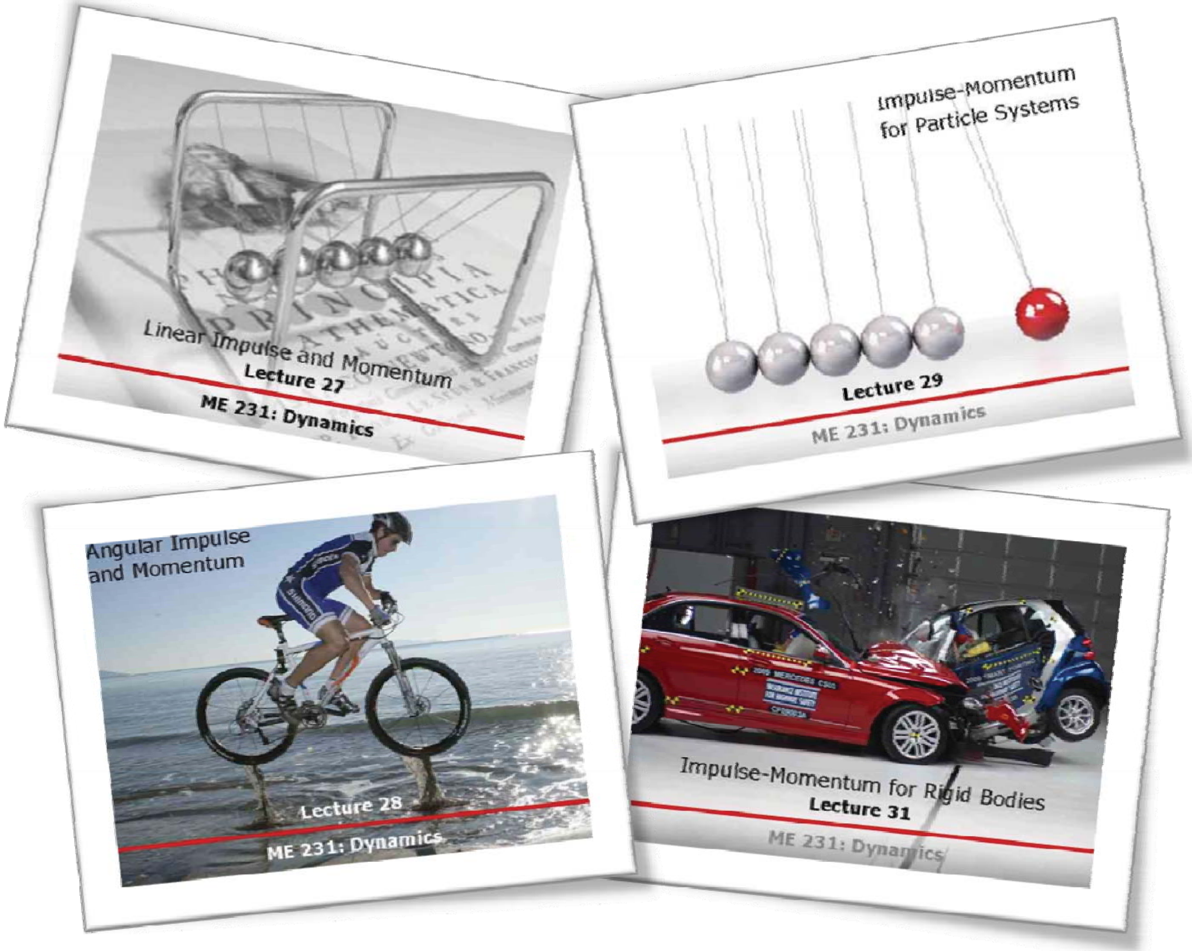

Kinetics: Imp-Mom (Ch. 5 & 8) Review

Lecture 34

ME 231: Dynamics

Question of the Day

How do we go from **F**=m**a** to impulse and momentum?

Integrate equations of motion with respect to ***time***

What is the relationship between impulse and momentum?

Impulse **(*F*t* or *M*t*)** on a particle or body equals change in ***momentum*** **(*G* or *H*)**

Why use impulse and momentum to solve dynamics problems?

Facilitates the ***solution*** of problems where ***forces*** act over ***specified time*** interval or during extremely ***short periods of time*** (e.g., ***impact***)

Outline for Today

- Question of the day
- Impulse-momentum principles
- Linear momentum for rigid bodies
- Angular momentum for rigid bodies
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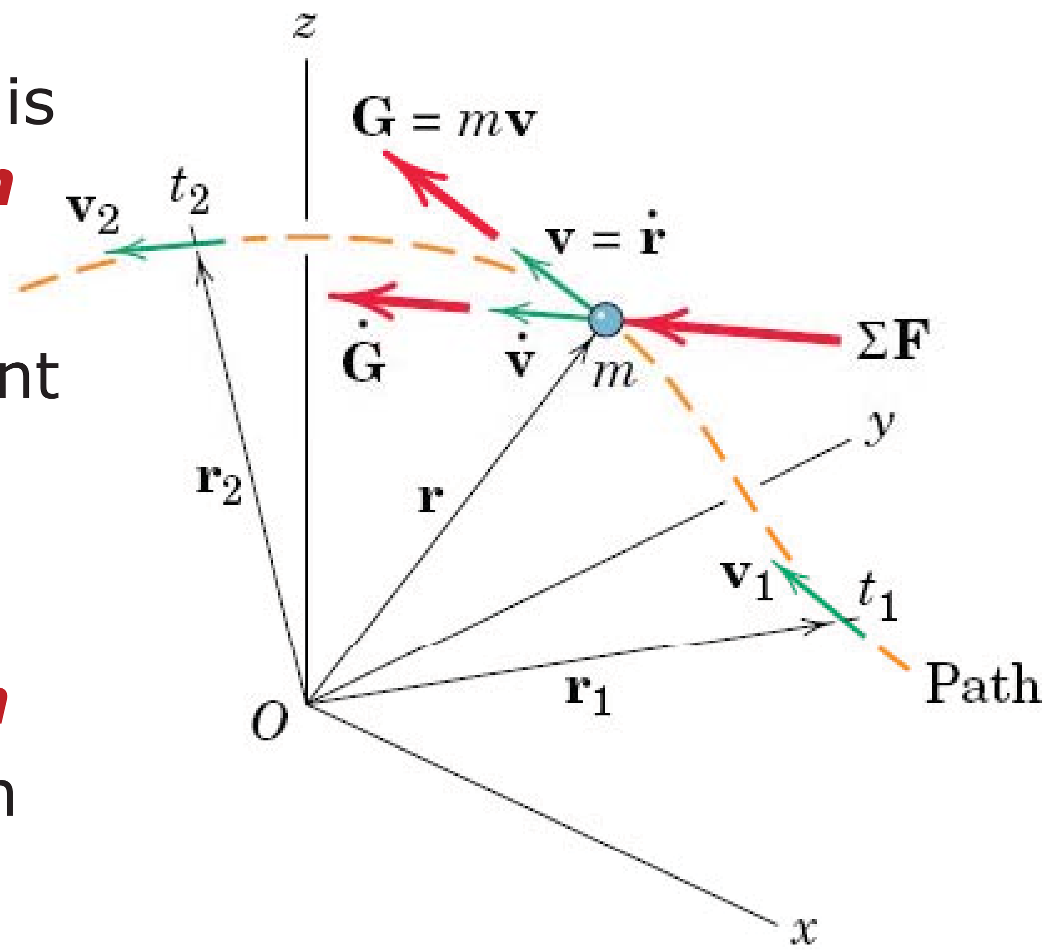
- Exam 2a grades…

Linear Momentum

$$\mathbf{G} = m\mathbf{v}$$

- Particle of ***mass m*** is located by ***position vector*** $\mathbf{r}$
- ***Velocity*** $\mathbf{v}$ is tangent to its path
- The ***mass*** times ***velocity*** is the ***linear momentum***
- ***Tangent*** to its path



Impulse-Momentum Principle: Linear

$$\mathbf{G} = m\mathbf{v}$$

$$\Sigma\mathbf{F} = \dot{\mathbf{G}} \qquad \int_{t_1}^{t_2} \Sigma\mathbf{F}\, dt = \int_{t_1}^{t_2} \dot{\mathbf{G}}\, dt$$

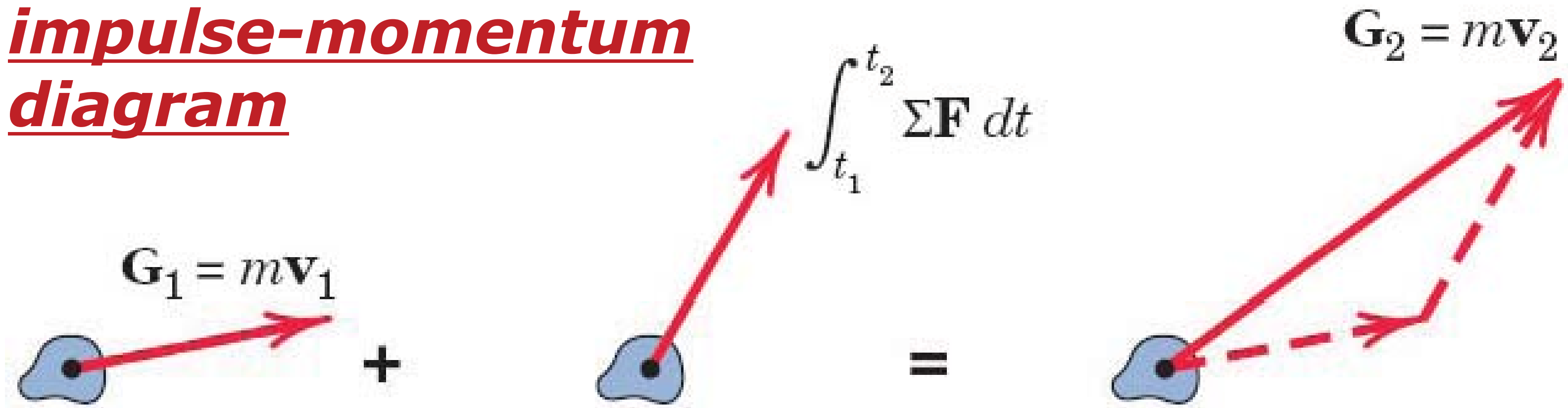
$$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma\mathbf{F}\, dt = \mathbf{G}_2$$

$$m(v_1)_x + \int_{t_1}^{t_2} \Sigma F_x\, dt = m(v_2)_x$$

$$m(v_1)_y + \int_{t_1}^{t_2} \Sigma F_y\, dt = m(v_2)_y$$

$$m(v_1)_z + \int_{t_1}^{t_2} \Sigma F_z\, dt = m(v_2)_z$$

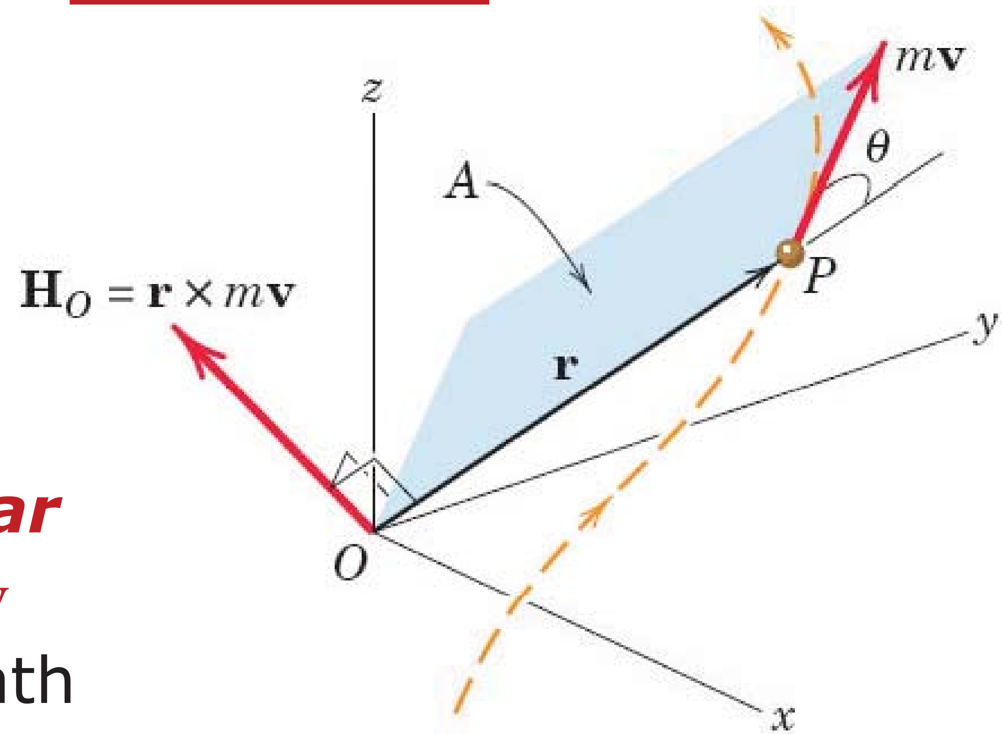
impulse-momentum diagram



- ***Integrate*** to describe the effect of the ***resultant force*** $\Sigma\mathbf{F}$ on ***linear momentum*** over a finite period of ***time***

Angular Momentum

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$$



- Particle of ***mass*** m is located by ***position vector*** $\mathbf{r}$
- ***Velocity*** $\mathbf{v}$ and ***linear momentum*** $\mathbf{G} = m\mathbf{v}$ are tangent to its path
- The ***moment*** of the ***linear momentum*** vector $m\mathbf{v}$ about ***point*** O is the ***angular momentum*** $\mathbf{H}_O$ ***of*** P ***about*** O
- ***Perpendicular*** to ***plane*** A defined by $\mathbf{r}$ and $\mathbf{v}$

Impulse-Momentum Principle: Angular

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$$

$$\sum \mathbf{M}_O = \dot{\mathbf{H}}_O$$

$$\int_{t_1}^{t_2} \Sigma \mathbf{M}_O \, dt = \int_{t_1}^{t_2} \dot{\mathbf{H}}_O \, dt$$

$$\left(\mathbf{H}_O\right)_1 + \int_{t_1}^{t_2} \Sigma \mathbf{M}_O \, dt = \left(\mathbf{H}_O\right)_2$$

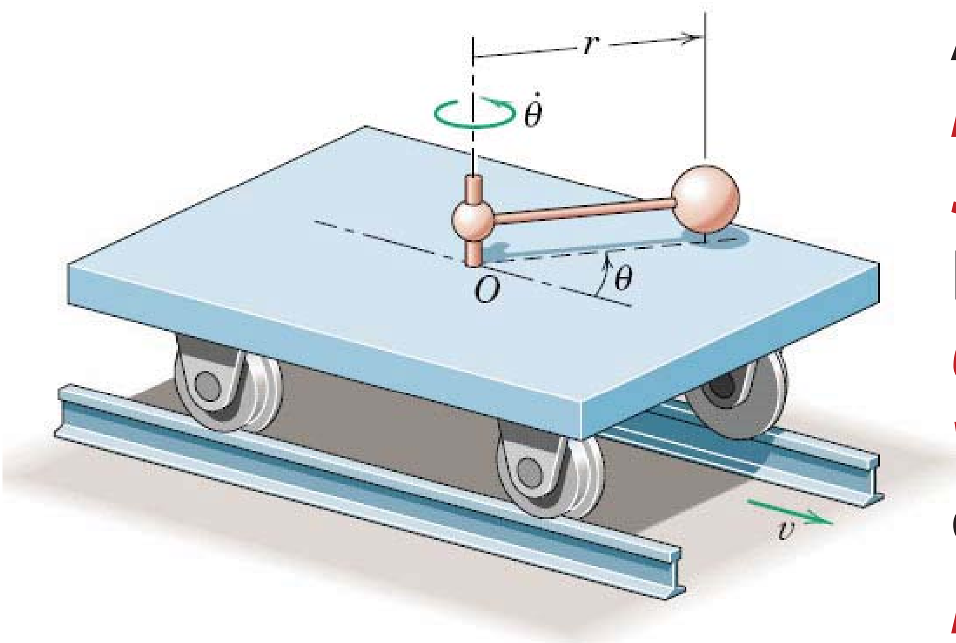
- ***Integrate*** to describe the effect of the ***angular impulse*** $\Sigma M_O {}^* t$ on ***angular momentum*** H_O ***of*** m ***about*** O over a finite period of ***time***

$$m(v_z y - v_y z)_1 + \int_{t_1}^{t_2} \Sigma \left(\mathbf{M}_O\right)_x \, dt = m(v_z y - v_y z)_2$$

$$m(v_x z - v_z x)_1 + \int_{t_1}^{t_2} \Sigma \left(\mathbf{M}_O\right)_y \, dt = m(v_x z - v_z x)_2$$

$$m(v_y x - v_x y)_1 + \int_{t_1}^{t_2} \Sigma \left(\mathbf{M}_O\right)_z \, dt = m(v_y x - v_x y)_2$$

Impulse-Momentum: Exercise



A small ***car*** with mass of ***20 kg*** rolls freely and carries a ***5-kg sphere*** mounted on a light rotating ***rod*** with ***r = 0.4 m*** and ***angular velocity*** of ***4 rad/s***. The car has a ***velocity v = 0.6 m/s*** when ***$\theta = 0^o$***.

Determine v when ***$\theta = 60^o$***.

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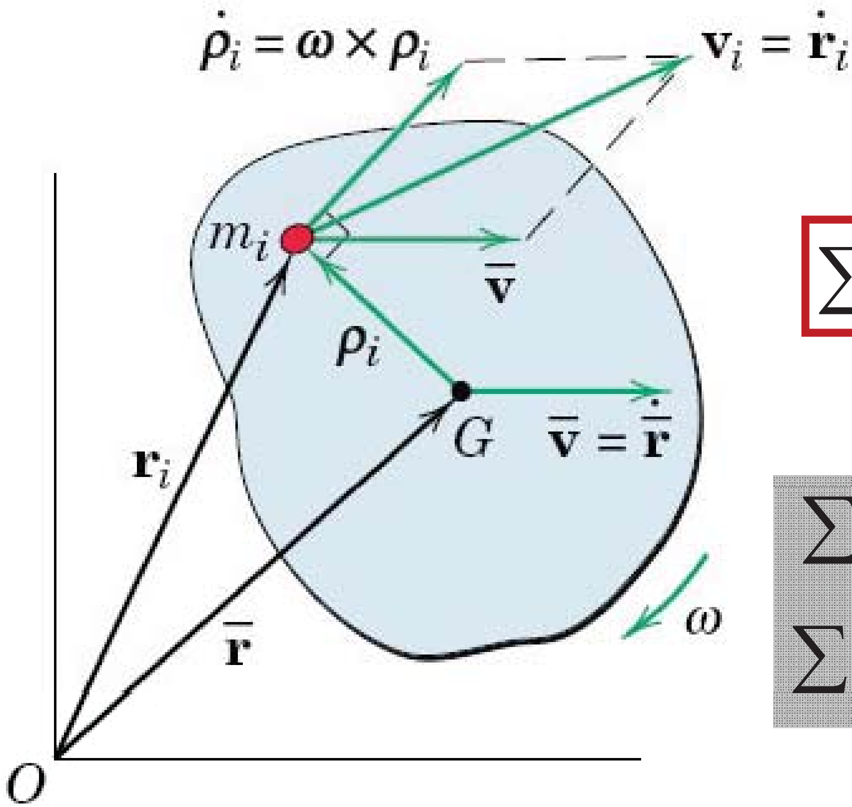
Linear Momentum for Rigid Bodies

- Special case of a general system of particles



$$\mathbf{G} = m\bar{\mathbf{v}}$$

$$\sum \mathbf{F} = \dot{\mathbf{G}}$$

$$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \mathbf{G}_2$$

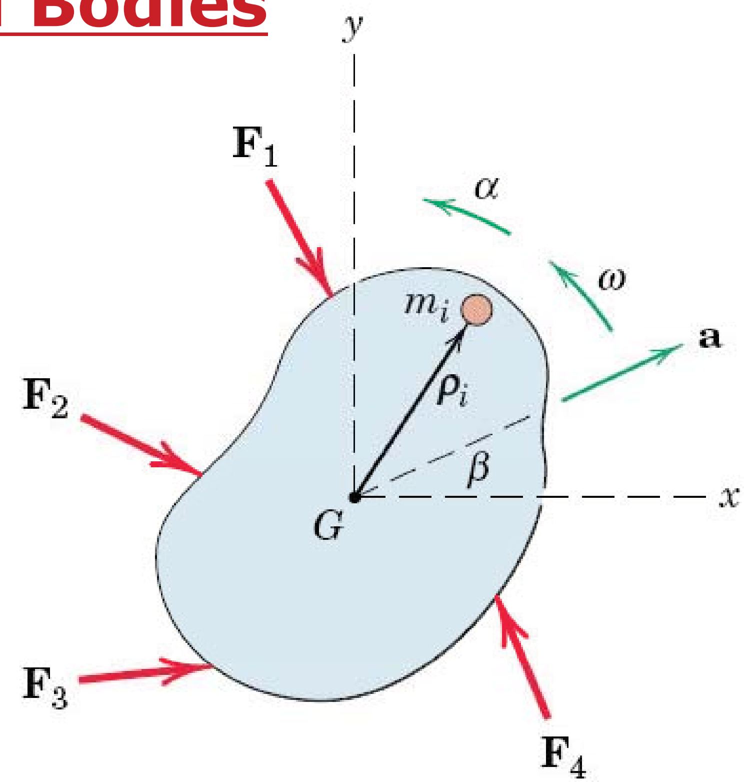
$$\sum F_x = \dot{G}_x \qquad (G_1)_x + \int_{t_1}^{t_2} \Sigma F_x\, dt = (G_2)_x$$

$$\sum F_y = \dot{G}_y \qquad (G_1)_y + \int_{t_1}^{t_2} \Sigma F_y\, dt = (G_2)_y$$

Angular Momentum for Rigid Bodies

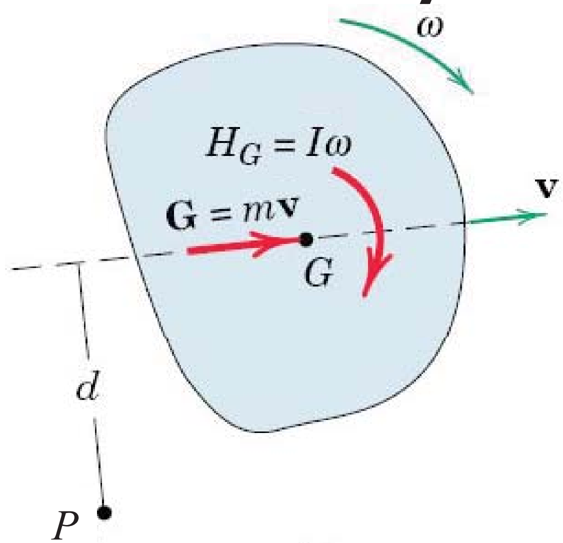
About the Mass Center G

linear	*angular*
$\mathbf{G} = m\bar{\mathbf{v}}$	$\mathbf{H}_G = I_G \boldsymbol{\omega}$
$\sum \mathbf{F} = \dot{\mathbf{G}}$	$\sum \mathbf{M}_G = \dot{\mathbf{H}}_G$
$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \mathbf{G}_2$	$(\mathbf{H}_G)_1 + \int_{t_1}^{t_2} \Sigma \mathbf{M}_G\, dt = (\mathbf{H}_G)_2$



Alternate Angular Momentum for Rigid Bodies

About an Arbitrary Point P

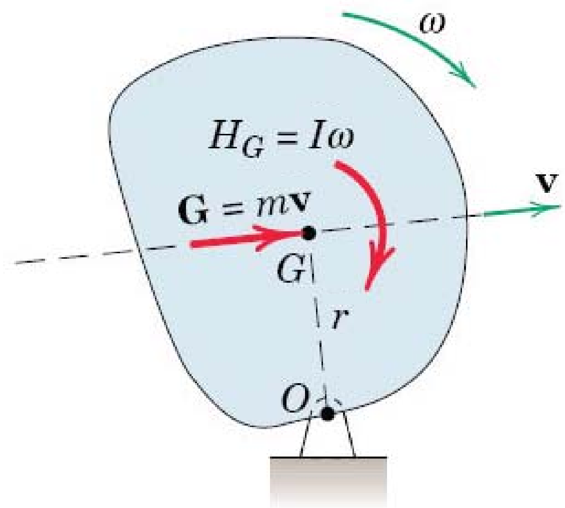


$$H_P = I_G\omega + mvd$$

$$\sum \mathbf{M}_P = \dot{\mathbf{H}}_P$$

$$\left(\mathbf{H}_P\right)_1 + \int_{t_1}^{t_2} \Sigma \mathbf{M}_P \, dt = \left(\mathbf{H}_P\right)_2$$

About a Fixed Point O



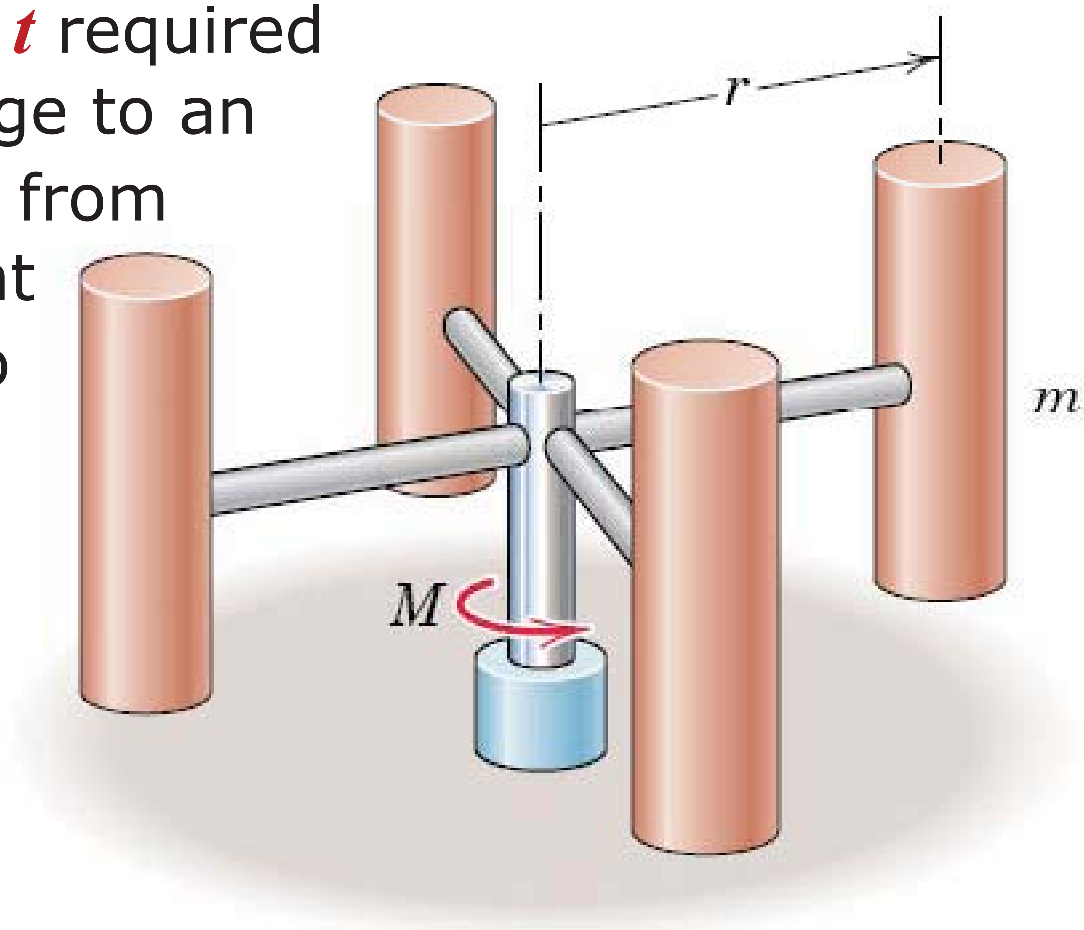
$$H_O = I_O\omega$$

$$\sum \mathbf{M}_O = \dot{\mathbf{H}}_O$$

$$\left(\mathbf{H}_O\right)_1 + \int_{t_1}^{t_2} \Sigma \mathbf{M}_O \, dt = \left(\mathbf{H}_O\right)_2$$

Impulse-Momentum: Exercise

Determine the ***time t*** required to bring the centrifuge to an ***angular velocity ω*** from rest under a constant ***torque M*** applied to the shaft.



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Equations, Equations, Equations…
Particle Kinetics: F=ma

Lecture	Equations	
18. Newton 2nd Law 19. Eqs. of Motion 20. Rectilinear	$\sum \mathbf{F} = m\mathbf{a}$ $\sum F_x = ma_x = m\ddot{x}$	$\sum F_y = ma_y = m\ddot{y}$ $\sum F_z = ma_z = m\ddot{z}$
21. Curvilinear	$\sum F_r = ma_r$ $\sum F_\theta = ma_\theta$	$\sum F_n = ma_n$ $\sum F_t = ma_t$
27. Lin. Imp. Mom.	$\mathbf{G} = m\mathbf{v}$ $\Sigma\mathbf{F} = \dot{\mathbf{G}}$	$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma\mathbf{F}\,dt = \mathbf{G}_2$ $\Delta\mathbf{G} = \mathbf{0}$
28. Ang. Imp. Mom.	$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$ $\sum \mathbf{M}_O = \dot{\mathbf{H}}_O$	$(\mathbf{H}_O)_1 + \int_{t_1}^{t_2} \Sigma\mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ $\Delta\mathbf{H}_O = \mathbf{0}$
29. Sys. Imp. Mom.	$\mathbf{G} = m\bar{\mathbf{v}}$ $\mathbf{H}_O = \sum(\mathbf{r}_i \times m_i\mathbf{v}_i)$	$\mathbf{H}_G = \sum(\boldsymbol{\rho}_i \times m_i\dot{\boldsymbol{\rho}}_i)$ $\quad \mathbf{H}_P = \mathbf{H}_G + \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{v}}$ $\sum \mathbf{M}_G = \dot{\mathbf{H}}_G$ $\quad \sum \mathbf{M}_P = \dot{\mathbf{H}}_G + \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{a}}$

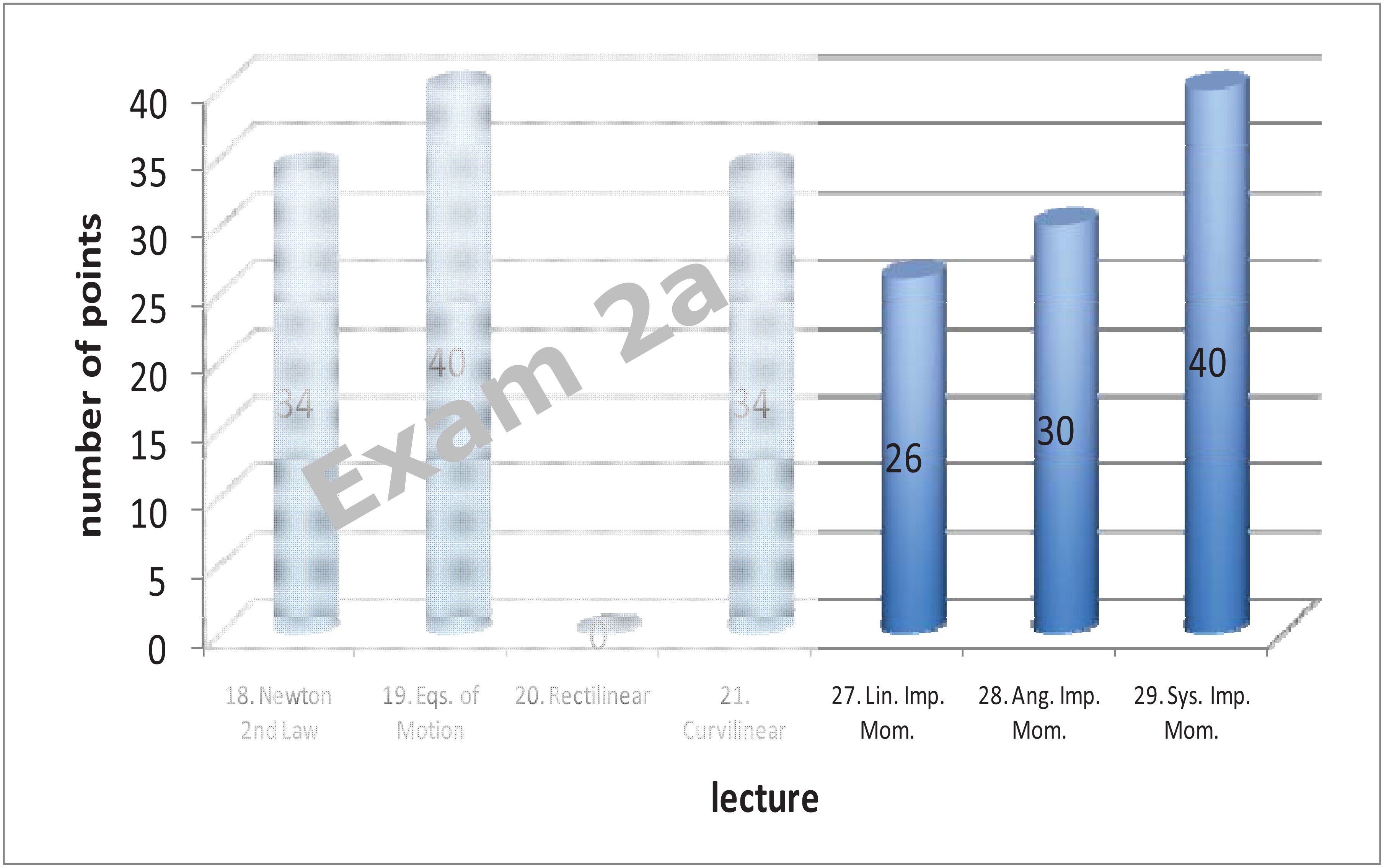
Exam 2a

Equations, Equations, Equations…
Rigid Body Kinetics: F=ma

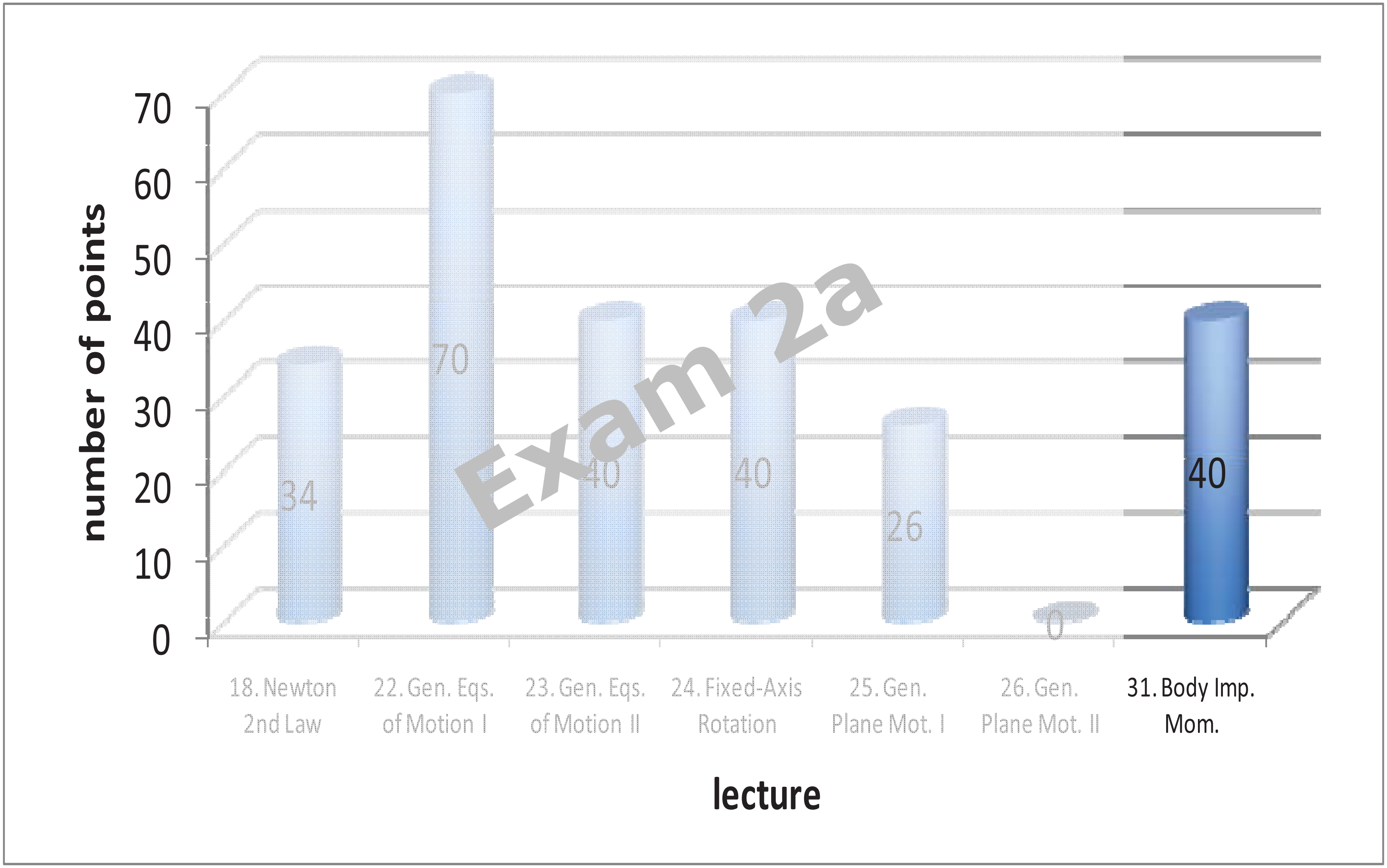
Lecture	Equations	
18. Newton 2nd Law 22. Gen. Eqs. Mot. I 23. Gen. Eqs. Mot. II	$\sum \mathbf{M}_G = \dot{\mathbf{H}}_G$ $\dot{\mathbf{H}}_G = \sum \boldsymbol{\rho}_i \times \mathbf{F}_i$ $\sum \mathbf{F} = m\mathbf{a}$ $\quad \sum \mathbf{M}_G = I_G \boldsymbol{\alpha}$	$\sum M_P = I_G \alpha + mad$ $\sum \mathbf{M}_P = I_P \boldsymbol{\alpha} + \boldsymbol{\rho} \times m\mathbf{a}_P$ $\sum \mathbf{M}_O = I_O \boldsymbol{\alpha}$
24. Fixed-Axis Rot.		$I_O = I_G + mr^2 \quad I_O = mk_O^2$
25. Gen. Plane Mot. I	$\sum \mathbf{M}_P = I_P \boldsymbol{\alpha} + \boldsymbol{\rho} \times m\mathbf{a}_P$	$\sum M_P = I_G \alpha + mad$
26. Gen. Plane Mot. II	$\sum \mathbf{F} = m\mathbf{a} \quad \sum \mathbf{M}_G = I_G \boldsymbol{\alpha}$	
31. Body Imp. Mom.	$\mathbf{G} = m\bar{\mathbf{v}} \quad \mathbf{H}_G = I_G \boldsymbol{\omega} \quad H_P = I_G \omega + mvd \quad H_O = I_O \omega$ $\sum \mathbf{F} = \dot{\mathbf{G}} \quad \sum \mathbf{M}_G = \dot{\mathbf{H}}_G \quad \sum \mathbf{M}_P = \dot{\mathbf{H}}_P \quad \sum \mathbf{M}_O = \dot{\mathbf{H}}_O$ $\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \mathbf{G}_2 \quad (\mathbf{H}_G)_1 + \int_{t_1}^{t_2} \Sigma \mathbf{M}_G\, dt = (\mathbf{H}_G)_2$	

Exam 2a

Exam 2a Breakdown (particle kinetics: F=ma)



Exam 2 Breakdown (rigid body kinetics: F=ma)



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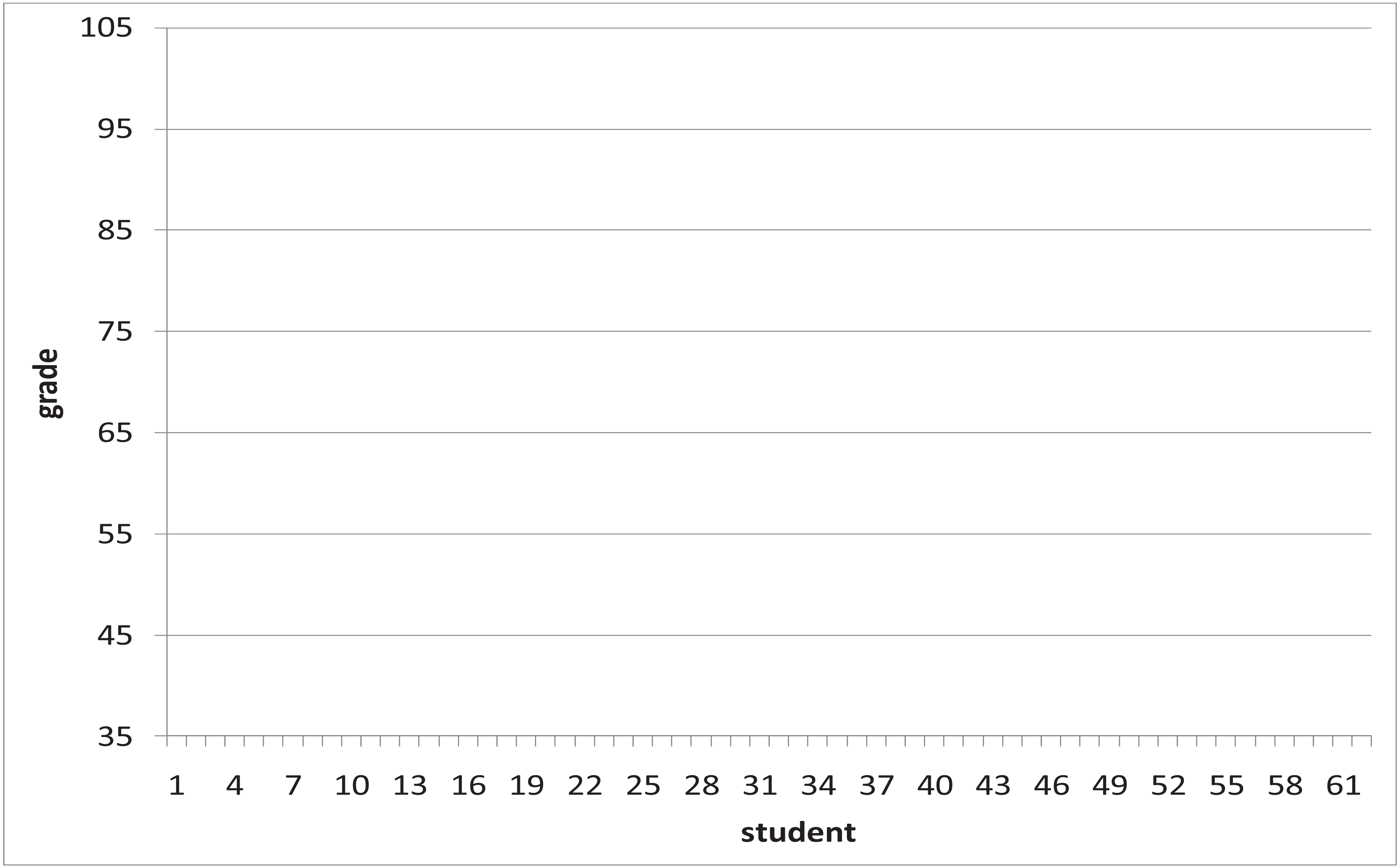
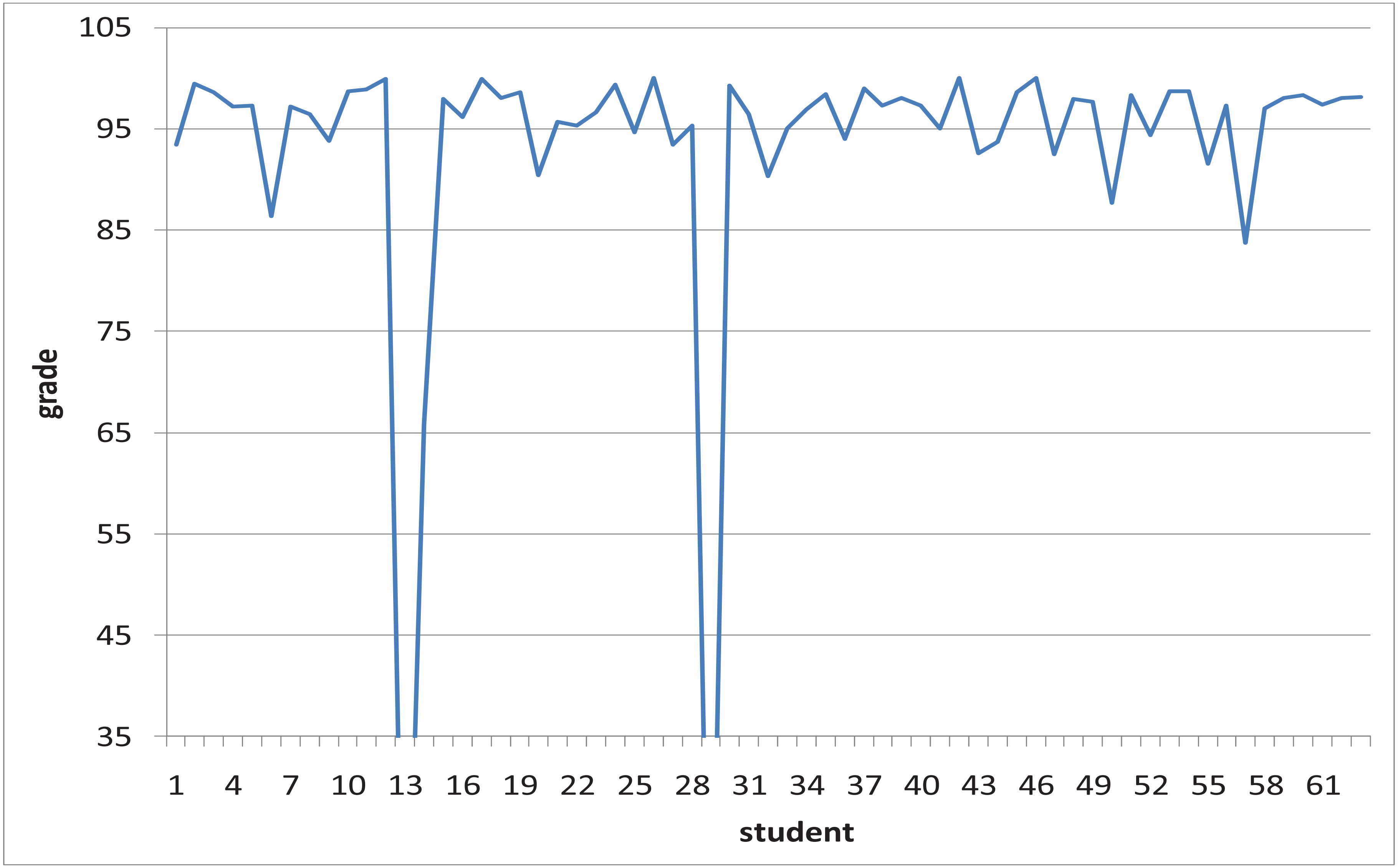
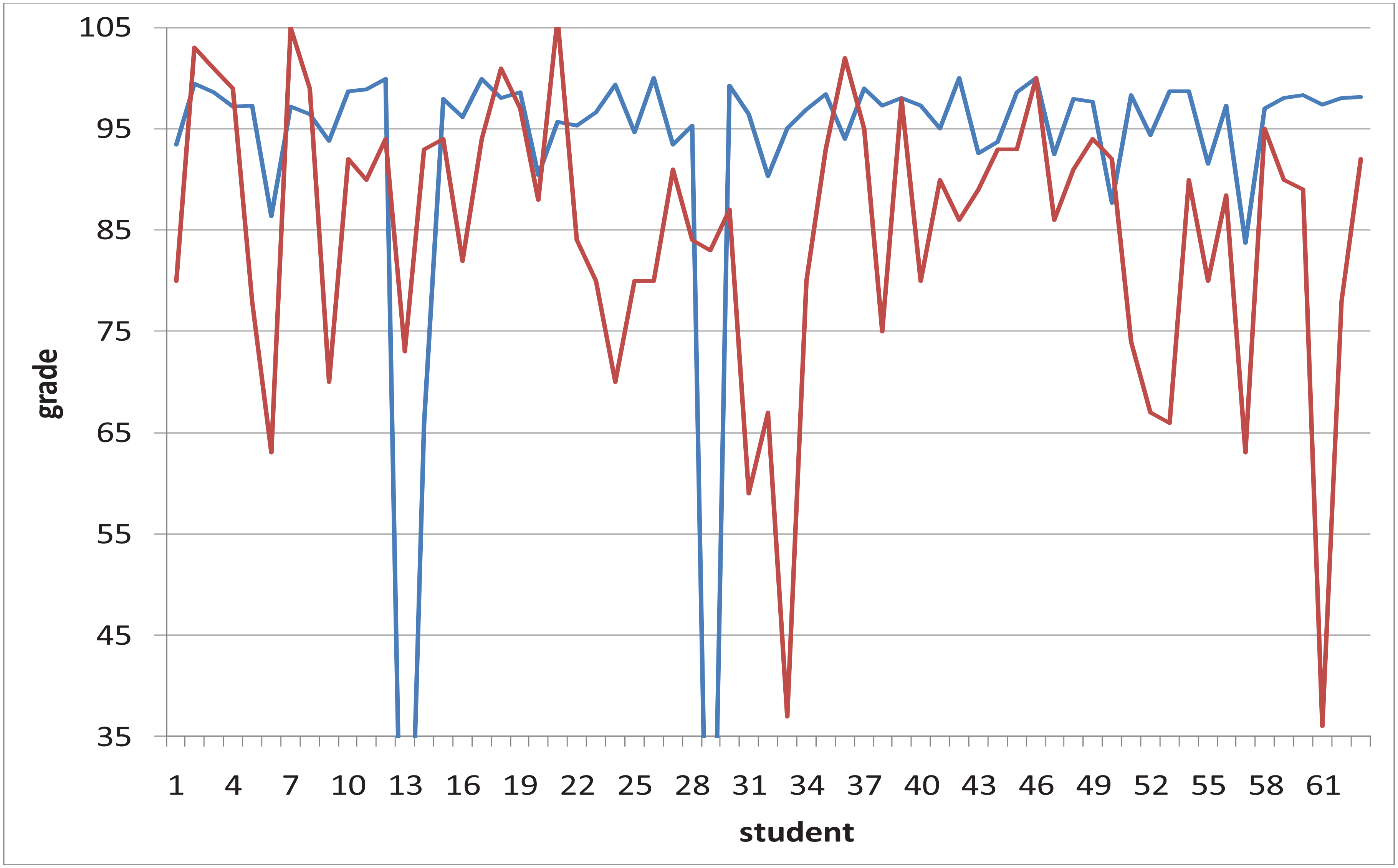
Grades


105
95
85
75
65
55
45
35
grade
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61
student

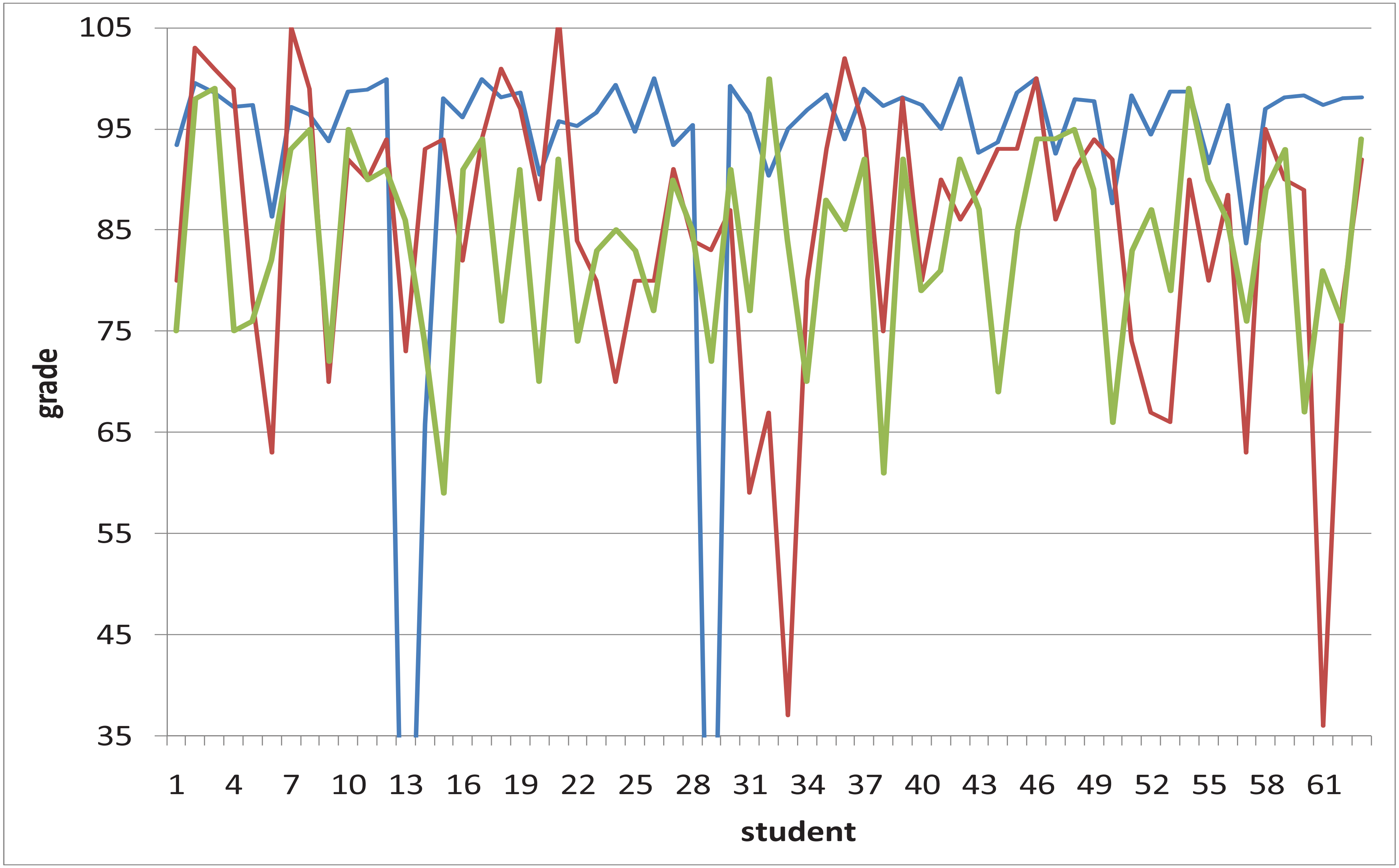
Homework Grades



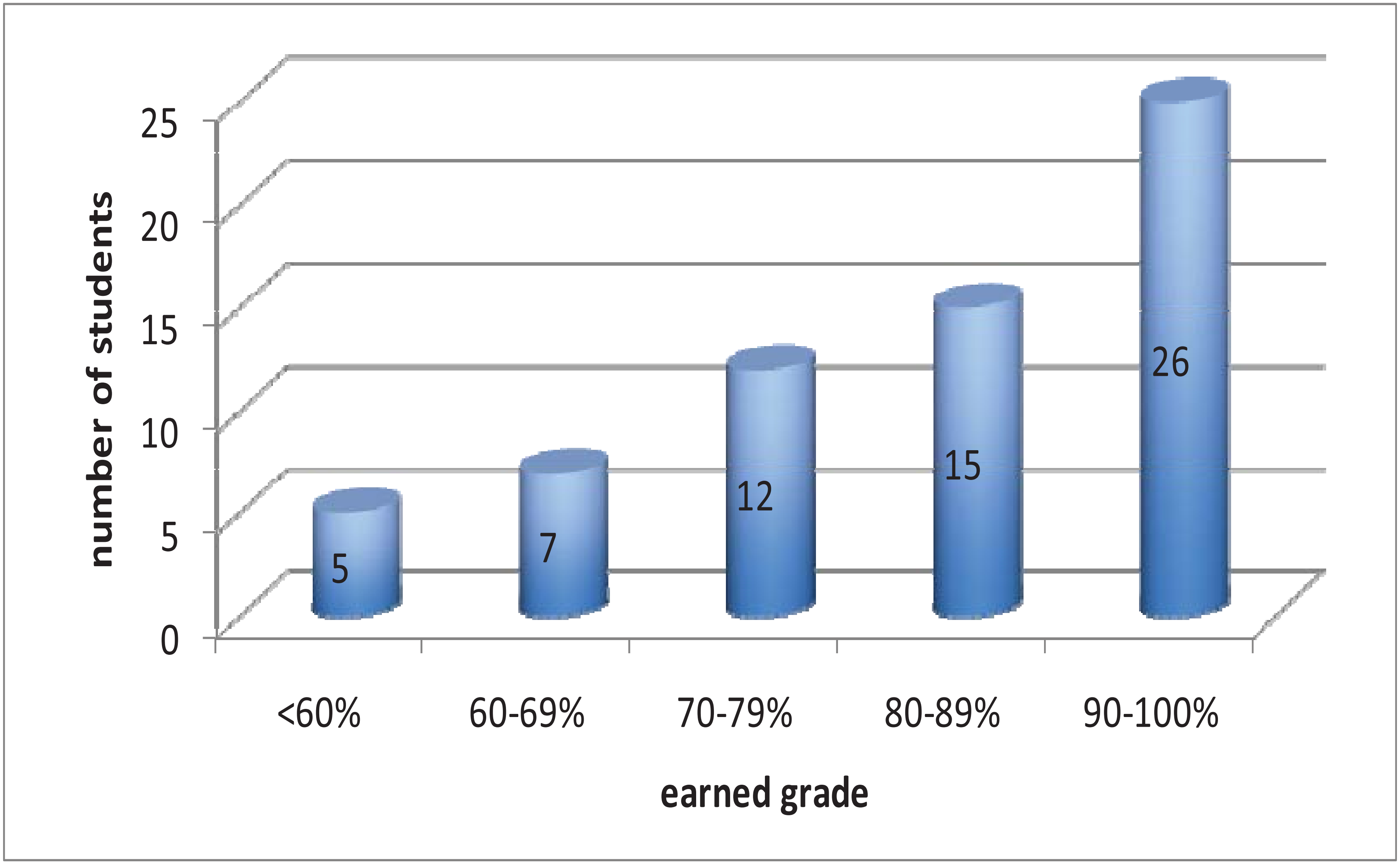
Homework and Exam 1 Grades



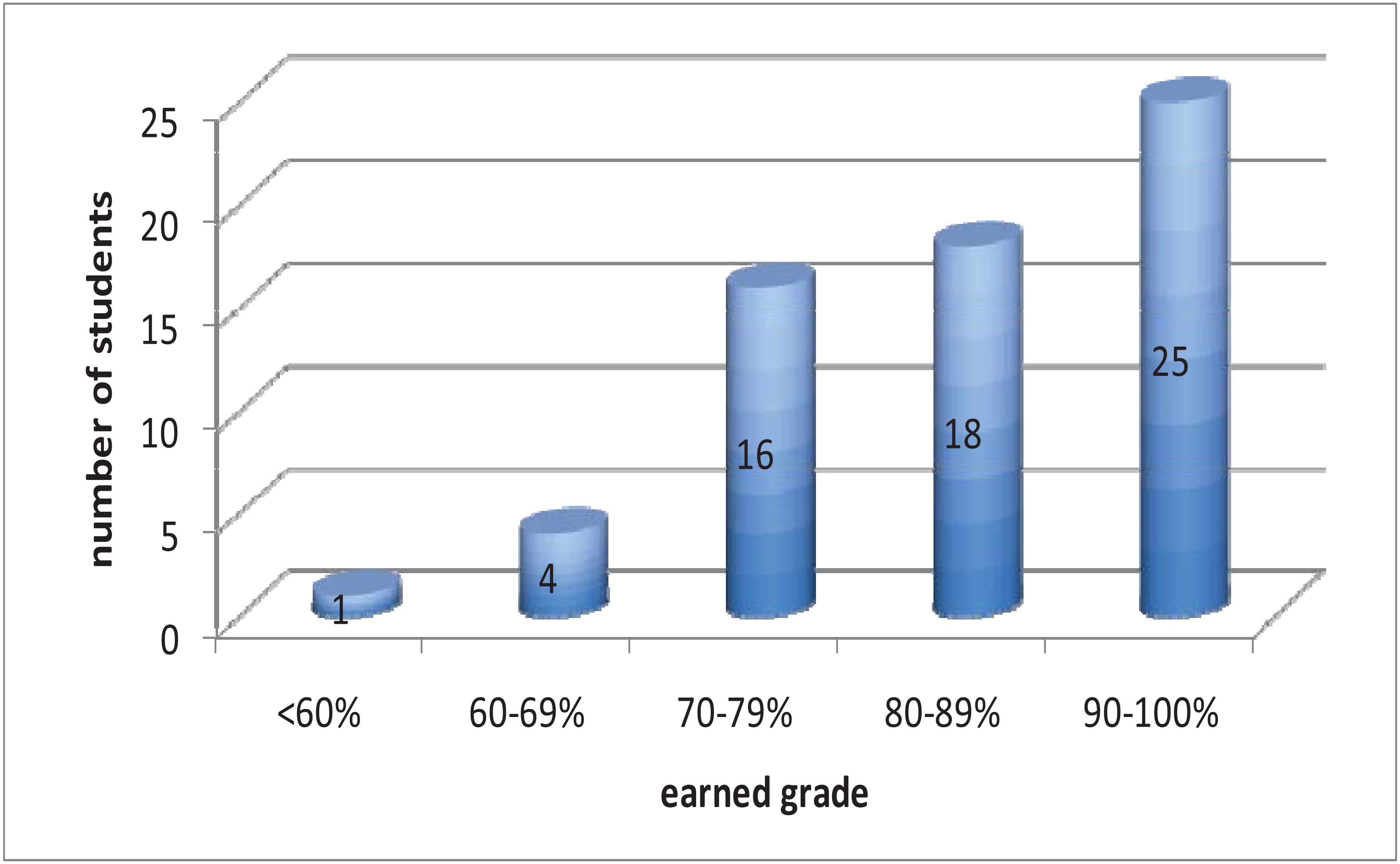
Homework, Exam 1, and Exam 2a Grades



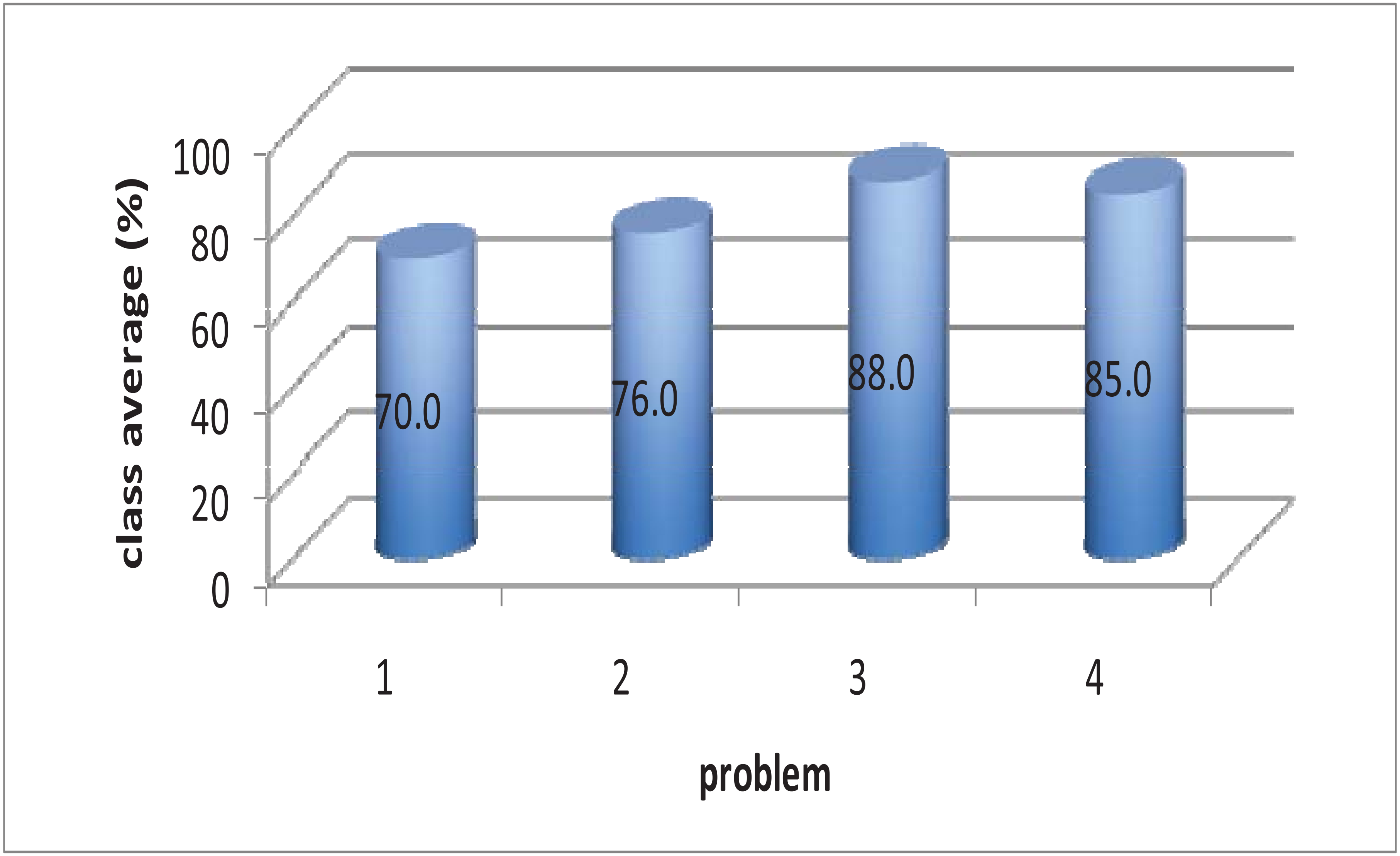
Exam 1 Grades



Exam 2a Grades



Exam 2a Problem Grades



For Next Time…

- Review Chapters 5 & 8
- Review Lectures slides
  - http://rrg.utk.edu/resources/ME231/lectures.html
- Review Examples from class
  - http://rrg.utk.edu/resources/ME231/examples.html

- *Exam #2b on Friday (11/16)*