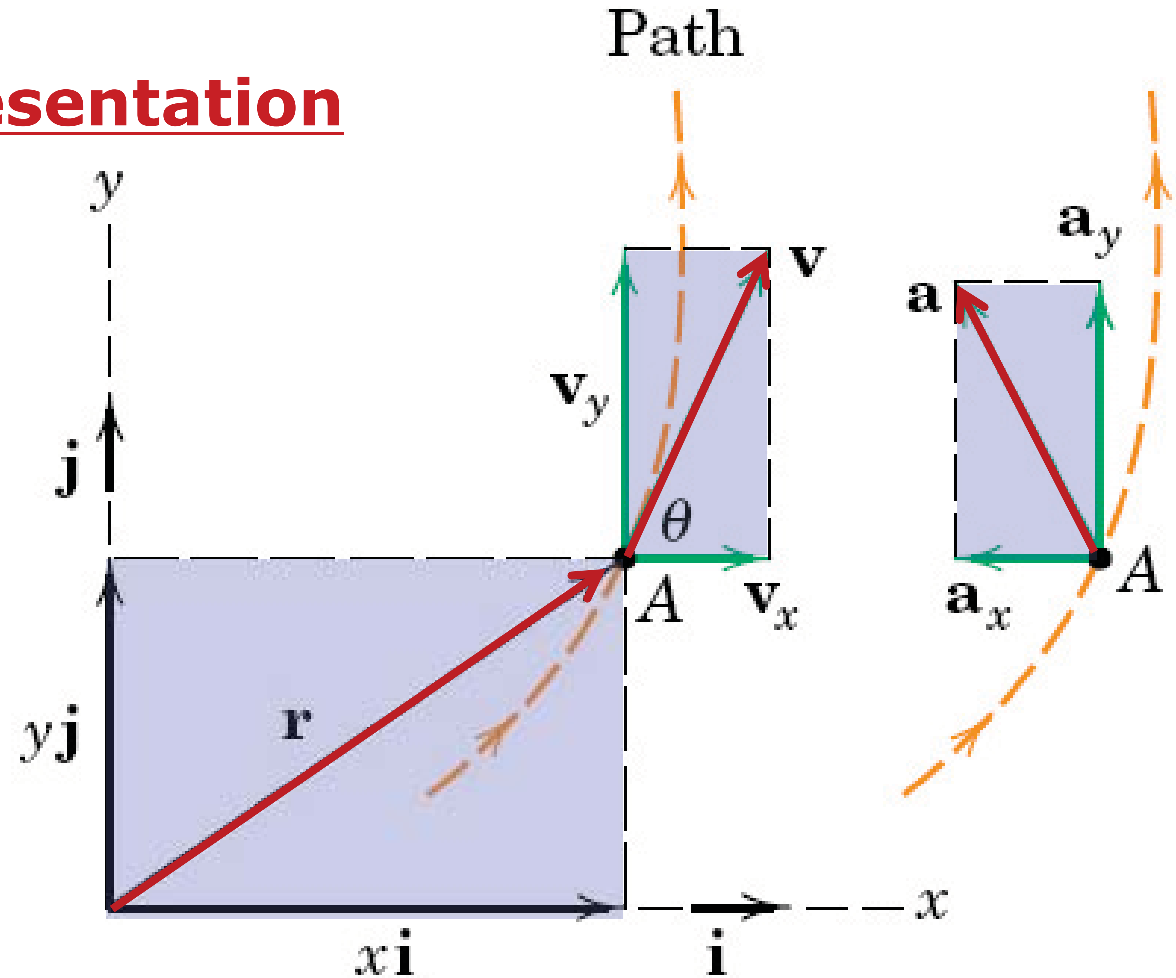


X-Y Vector Representation

$$\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j}$$

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\,\mathbf{i} + \dot{y}\,\mathbf{j}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \ddot{x}\,\mathbf{i} + \ddot{y}\,\mathbf{j}$$



- The x- and y-components are independent
- Resulting motion is a vector combination of x- and y-components

X-Y Vector Representation: Exercise

A particle moving in two-dimensions has a position vector ($\mathbf{r}$) as a function of time (t) with coordinates given by

$$x(t) = t^2 - 4t + 20 \quad , \quad y(t) = 3\sin(2t)$$

where $\mathbf{r}$ is measured in inches and t is in seconds.

Determine the magnitude of the ***velocity*** ($\mathbf{v}$) and the ***acceleration*** ($\mathbf{a}$) at time t =3 s.

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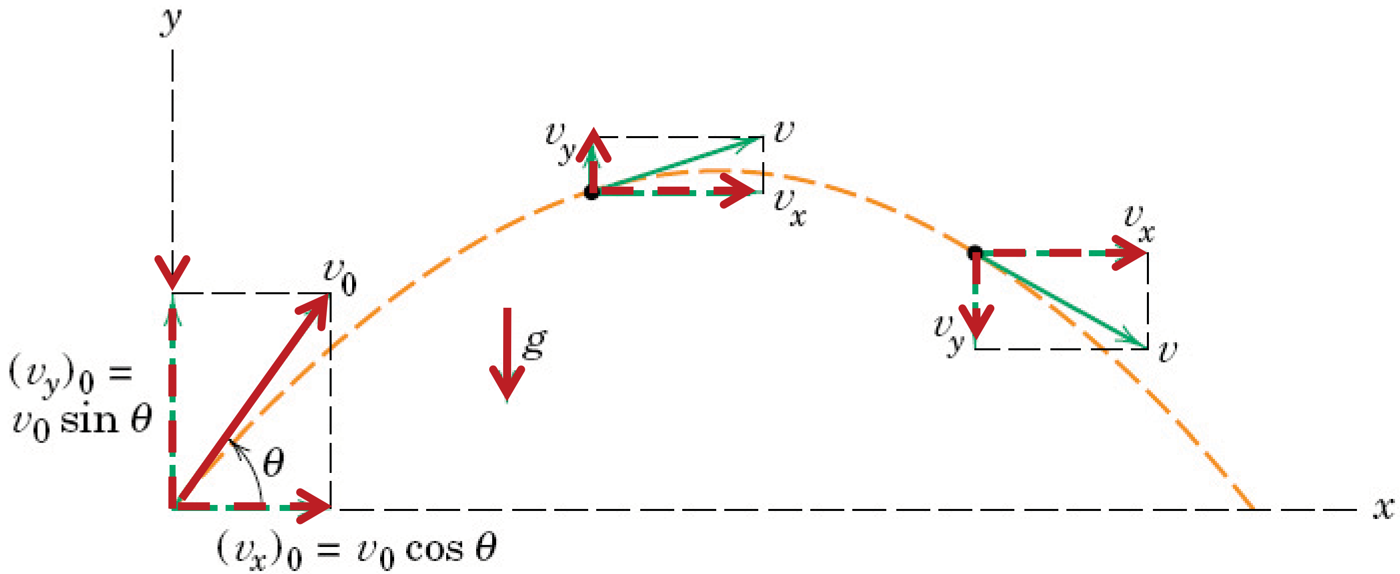
Projectile Motion

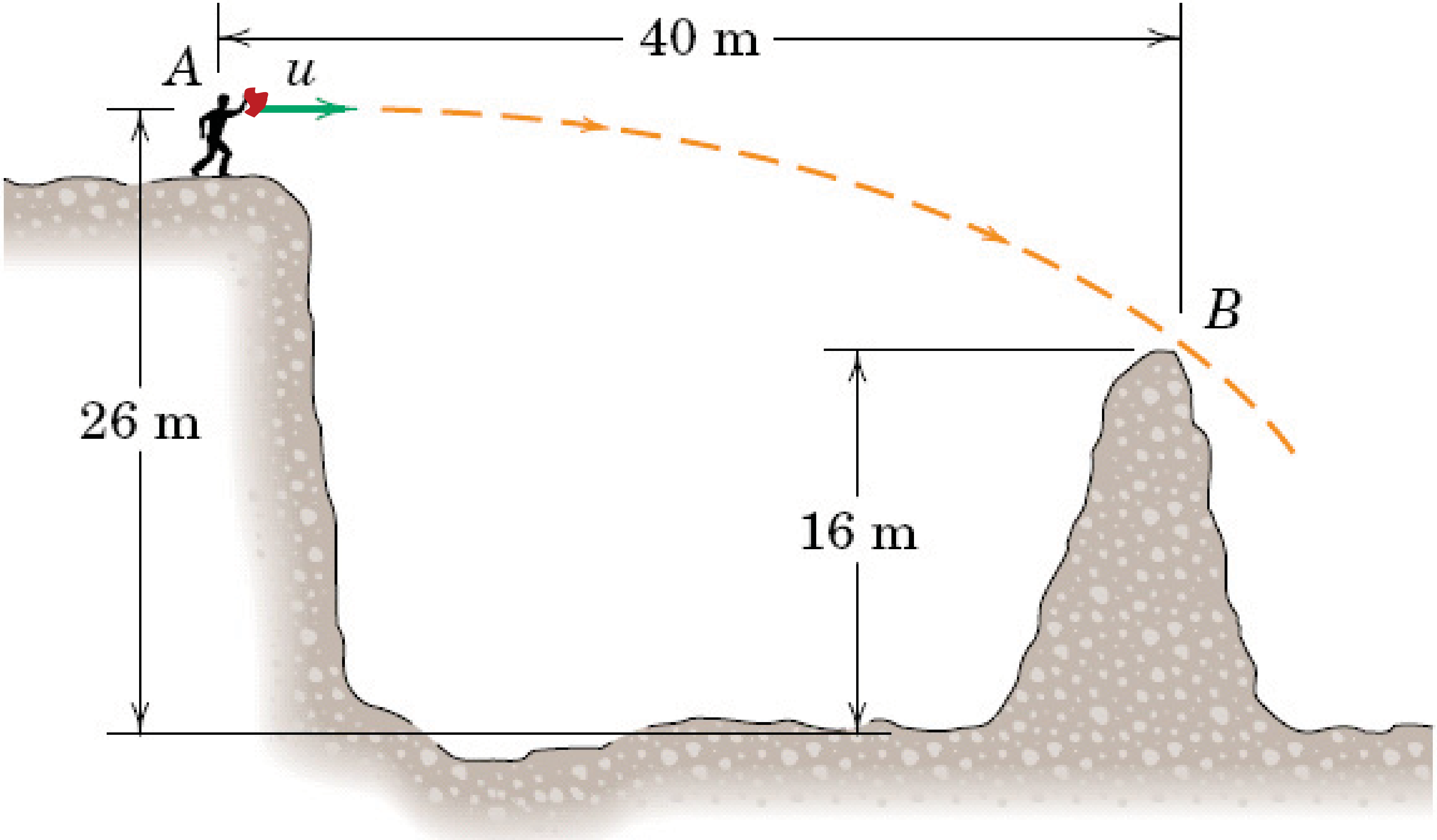


$$a_x = 0 \qquad a_y = -g$$

$$v_x = (v_x)_0 \qquad v_y = (v_y)_0 - gt$$

$$x = x_0 + (v_x)_0 t \qquad y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$$

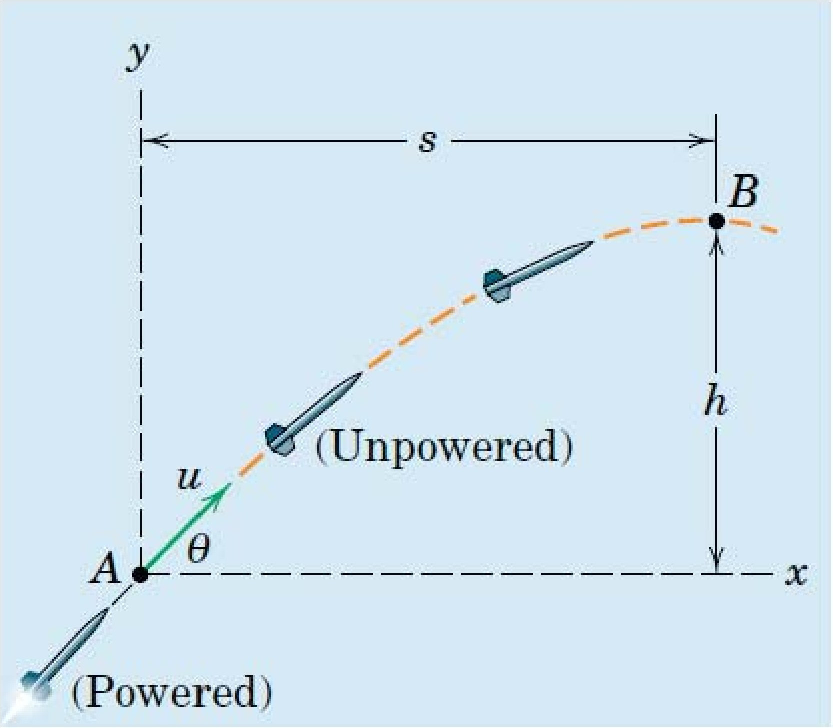
Projectile Motion: Exercise



What is the minimum horizontal ***velocity*** (***u***) a boy can throw a rock at *A* and have it clear the obstruction at *B*?

Projectile Motion: Exercise

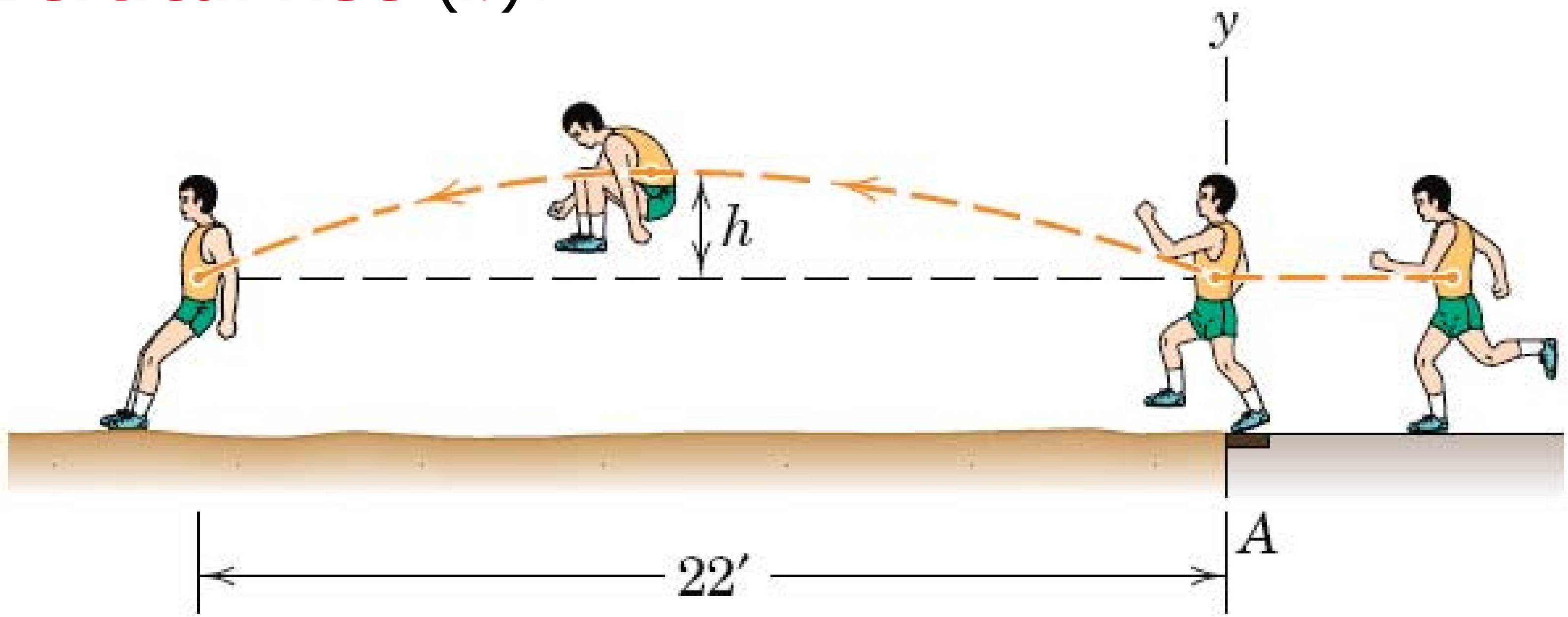
A rocket has expended all its fuel when it reaches ***position*** A, where it has a ***velocity*** of u at an angle θ with respect to the horizontal. It attains an additional ***height*** h at ***position*** B after traveling a ***distance*** s from A.



Determine expressions for h, s, and the ***time*** t of flight from A to B.

Projectile Motion: Exercise

With a horizontal ***velocity*** (v_x=30 ft/s), what is the vertical ***velocity*** (v_y) of the long jumper at takeoff to make the jump shown? What is the ***vertical rise*** (h)?



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For Next Time…

- Complete Homework #1 due on Wednesday (8/29) at the ***beginning of class***
- Read Chapter 2, Section 2.5