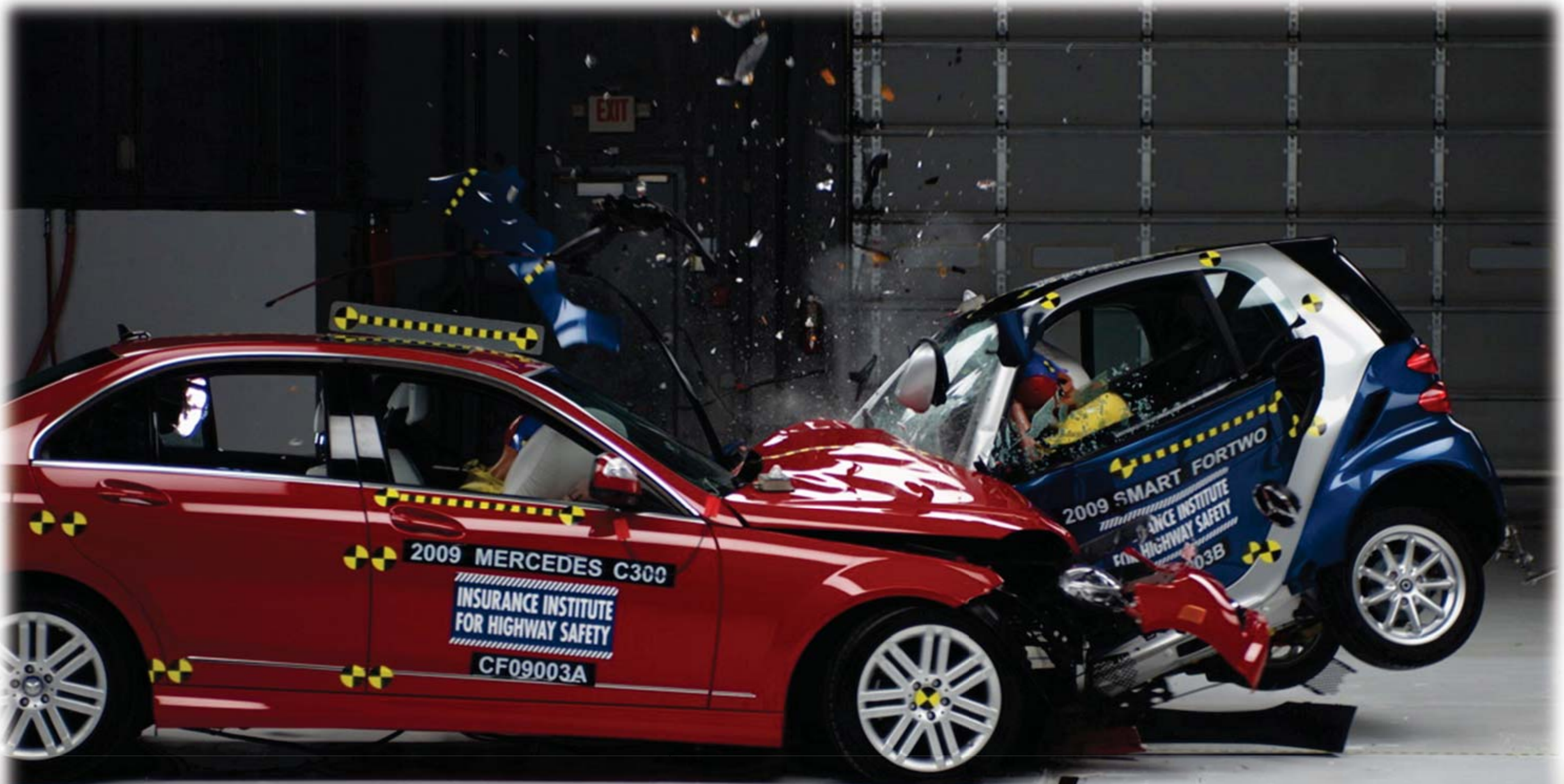


Impulse-Momentum for Rigid Bodies

Lecture 31

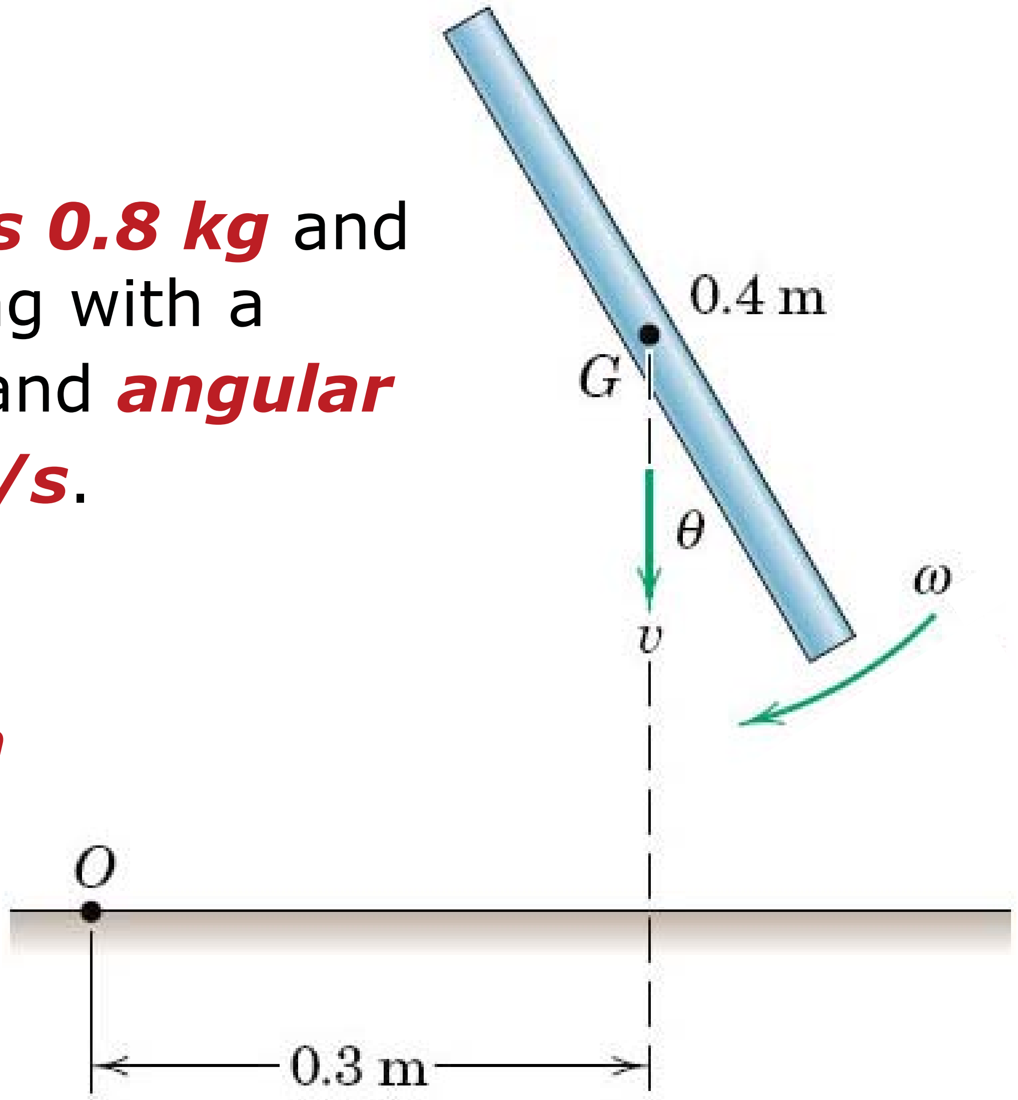
ME 231: Dynamics

$$H_O = I_G\omega + mvd$$

Question of the Day

A slender bar of ***mass 0.8 kg*** and ***length 0.4 m*** is falling with a ***velocity*** v ***= 2 m/s*** and ***angular velocity*** ω ***= 10 rad/s***.

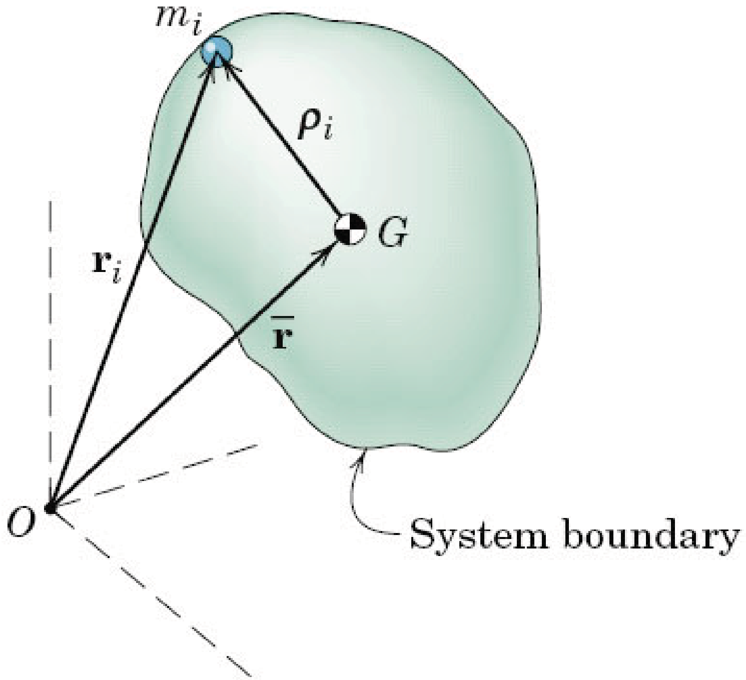
Determine the ***angular momentum*** H_O of the bar about ***point*** O.



Outline for Today

- Question of the day
- Linear momentum for rigid bodies
- Angular momentum for rigid bodies
- Interconnected rigid bodies
- Conservation of momentum for rigid bodies
- Answer your questions!

Recall: Linear Momentum for Particle Systems



$$\mathbf{G}_i = m_i \mathbf{v}_i$$

$$\mathbf{G} = \sum m_i \mathbf{v}_i$$

$$\mathbf{G} = \sum m_i \left(\bar{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i \right)$$

$$\mathbf{G} = \sum m_i \bar{\mathbf{v}} + \frac{d}{dt} \sum m_i \boldsymbol{\rho}_i$$

$$m\bar{\boldsymbol{\rho}} = \mathbf{0}$$

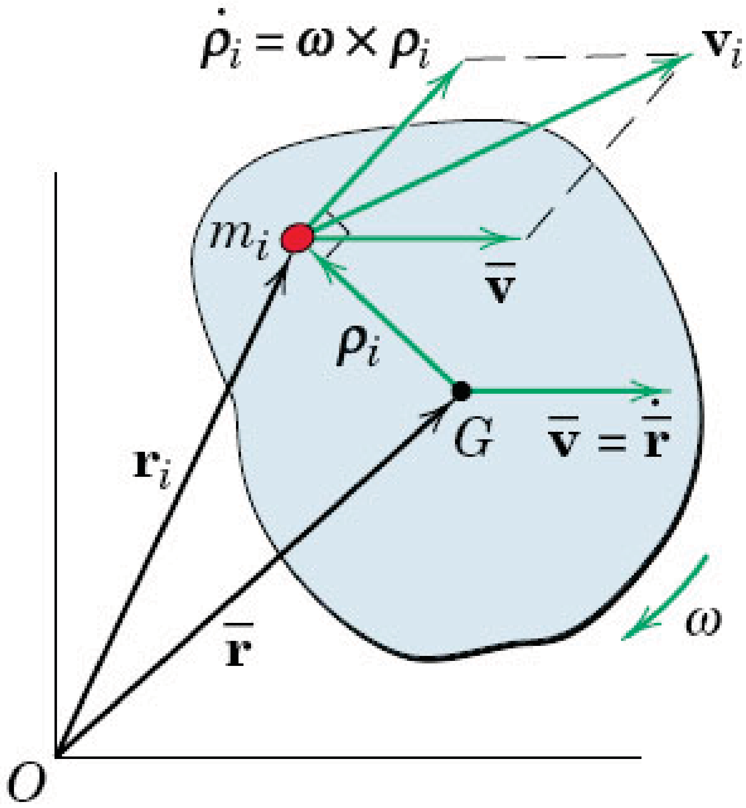
$$\mathbf{G} = m\bar{\mathbf{v}}$$

$$\dot{\mathbf{G}} = m\dot{\bar{\mathbf{v}}} = m\bar{\mathbf{a}}$$

$$\sum \mathbf{F} = \dot{\mathbf{G}}$$

Linear Momentum for Rigid Bodies

- Special case of a general system of particles



$$\mathbf{G} = m\bar{\mathbf{v}}$$

$$\sum \mathbf{F} = \dot{\mathbf{G}}$$

$$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \mathbf{G}_2$$

$$\sum F_x = \dot{G}_x$$

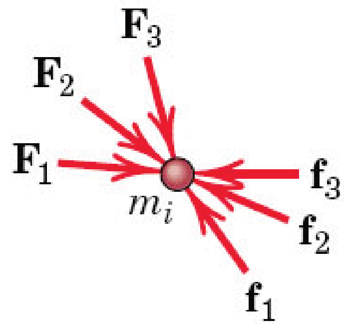
$$\sum F_y = \dot{G}_y$$

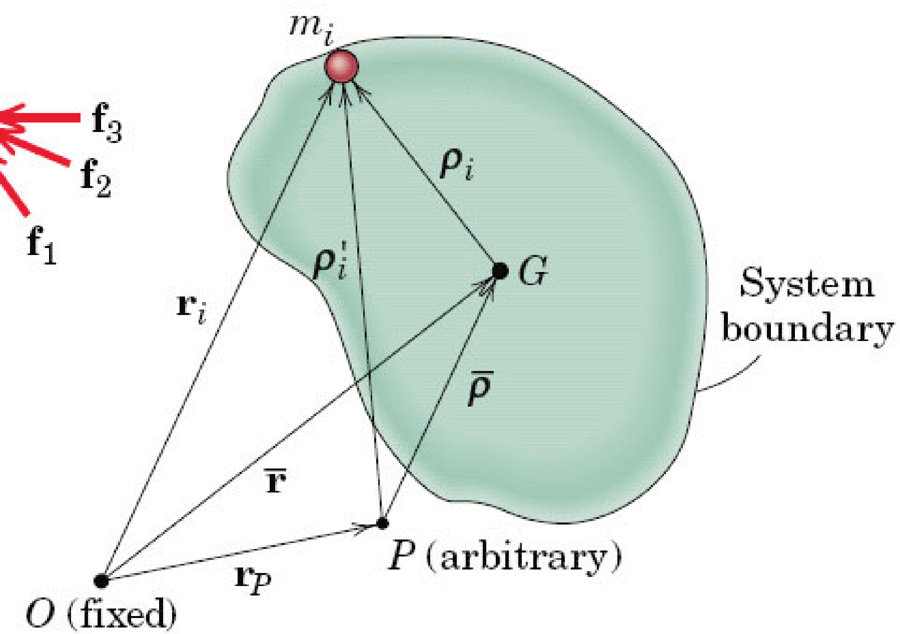
$$(G_1)_x + \int_{t_1}^{t_2} \Sigma F_x\, dt = (G_2)_x$$

$$(G_1)_y + \int_{t_1}^{t_2} \Sigma F_y\, dt = (G_2)_y$$

Recall: Angular Momentum for Particle Systems

About the Mass Center *G*





$$(\mathbf{H}_G)_i = \boldsymbol{\rho}_i \times m_i \dot{\mathbf{r}}_i$$

$$\mathbf{H}_G = \sum(\boldsymbol{\rho}_i \times m_i \dot{\mathbf{r}}_i)$$

$$\mathbf{H}_G = \sum[\boldsymbol{\rho}_i \times m_i(\dot{\bar{\mathbf{r}}} + \dot{\boldsymbol{\rho}}_i)]$$

$$\mathbf{H}_G = \sum(\boldsymbol{\rho}_i \times m_i \dot{\bar{\mathbf{r}}}) + \sum(\boldsymbol{\rho}_i \times m_i \dot{\boldsymbol{\rho}}_i)$$

$$\mathbf{H}_G = (-\dot{\bar{\mathbf{r}}} \times \sum m_i \boldsymbol{\rho}_i) + \sum(\boldsymbol{\rho}_i \times m_i \dot{\boldsymbol{\rho}}_i)$$

$\sum m_i \boldsymbol{\rho}_i \rightarrow \mathbf{0}$

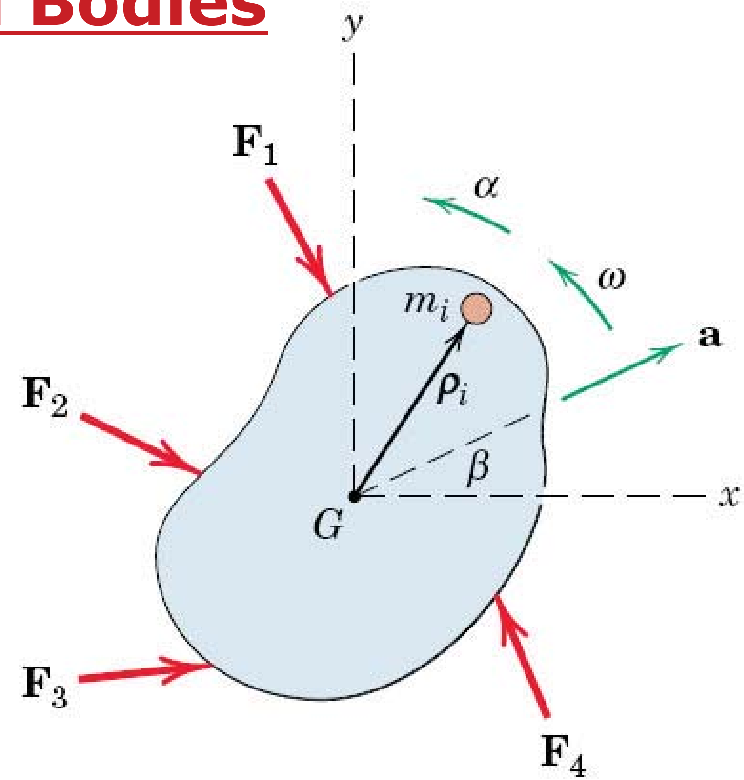
$$\mathbf{H}_G = \sum(\boldsymbol{\rho}_i \times m_i \dot{\boldsymbol{\rho}}_i)$$

$$\dot{\mathbf{H}}_G = \sum(\boldsymbol{\rho}_i \times \mathbf{F}_i)$$

$$\boxed{\sum \mathbf{M}_G = \dot{\mathbf{H}}_G}$$

Angular Momentum for Rigid Bodies

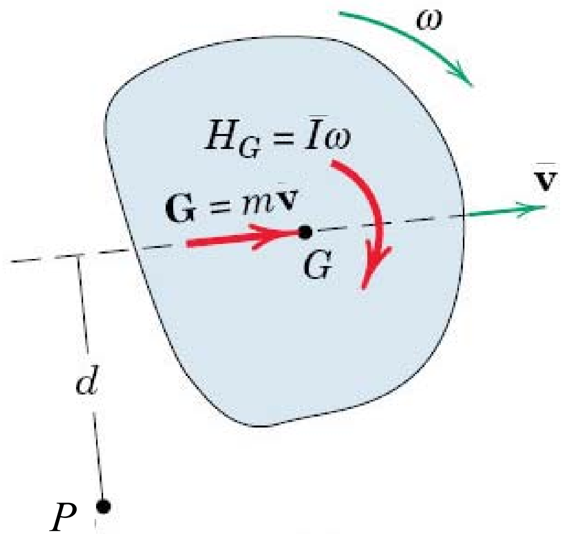
About the Mass Center G



linear	***angular***
$\mathbf{G} = m\overline{\mathbf{v}}$	$\mathbf{H}_G = I_G \boldsymbol{\omega}$
$\sum \mathbf{F} = \dot{\mathbf{G}}$	$\sum \mathbf{M}_G = \dot{\mathbf{H}}_G$
$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \mathbf{G}_2$	$\left(\mathbf{H}_G\right)_1 + \int_{t_1}^{t_2} \Sigma \mathbf{M}_G\, dt = \left(\mathbf{H}_G\right)_2$

Alternate Angular Momentum for Rigid Bodies

About an Arbitrary Point P

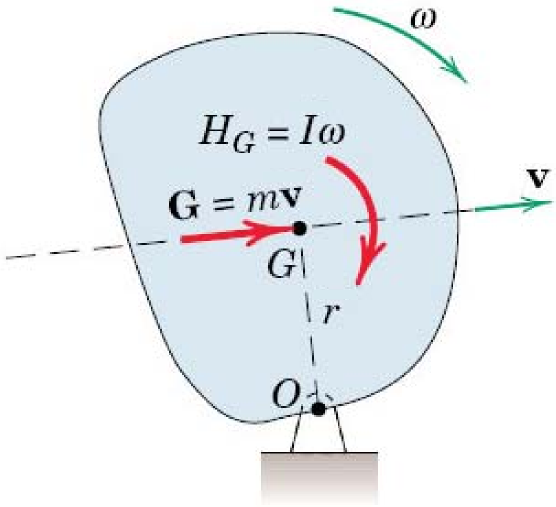


$$H_P = I_G\omega + mvd$$

$$\sum \mathbf{M}_P = \dot{\mathbf{H}}_P$$

$$\left(\mathbf{H}_P\right)_1 + \int_{t_1}^{t_2} \Sigma \mathbf{M}_P \, dt = \left(\mathbf{H}_P\right)_2$$

About a Fixed Point O



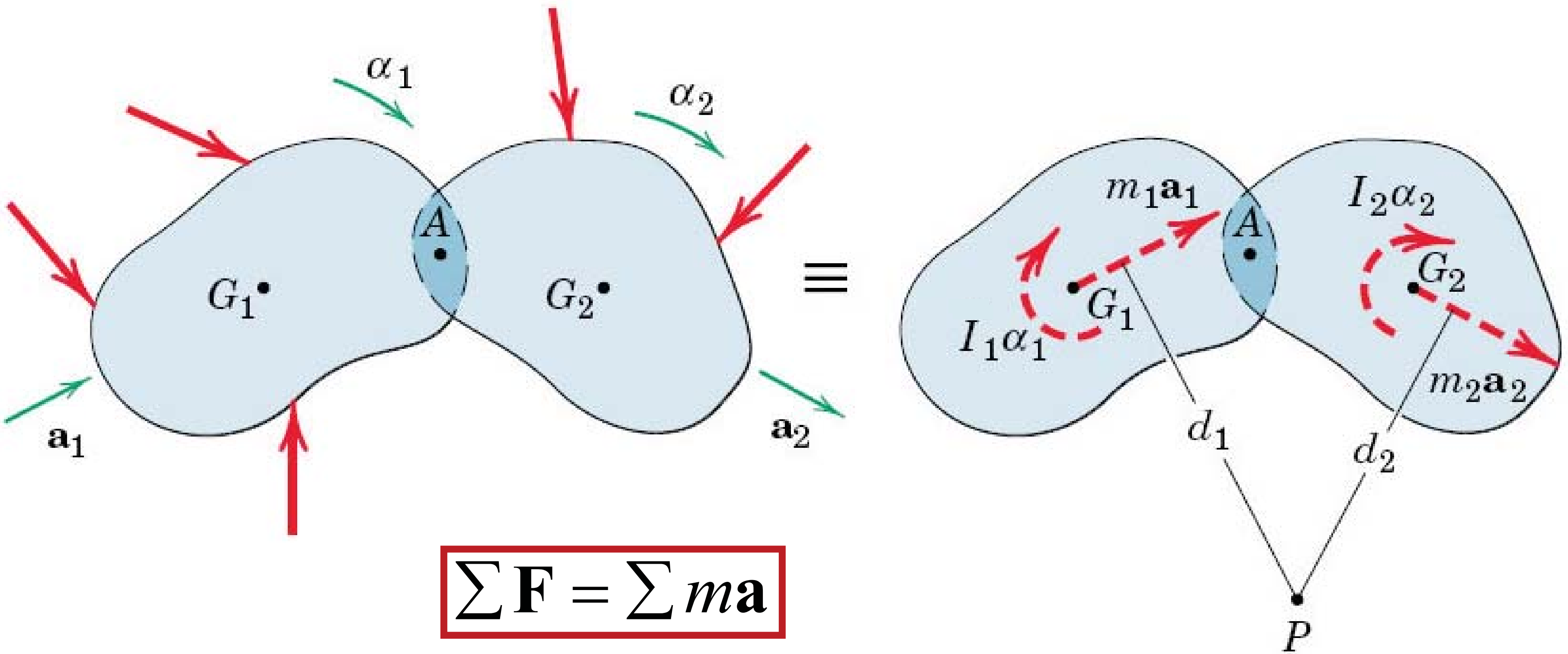
$$H_O = I_O\omega$$

$$\sum \mathbf{M}_O = \dot{\mathbf{H}}_O$$

$$\left(\mathbf{H}_O\right)_1 + \int_{t_1}^{t_2} \Sigma \mathbf{M}_O \, dt = \left(\mathbf{H}_O\right)_2$$

Recall: Systems of Interconnected Bodies

free-body diagram ***kinetic diagram***



$$\sum \mathbf{F} = \sum m\mathbf{a}$$

$$\sum M_P = \sum I_G \alpha + \sum mad$$

Interconnected Rigid Bodies

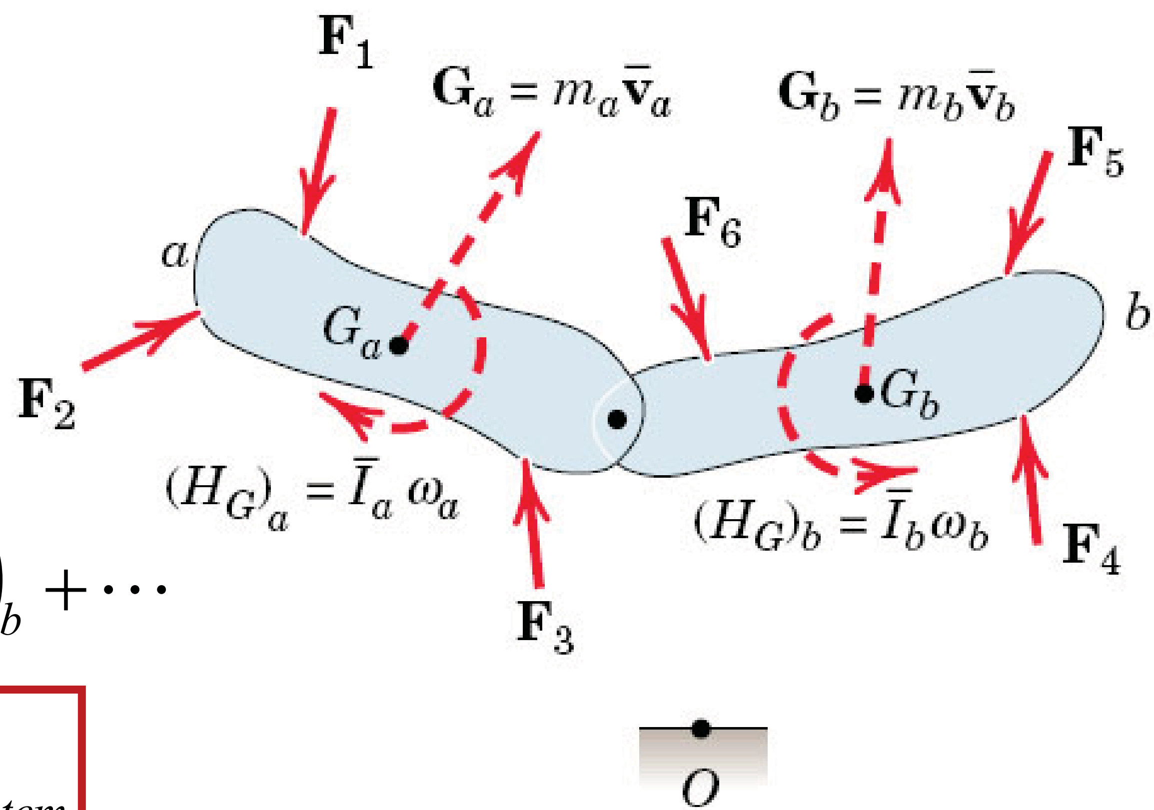
$$\sum \mathbf{F} = \dot{\mathbf{G}}_a + \dot{\mathbf{G}}_b + \cdots$$

$$\int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \left(\Delta \mathbf{G}\right)_{system}$$

$$\sum \mathbf{M}_O = \left(\dot{\mathbf{H}}_O\right)_a + \left(\dot{\mathbf{H}}_O\right)_b + \cdots$$

$$\int_{t_1}^{t_2} \Sigma \mathbf{M}_O\, dt = \left(\Delta \mathbf{H}_O\right)_{system}$$

combined free-body and momentum diagrams



Recall: Conservation of Momentum for Systems

$$\mathbf{G}_1 = \mathbf{G}_2$$

$$\left(\mathbf{H}_O\right)_1 = \left(\mathbf{H}_O\right)_2$$

$$\left(\mathbf{H}_G\right)_1 = \left(\mathbf{H}_G\right)_2$$

- If the ***resultant external force*** $\Sigma\mathbf{F}$ is zero, then ***linear momentum*** is ***conserved***
- If the ***resultant moment about*** a fixed ***point*** O or ***mass center*** G is zero, then ***angular momentum*** is ***conserved***

Conservation of Momentum for Rigid Bodies

Exactly the same as for particle systems!

$$\mathbf{G}_1 = \mathbf{G}_2$$

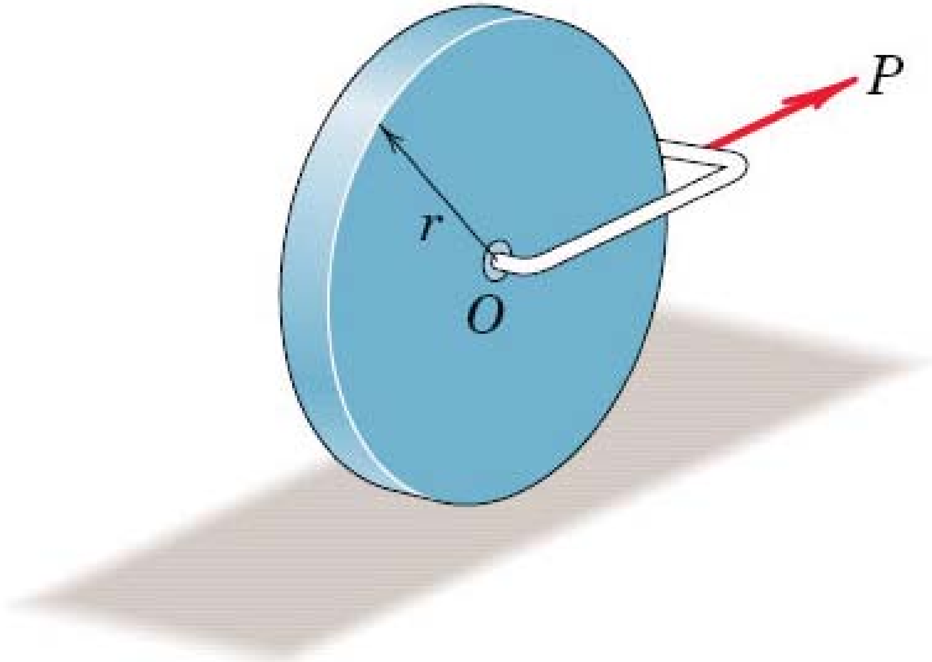
$$\left(\mathbf{H}_O\right)_1 = \left(\mathbf{H}_O\right)_2$$

$$\left(\mathbf{H}_G\right)_1 = \left(\mathbf{H}_G\right)_2$$

- If the ***resultant external force*** $\Sigma\mathbf{F}$ is zero, then ***linear momentum*** is ***conserved***
- If the ***resultant moment about*** a fixed ***point*** O or ***mass center*** G is zero, then ***angular momentum*** is ***conserved***

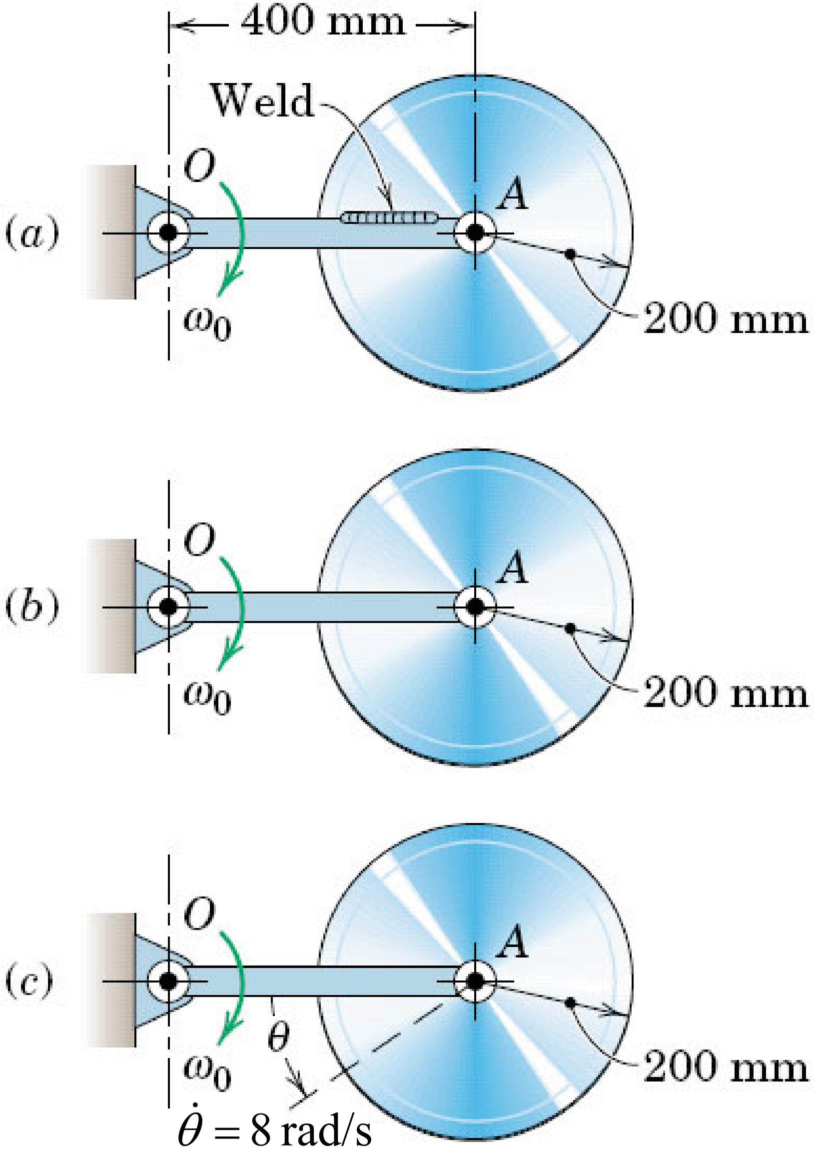
Impulse-Momentum for Rigid Bodies: Exercise 1

A constant horizontal ***force*** P is applied the light yoke attached to the ***center*** O of a uniform circular disk of ***mass*** m, which is initially at rest and rolls without slipping.



Determine the ***velocity*** v of the ***center*** O in terms of t.

Impulse-Momentum for Rigid Bodies: Exercise 2



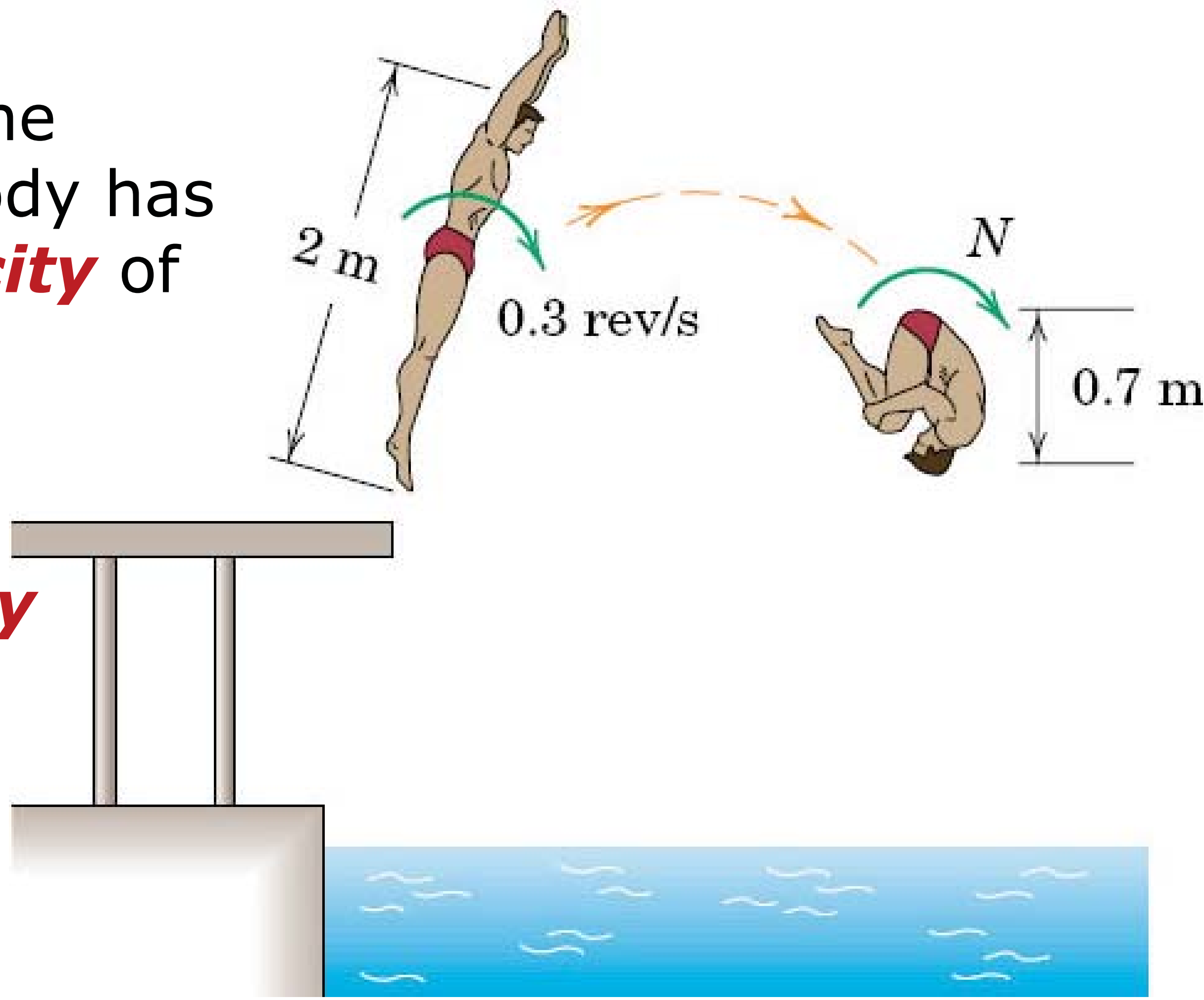
A uniform circular disk has a ***mass*** of ***25 kg*** and is mounted to a rotating bar in three different ways and $\omega_O = 4$ ***rad/s***

Determine the ***angular momentum*** H_O of the disk about ***point*** O for each case.

Impulse-Momentum for Rigid Bodies: Exercise 3

Fully extended the diver's ***80-kg*** body has a ***angular velocity*** of ***0.3 rev/s***.

Determine the ***angular velocity*** ω later when the diver assumes a tuck position.



Outline for Today

- Question of the day
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For Next Time…

- ***Exam 2.a on Friday (11/9)***
- Homework #10 due on ***Monday (11/12)***
- Homework #11 due on ***Wednesday(11/14)***
- Read Chapter 8, Sections 8.2 & 8.3