Kinetics: **F**=m**a** (Ch. 3 & 7) Review

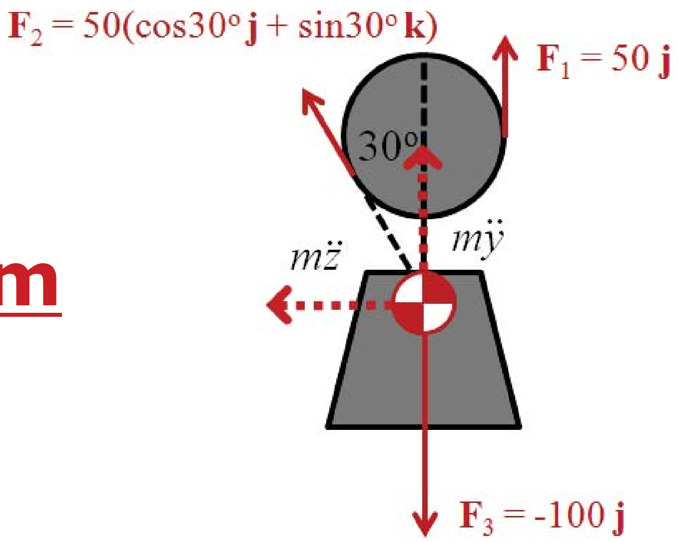
Lecture 32

ME 231: Dynamics

Question of the Day

What is the most important concept in ***mechanics***?

Free Body Diagram



What is the most important concept in ***dynamics***?

Equations of Motion

$$\sum \mathbf{F} = m\mathbf{a}$$

$$\sum F_x = ma_x = m\ddot{x}$$

$$\sum F_y = ma_y = m\ddot{y}$$

$$\sum F_z = ma_z = m\ddot{z}$$

Outline for Today

- Question of the day
- Where are we in the course?
- Inverse vs. forward dynamics
- Kinetics: cause of motion
- Possible solutions to kinetics problems
- Direct application of Newton's 2^{nd} Law
- Plane motion types for rigid bodies
- Equations, equations, equations…
- Exam 2a breakdown (kinetics: $\mathbf{F}=m\mathbf{a}$)

Where are we in the course?

Concept: What is dynamics?

Chapters 1, 2, 6 — **Kinematics**

Chapters 3, 5, 7, 8 — **Kinetics**

Kinematics + **Kinetics** = **Dynamics**

Relationship among ***position***, ***velocity***, and ***acceleration***

Relationship among ***forces (and moments)*** and ***acceleration***

Where are we in the course?

Calculation: How do we use dynamics?

Newton's 2nd Law

Force. A push or pull exerted on a body, characterized by:

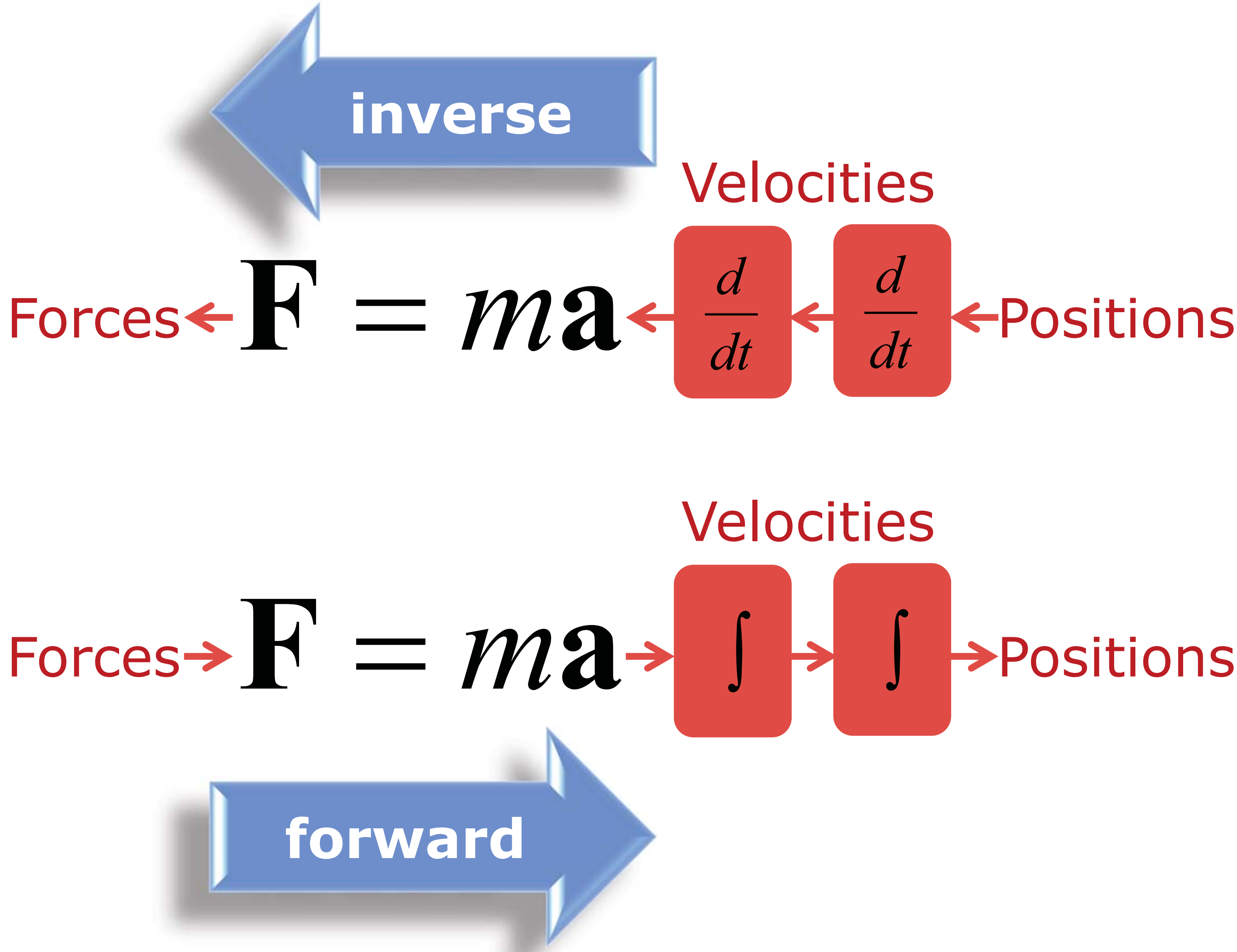
- magnitude
- direction
- point of application

$$F = m\,a$$

Mass. Measure of the resistance of a body to linear acceleration.

Acceleration. Velocity rate of change with respect to time

Inverse vs. Forward Dynamics



Kinetics: Cause of Motion?

Concept: What is kinetics?

ME 202 | **Chapters 1, 2, 6** | **Chapters 3, 5, 7, 8**

Relationship among ***forces (and moments)*** and ***equilibrium***

Relationship among ***position***, ***velocity***, and ***acceleration***

Relationship among ***forces (and moments)***and ***acceleration***

Possible Solutions to Kinetics Problems

- Direct application of ***Newton's 2nd Law***
  - force-mass-acceleration method
  - *Chapters 3 and 7*
- Use of ***impulse*** and ***momentum*** methods
  - *Chapters 5 and 8*
- Use of ***work*** and ***energy*** principles
  - *Chapter 4*

Step-by-Step Solution Process

1. ***Kinematics***
   - Identify type of ***motion***
   - Solve for ***linear*** and ***angular accelerations***
2. ***Diagram***
   - Assign ***inertial coordinate system***
   - Draw complete ***free-body diagram***
   - Draw ***kinetic diagram*** to clarify equations
3. ***Equations of motion***
   - Apply ***2 linear*** and ***1 angular equations***
   - Maintain ***consistent sense***
   - Solve for no more than 5 scalar unknowns (***3 scalar equations of motion*** and ***2 scalar relations*** from the ***relative-acceleration equation***)

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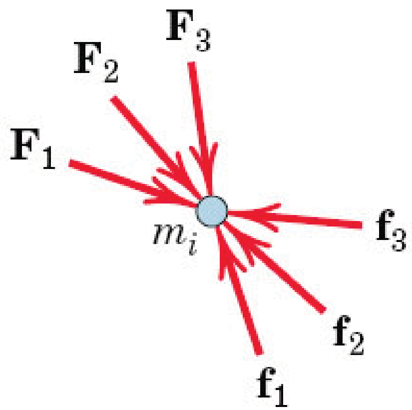
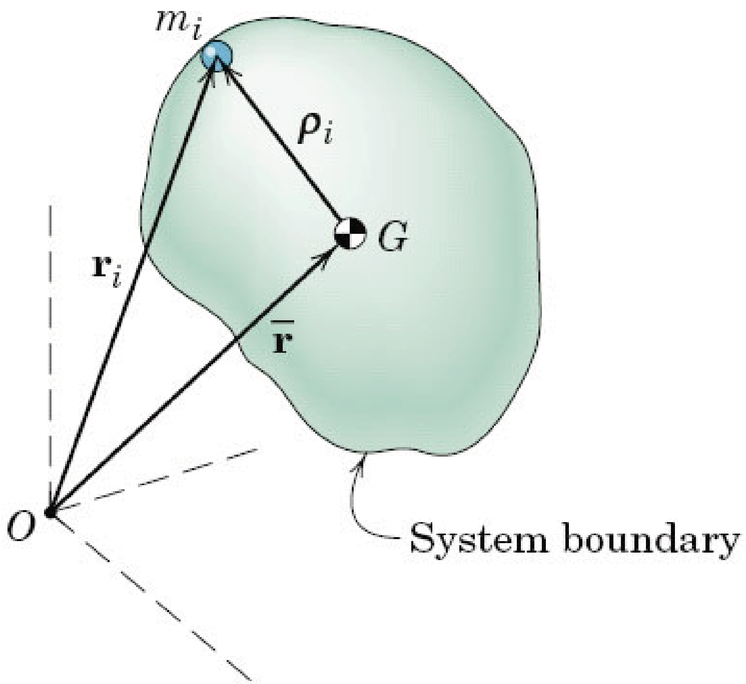
Direct Application of Newton's 2nd Law



$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots + \mathbf{f}_1 + \mathbf{f}_2 + \mathbf{f}_3 + \cdots = m_i \ddot{\mathbf{r}}_i$$

$$\sum \mathbf{F} + \sum \mathbf{f} = \sum m_i \ddot{\mathbf{r}}_i$$



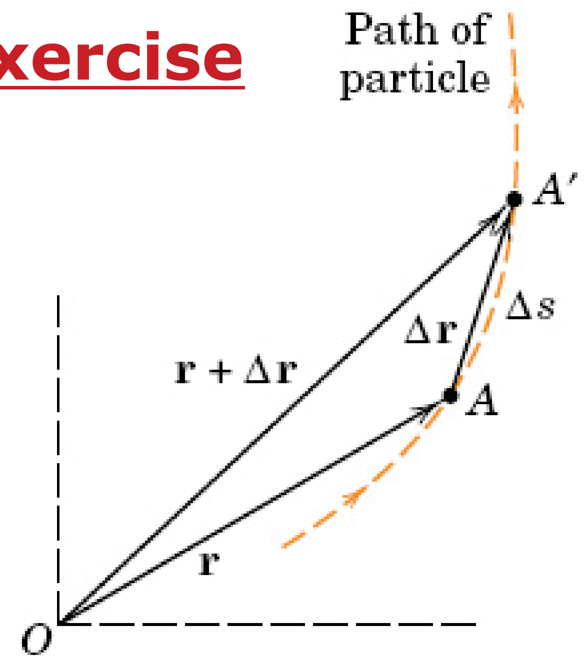
$$\boxed{\sum \mathbf{F} = m\ddot{\bar{\mathbf{r}}}} \quad \text{or} \quad \boxed{\sum \mathbf{F} = m\bar{\mathbf{a}}}$$

$$\sum F_x = m\bar{a}_x$$

$$\sum F_y = m\bar{a}_y$$

$$\sum F_z = m\bar{a}_z$$

Rectangular (x-y) Coordinates: Exercise



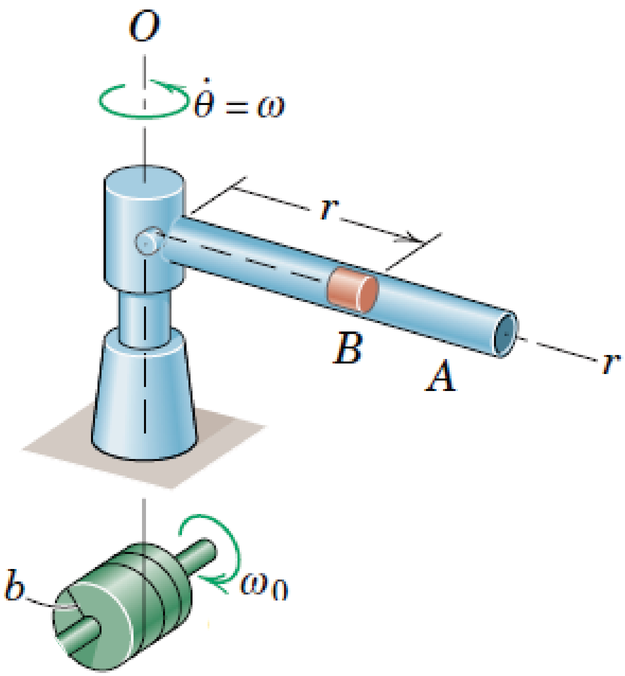
A particle with ***mass*** of ***10 slugs*** moving in two-dimensions has a position vector ($\mathbf{r}$) as a function of time (t) with coordinates given by

$$x(t) = t^2 - 4t + 20 \quad , \quad y(t) = 3\sin(2t)$$

where $\mathbf{r}$ is measured in feet and t is in seconds.

Determine the magnitude of the net ***force*** ($\mathbf{F}$) ***accelerating*** the particle at time t **= 3 s**.

Polar (r-θ) Coordinates: Exercise

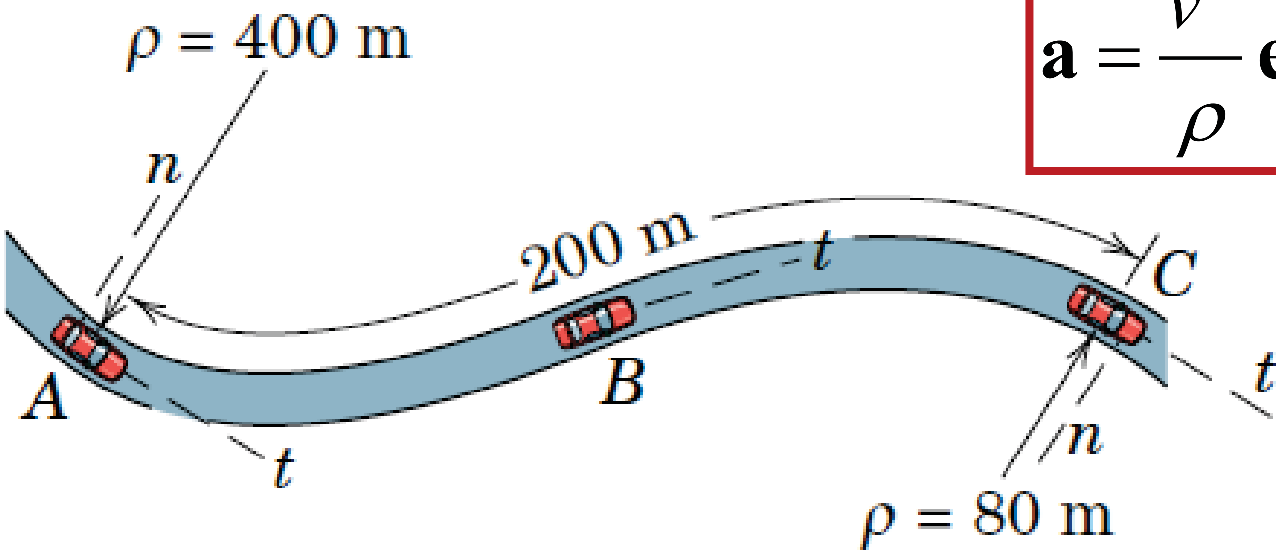


$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta$$

Tube A rotates about the vertical ***O-axis*** with constant ***angular velocity*** ω and contains a small ***cylinder*** B of ***mass*** m whose radial position is controlled by a cord passing through the tube and wound around a ***drum*** of ***radius*** b.

Determine the ***tension*** T in the cord and ***θ-component*** of ***force*** F_θ if the drum has a constant angular rate of rotation of ω_0 as shown.

Normal and Tangential (*n*-*t*) Coordinates: Exercise



$$\mathbf{a} = \frac{v^2}{\rho}\mathbf{e}_n + \dot{v}\,\mathbf{e}_t$$

A ***1500-kg*** car enters an s-curve and slows down from ***100 km/h*** at A to a speed of ***50 km/h*** as it passes C.

Determine the total ***horizontal force*** exerted by the road on the tires at ***positions*** A, B, and C.

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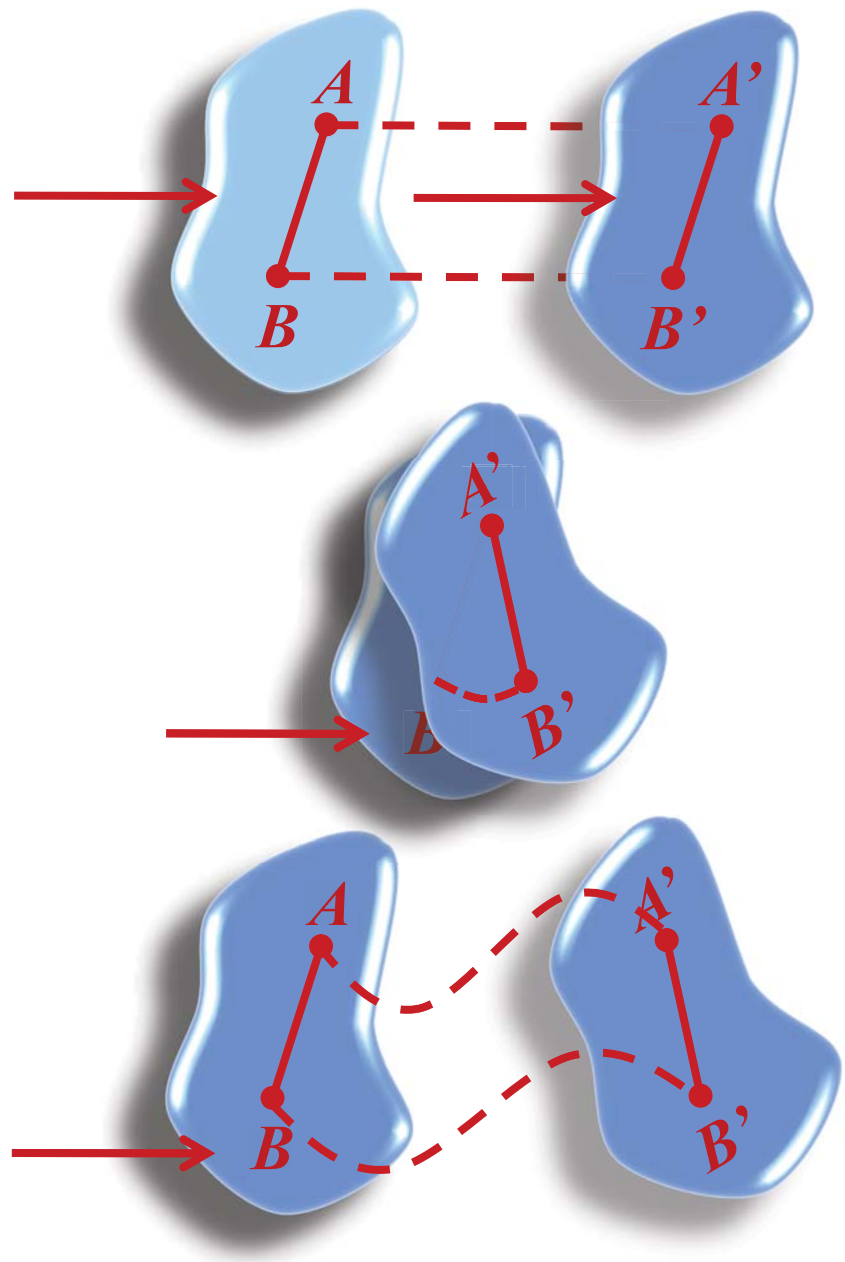
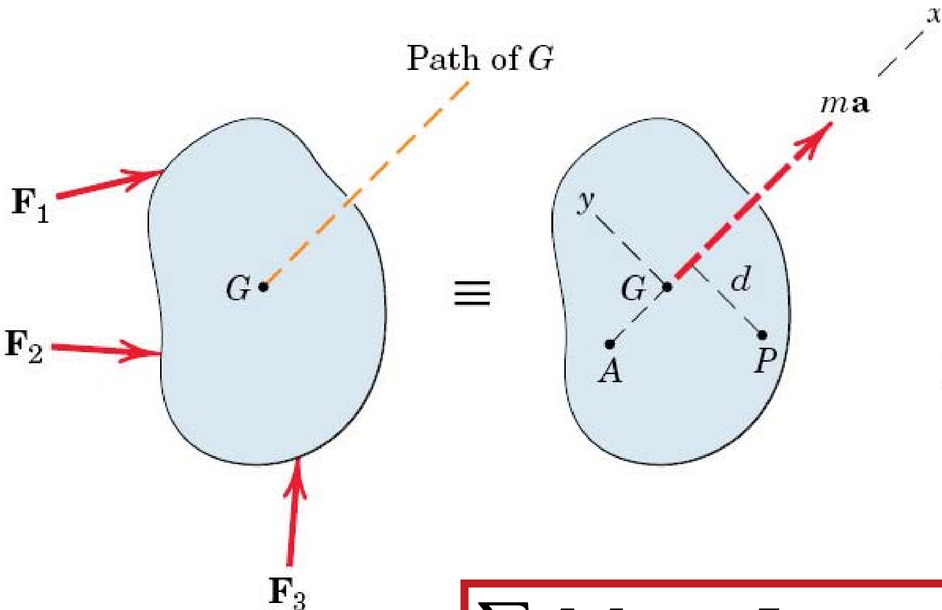
Plane Motion Types for Rigid Bodies

- Translation
- Fixed-axis rotation
- General plane motion



Rigid-Body Translation

rectilinear



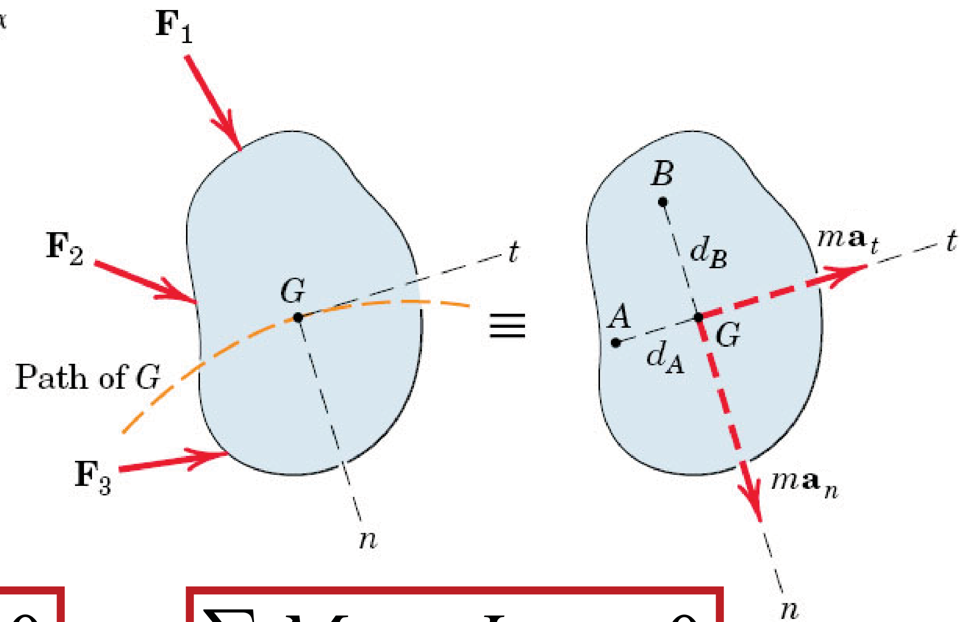
$$\sum \mathbf{F} = m\mathbf{a}$$

$$\alpha = 0$$

$$\omega = 0$$

$$\sum M_G = I_G\alpha = 0$$
$$\sum M_P = mad$$
$$\sum M_A = 0$$

curvilinear

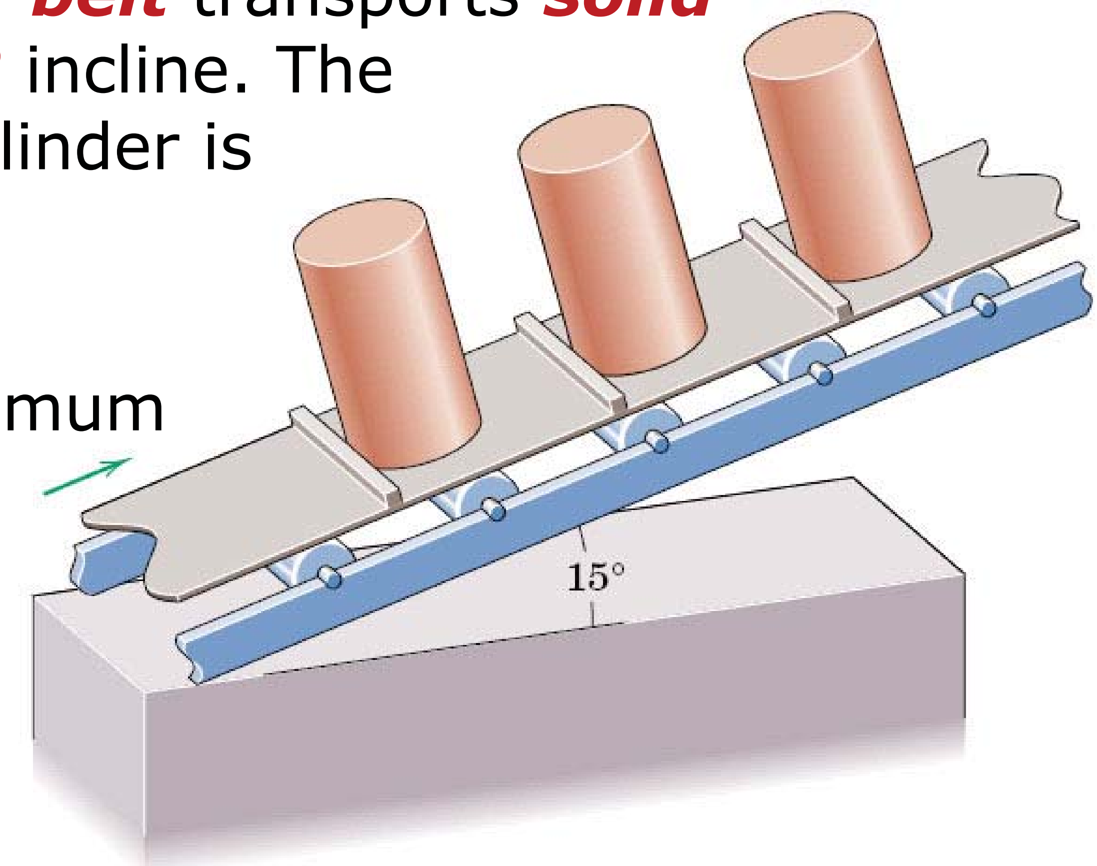


$$\sum M_G = I_G\alpha = 0$$
$$\sum M_A = ma_n d_A$$
$$\sum M_B = ma_t d_B$$

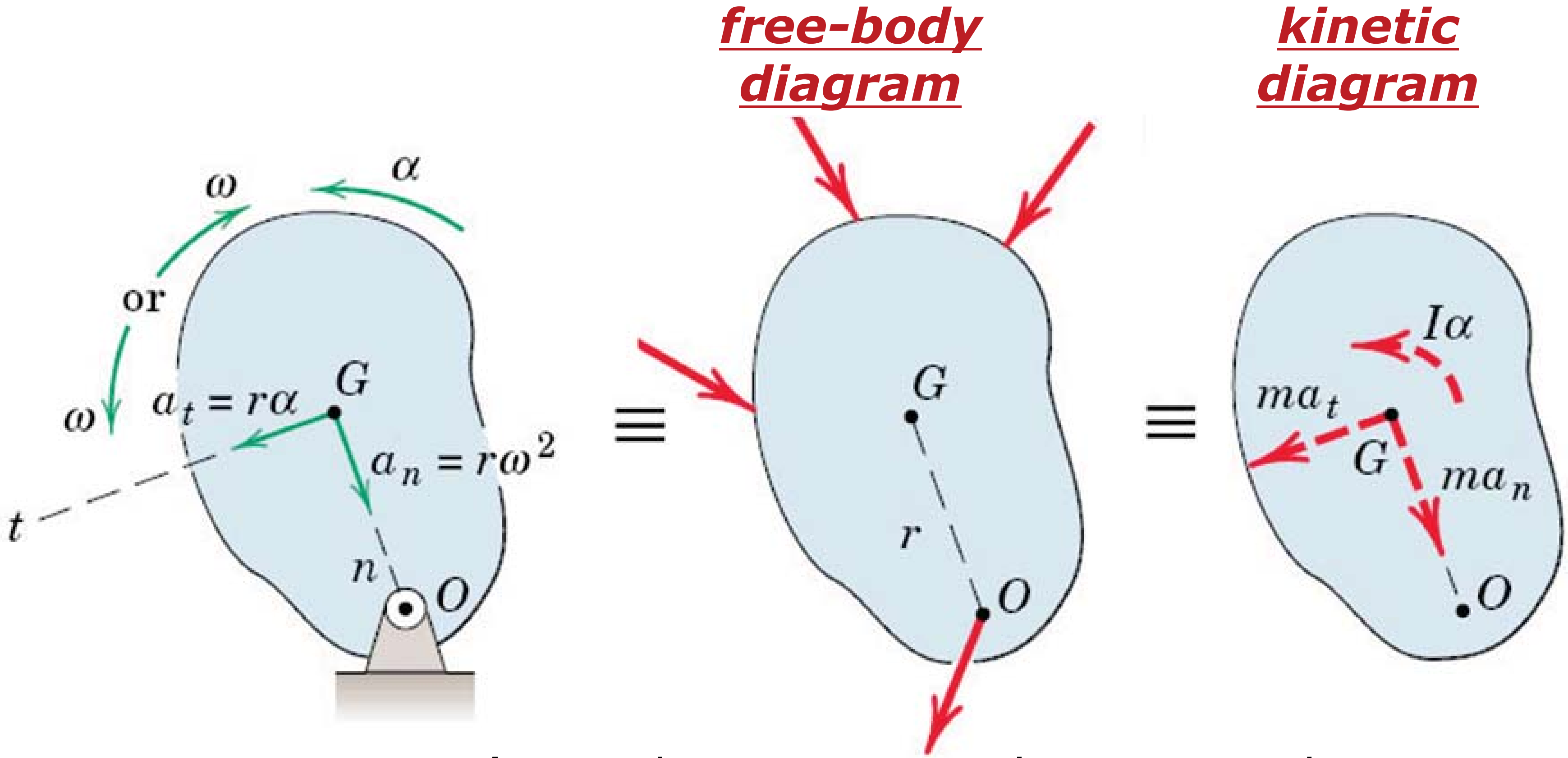
Rigid-Body Translation: Exercise

A cleated ***conveyor belt*** transports ***solid cylinders*** up a ***15º*** incline. The diameter of each cylinder is half its height.

Determine the maximum ***acceleration*** for the ***belt*** without tipping the ***cylinders*** as it starts.

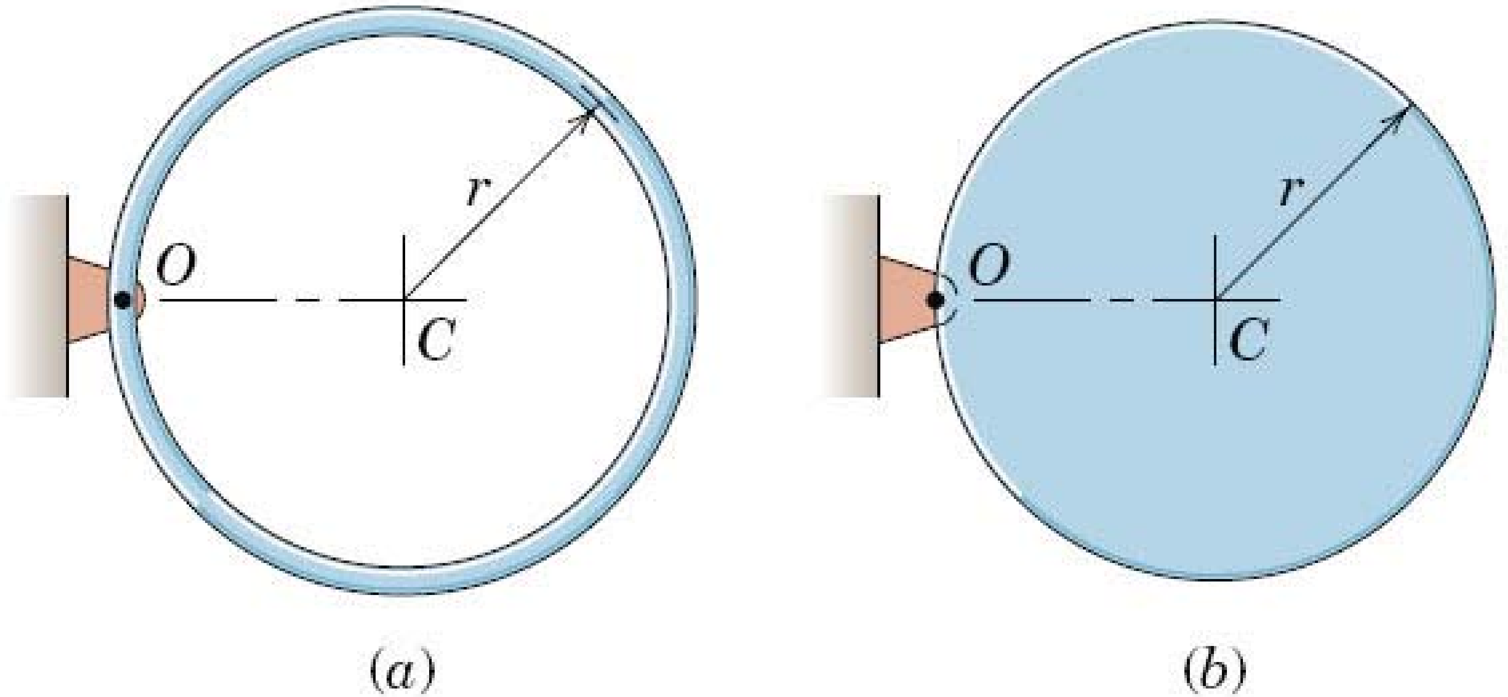


Fixed-Axis Rotation



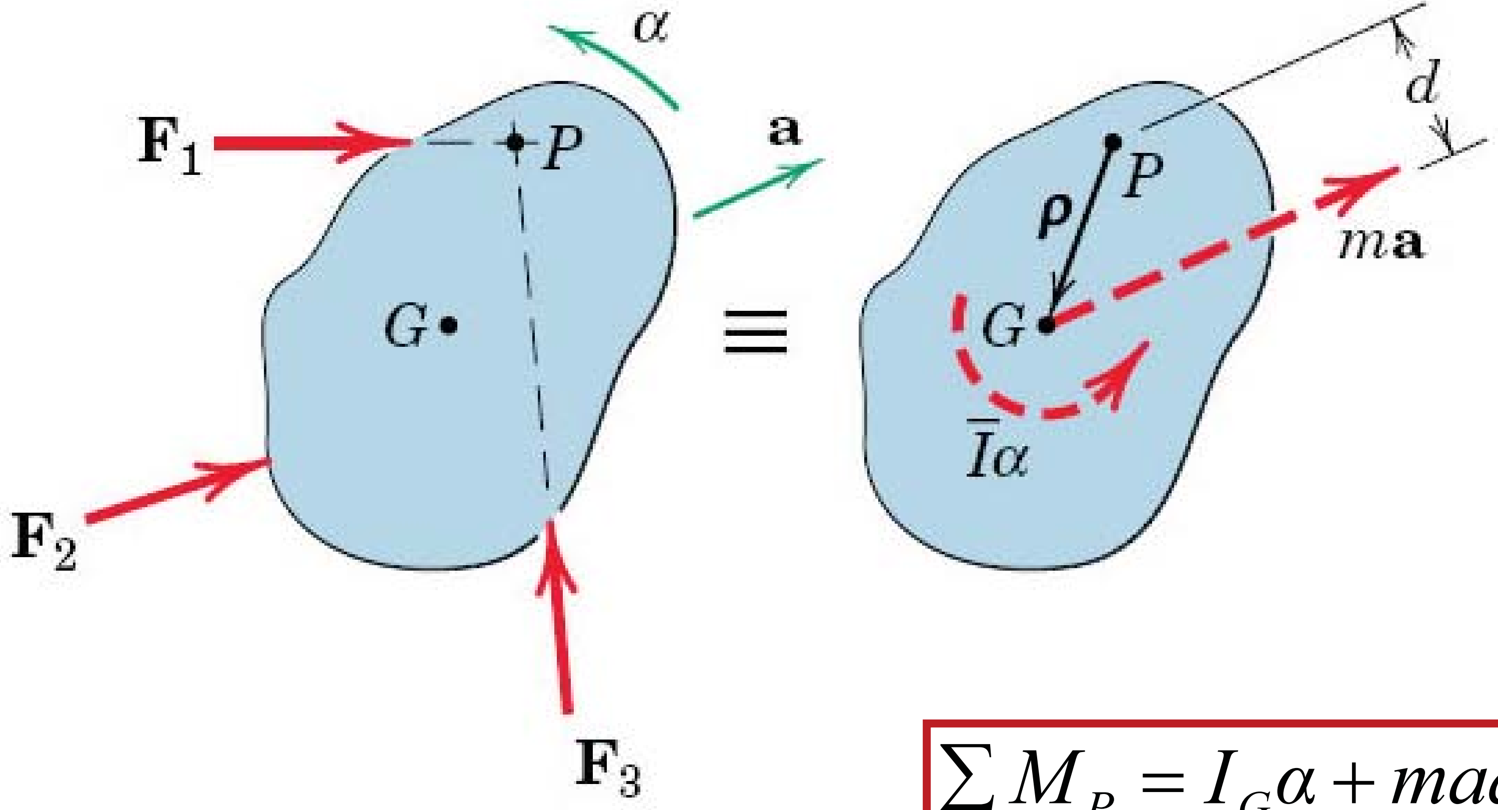
- Mass center's circular motion easily expressed in n-t coordinates
- Plane-motion equations: $\sum \mathbf{F} = m\mathbf{a}$ $\sum \mathbf{M}_G = I_G\boldsymbol{\alpha}$ $\sum \mathbf{M}_O = I_O\boldsymbol{\alpha}$

Fixed-Axis Rotation: Exercise



Determine the ***angular acceleration*** and the ***force*** on the ***bearing*** at O for (a) the narrow ***ring*** of ***mass*** m and (b) the flat circular ***disk*** of ***mass*** m immediately after each is released from rest with OC horizontal.

General Plane Motoin:
Combined Translation and Rotation



$$\sum M_P = I_G \alpha + mad$$

$$\sum \mathbf{F} = m\mathbf{a}$$

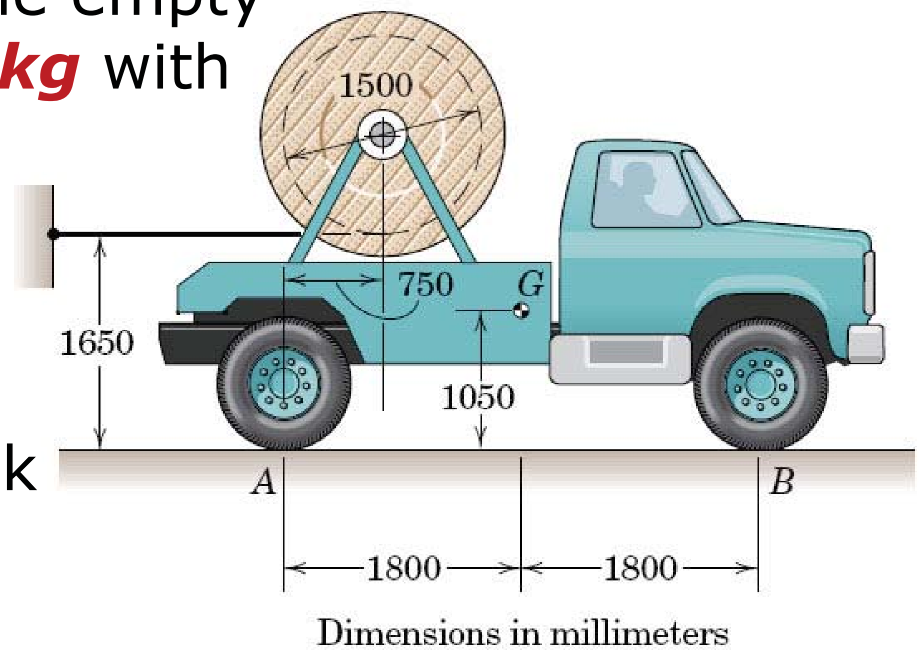
$$\sum \mathbf{M}_G = I_G \boldsymbol{\alpha}$$

$$\sum \mathbf{M}_P = I_P \boldsymbol{\alpha} + \boldsymbol{\rho} \times m\mathbf{a}_P$$

General Plane Motion: Exercise

A truck has a ***mass*** of ***2030 kg*** and carries a ***1500-mm-diameter spool*** of cable with a ***mass*** of ***0.75 kg per meter of length***. There are ***150 turns*** on the full spool. The empty spool has a ***mass*** of ***140 kg*** with ***radius of gyration*** of ***530 mm***.

Determine the ***tension*** T in the cable when the truck starts from rest with an ***acceleration*** of **0.2*g***.



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Equations, Equations, Equations…
Particle Kinetics: F=ma

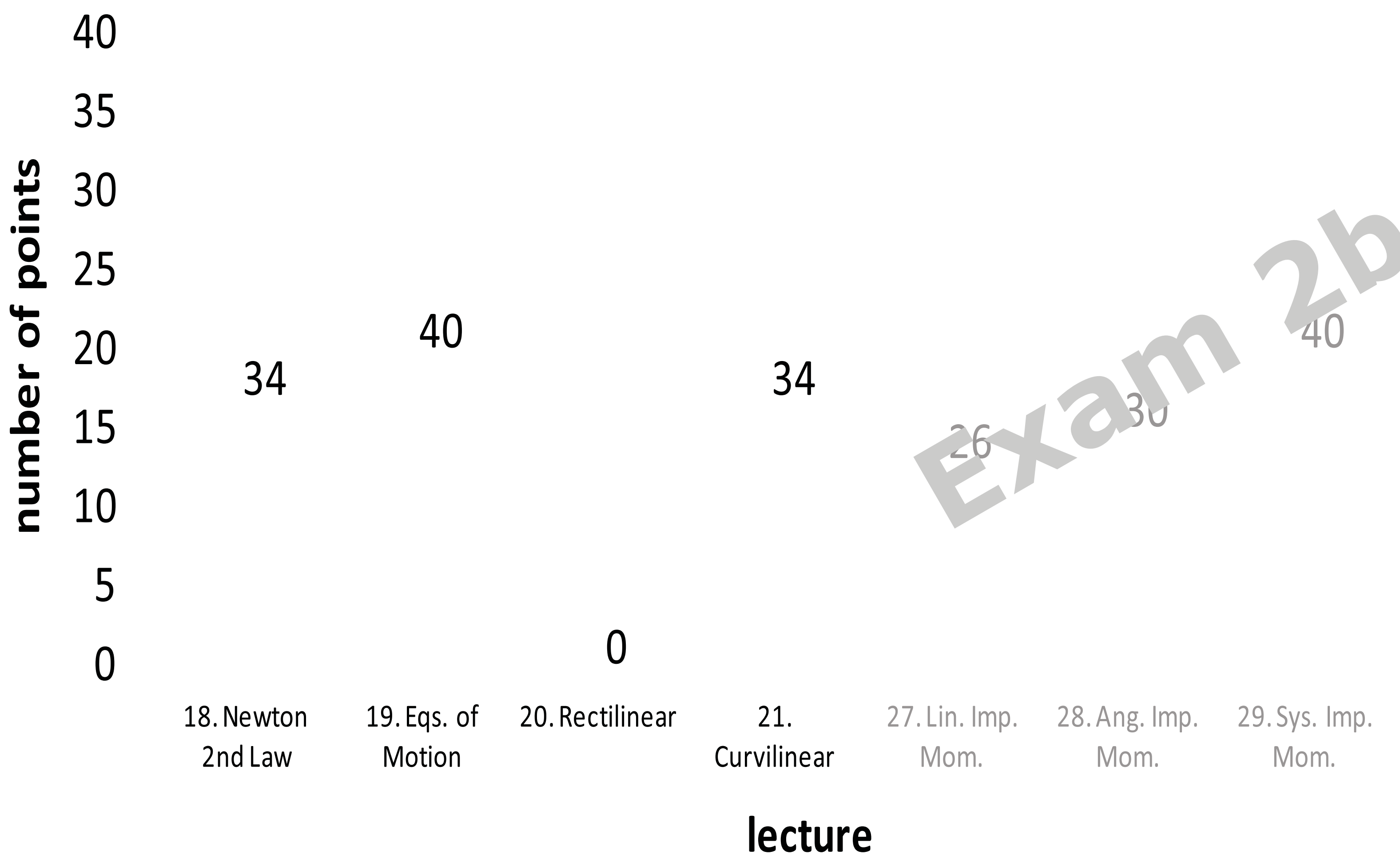
Lecture	Equations	
18. Newton 2nd Law 19. Eqs. of Motion 20. Rectilinear	$\sum \mathbf{F} = m\mathbf{a}$ $\sum F_x = ma_x = m\ddot{x}$	$\sum F_y = ma_y = m\ddot{y}$ $\sum F_z = ma_z = m\ddot{z}$
21. Curvilinear	$\sum F_r = ma_r$ $\sum F_\theta = ma_\theta$	$\sum F_n = ma_n$ $\sum F_t = ma_t$
27. Lin. Imp. Mom.	$\mathbf{G} = m\mathbf{v}$ $\Sigma\mathbf{F} = \dot{\mathbf{G}}$	$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma\mathbf{F}\, dt = \mathbf{G}_2$ $\Delta\mathbf{G} = \mathbf{0}$
28. Ang. Imp. Mom.	$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$ $\sum \mathbf{M}_O = \dot{\mathbf{H}}_O$	$(\mathbf{H}_O)_1 + \int_{t_1}^{t_2} \Sigma\mathbf{M}_O\, dt = (\mathbf{H}_O)_2$ $\Delta\mathbf{H}_O = \mathbf{0}$
29. Sys. Imp. Mom.	$\mathbf{G} = m\bar{\mathbf{v}}$ $\mathbf{H}_G = \sum(\boldsymbol{\rho}_i \times m_i\dot{\boldsymbol{\rho}}_i)$ $\mathbf{H}_O = \sum(\mathbf{r}_i \times m_i\mathbf{v}_i)$ $\sum \mathbf{M}_G = \dot{\mathbf{H}}_G$	$\mathbf{H}_P = \mathbf{H}_G + \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{v}}$ $\sum \mathbf{M}_P = \dot{\mathbf{H}}_G + \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{a}}$

Exam 2b

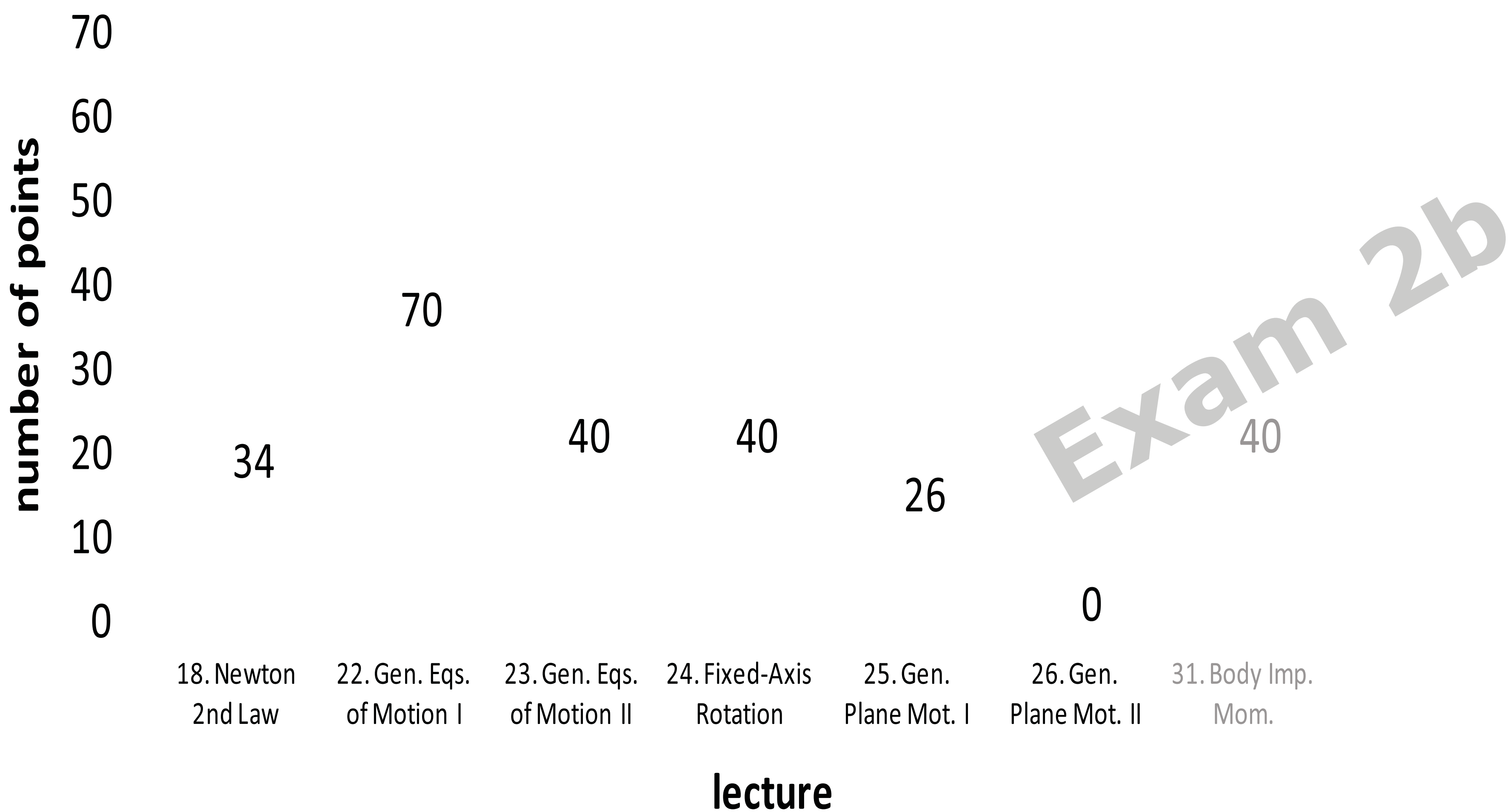
Equations, Equations, Equations…
Rigid Body Kinetics: F=ma

Lecture	Equations	
18. Newton 2nd Law 22. Gen. Eqs. Mot. I 23. Gen. Eqs. Mot. II	$\sum \mathbf{M}_G = \dot{\mathbf{H}}_G$ $\dot{\mathbf{H}}_G = \sum \boldsymbol{\rho}_i \times \mathbf{F}_i$	$\sum M_P = I_G \alpha + mad$ $\sum \mathbf{M}_P = I_P \boldsymbol{\alpha} + \boldsymbol{\rho} \times m\mathbf{a}_P$
24. Fixed-Axis Rot.	$\sum \mathbf{F} = m\mathbf{a}$ $\quad \sum \mathbf{M}_G = I_G \boldsymbol{\alpha}$	$\sum \mathbf{M}_O = I_O \boldsymbol{\alpha}$ $I_O = I_G + mr^2$ $\quad I_O = mk_O^2$
25. Gen. Plane Mot. I 26. Gen. Plane Mot. II	$\sum \mathbf{M}_P = I_P \boldsymbol{\alpha} + \boldsymbol{\rho} \times m\mathbf{a}_P$ $\sum \mathbf{F} = m\mathbf{a}$ $\quad \sum \mathbf{M}_G = I_G \boldsymbol{\alpha}$	$\sum M_P = I_G \alpha + mad$
31. Body Imp. Mom.	$\mathbf{G} = m\bar{\mathbf{v}}$ $\quad \mathbf{H}_G = I_G \boldsymbol{\omega}$ $\quad H_P = I_G \omega + mvd$ $\quad H_O = I_O \omega$ $\sum \mathbf{F} = \dot{\mathbf{G}}$ $\quad \sum \mathbf{M}_G = \dot{\mathbf{H}}_G$ $\quad \sum \mathbf{M}_P = \dot{\mathbf{H}}_P$ $\quad \sum \mathbf{M}_O = \dot{\mathbf{H}}_O$ $\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \mathbf{G}_2$ $\quad (\mathbf{H}_G)_1 + \int_{t_1}^{t_2} \Sigma \mathbf{M}_G\, dt = (\mathbf{H}_G)_2$	

Exam 2a Breakdown (particle kinetics: F=ma)



Exam 2 Breakdown (rigid body kinetics: F=ma)



For Next Time…

- Review Chapters 3 & 7
- Review Lectures slides
  - http://rrg.utk.edu/resources/ME231/lectures.html
- Review Examples from class
  - http://rrg.utk.edu/resources/ME231/examples.html

- *Exam #2a on Friday (11/9)*