Impulse-Momentum for Rigid Bodies

Lecture 31

ME 231: Dynamics
A slender bar of mass 0.8 kg and length 0.4 m is falling with a velocity $v = 2 \text{ m/s}$ and angular velocity $\omega = 10 \text{ rad/s}$.

Determine the angular momentum $H_O$ of the bar about point $O$. 

\[ H_O = I_G \omega + mv\ell \]
Outline for Today

- Question of the day
- Linear momentum for rigid bodies
- Angular momentum for rigid bodies
- Interconnected rigid bodies
- Conservation of momentum for rigid bodies
- Answer your questions!
Recall: Linear Momentum for Particle Systems

\[
\begin{align*}
G_i &= m_i \mathbf{v}_i \\
G &= \sum m_i \mathbf{v}_i \\
G &= \sum m_i (\mathbf{v} + \mathbf{\dot{r}}_i) \\
G &= \sum m_i \mathbf{v} + \frac{d}{dt} \sum m_i \mathbf{\dot{r}}_i \\
G &= m \mathbf{\bar{v}} \\
\dot{G} &= m \mathbf{\bar{v}} = m \mathbf{\bar{a}} \\
\sum \mathbf{F} &= \dot{G}
\end{align*}
\]
Linear Momentum for Rigid Bodies

- Special case of a general system of particles

\[ \sum \mathbf{F} = \dot{\mathbf{G}} \]

\[ \mathbf{G}_1 + \int_{t_1}^{t_2} \sum \mathbf{F} \, dt = \mathbf{G}_2 \]

\[ \sum F_x = \dot{G}_x \]

\[ \sum F_y = \dot{G}_y \]

\[ \left(G_1\right)_x + \int_{t_1}^{t_2} \sum F_x \, dt = \left(G_2\right)_x \]

\[ \left(G_1\right)_y + \int_{t_1}^{t_2} \sum F_y \, dt = \left(G_2\right)_y \]
Recall: Angular Momentum for Particle Systems

About the Mass Center $G$

\[
(H_G)_i = \rho_i \times m_i \dot{\mathbf{r}}_i
\]

\[
H_G = \sum (\rho_i \times m_i \dot{\mathbf{r}}_i)
\]

\[
H_G = \sum \left[ \rho_i \times m_i \left( \dot{\mathbf{r}} + \dot{\rho}_i \right) \right]
\]

\[
H_G = \sum \left( \rho_i \times m_i \dot{\mathbf{r}} \right) + \sum \left( \rho_i \times m_i \dot{\rho}_i \right)
\]

\[
H_G = \left( -\dot{\mathbf{r}} \times \sum m_i \mathbf{\rho}_i \right) + \sum \left( \rho_i \times m_i \dot{\rho}_i \right)
\]

\[
H_G = \sum \left( \rho_i \times m_i \dot{\rho}_i \right)
\]

\[
\dot{H}_G = \sum (\rho_i \times \mathbf{F}_i)
\]

\[
\sum \mathbf{M}_G = \dot{H}_G
\]
Angular Momentum for Rigid Bodies
About the Mass Center $G$

**linear**

$$G = m\bar{v}$$

$$\sum \mathbf{F} = \mathbf{G}$$

**angular**

$$\mathbf{H}_G = I_G \mathbf{\omega}$$

$$\sum \mathbf{M}_G = \dot{\mathbf{H}}_G$$

$$G_1 + \int_{t_1}^{t_2} \sum \mathbf{F} \, dt = G_2$$

$$\left(\mathbf{H}_G\right)_1 + \int_{t_1}^{t_2} \sum \mathbf{M}_G \, dt = \left(\mathbf{H}_G\right)_2$$
Alternate Angular Momentum for Rigid Bodies

About an Arbitrary Point \( P \)

\[ H_P = I_G \omega + mvd \]

\[ \sum M_P = \dot{H}_P \]

\[ (H_P)_1 + \int_{t_1}^{t_2} \sum M_P \, dt = (H_P)_2 \]

About a Fixed Point \( O \)

\[ H_O = I_O \omega \]

\[ \sum M_O = \dot{H}_O \]

\[ (H_O)_1 + \int_{t_1}^{t_2} \sum M_O \, dt = (H_O)_2 \]
Recall: Systems of Interconnected Bodies

**free-body diagram**

**kinetic diagram**

\[ \sum \mathbf{F} = \sum m \mathbf{a} \]

\[ \sum M_P = \sum I_G \alpha + \sum m \mathbf{a}d \]
Interconnected Rigid Bodies

\[ \sum \mathbf{F} = \dot{\mathbf{G}}_a + \dot{\mathbf{G}}_b + \cdots \]

\[ \int_1^2 \sum \mathbf{F} \, dt = (\Delta \mathbf{G})_{\text{system}} \]

\[ \sum \mathbf{M}_O = (\dot{\mathbf{H}}_O)_a + (\dot{\mathbf{H}}_O)_b + \cdots \]

\[ \int_1^2 \sum \mathbf{M}_O \, dt = (\Delta \mathbf{H}_O)_{\text{system}} \]
Recall: Conservation of Momentum for Systems

- If the resultant external force $\Sigma F$ is zero, then linear momentum is conserved.
- If the resultant moment about a fixed point $O$ or mass center $G$ is zero, then angular momentum is conserved.
Conservation of Momentum for Rigid Bodies

Exactly the same as for particle systems!

\[ G_1 = G_2 \]

\[ \left( H_O \right)_1 = \left( H_O \right)_2 \]

\[ \left( H_G \right)_1 = \left( H_G \right)_2 \]

- If the **resultant external force** \( \Sigma F \) is zero, then **linear momentum** is conserved
- If the **resultant moment about** a fixed point \( O \) or mass center \( G \) is zero, then **angular momentum** is conserved
A constant horizontal force $P$ is applied to the light yoke attached to the center $O$ of a uniform circular disk of mass $m$, which is initially at rest and rolls without slipping.

Determine the velocity $v$ of the center $O$ in terms of $t$. 
A uniform circular disk has a **mass** of **25 kg** and is mounted to a rotating bar in three different ways and \( \omega_0 = 4 \text{ rad/s} \)

Determine the **angular momentum** \( H_0 \) of the disk about **point O** for each case.
Fully extended the diver’s 80-kg body has an angular velocity of 0.3 rev/s.

Determine the angular velocity $\omega$ later when the diver assumes a tuck position.
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For Next Time...

- **Exam 2.a on Friday (11/9)**
- Homework #10 due on **Monday (11/12)**
- Homework #11 due on **Wednesday (11/14)**
- Read Chapter 8, Sections 8.2 & 8.3