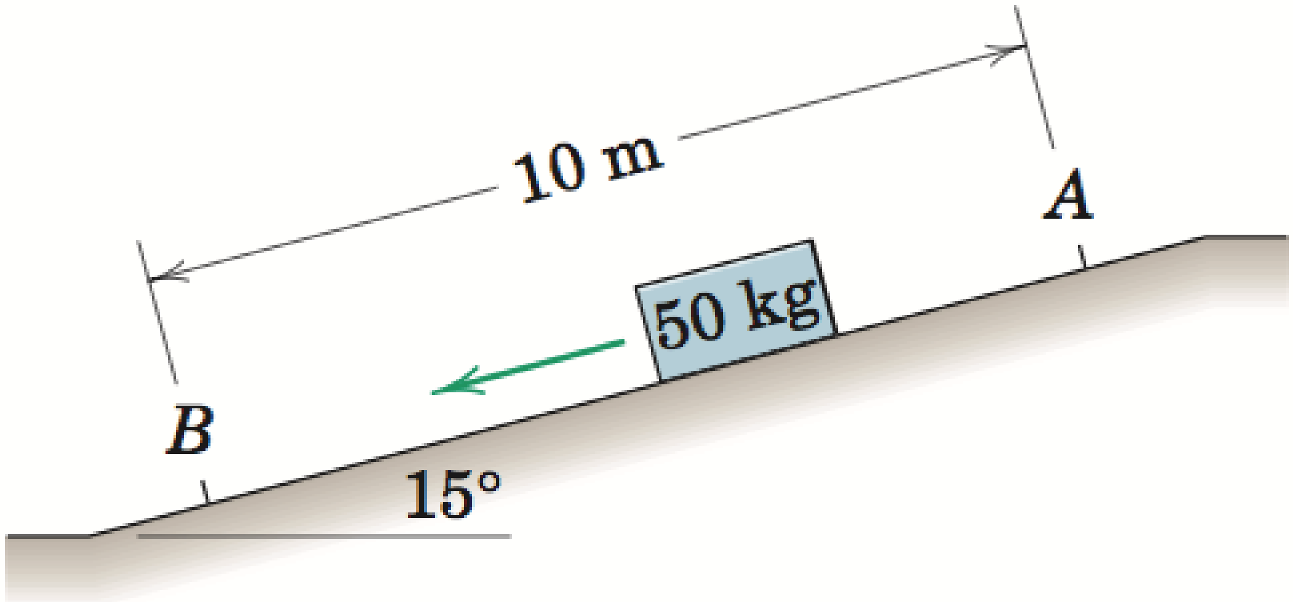
Work and Kinetic Energy

Lecture 35

ME 231: Dynamics

Question of the Day



A ***50-kg*** crate is given an initial ***velocity*** of ***4 m/s*** down the chute at *A*. The ***coefficient of kinetic friction*** is ***0.30***.

Determine the ***velocity*** *v* of the crate when it reaches the bottom of the chute at *B*.

Outline for Today

- Question of the day
- From **F**=*m***a** to work and energy
- Definition of work
- Units of work
- Calculation of work
- Examples of work
- Principle of work and kinetic energy
- Advantages of the work-energy method
- Answer your questions!

Recall: Possible Solutions to Kinetics Problems

- Direct application of ***Newton's 2nd Law***
  - force-mass-acceleration method
  - *Chapters 3 and 7*
- Use of ***impulse*** and ***momentum*** methods
  - *Chapters 5 and 8*
- Use of ***work*** and ***energy*** principles
  - *Chapters 4 and 8*

From F=*ma* to Work and Energy

- ***Integrate*** equations of motion with respect to ***displacement***
- **Work ($U_{1\text{-}2}$)** on m equals change in ***kinetic energy*** **(ΔT)** of m
- Facilitates the ***solution*** of problems where ***forces*** act over ***specified displacement*** interval

$$\Sigma \mathbf{F} = m\mathbf{a}$$

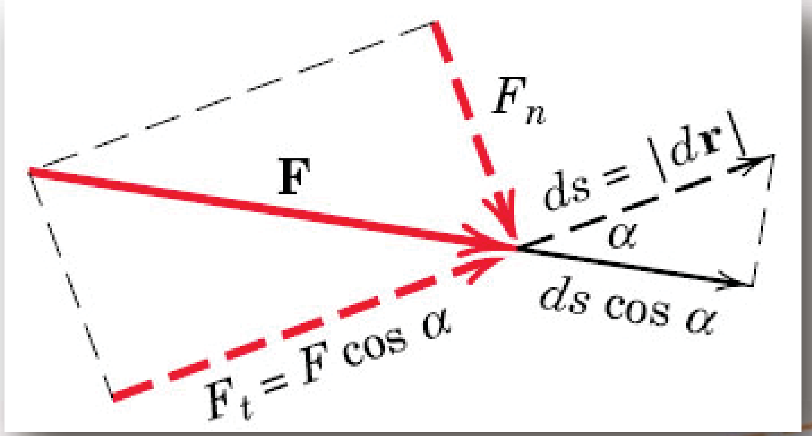
$$\int_1^2 \Sigma \mathbf{F} \cdot d\mathbf{r} = \int_1^2 m\mathbf{a} \cdot d\mathbf{r}$$

$$\int_{s_1}^{s_2} F_t \, ds = \int_{v_1}^{v_2} mv \, dv$$

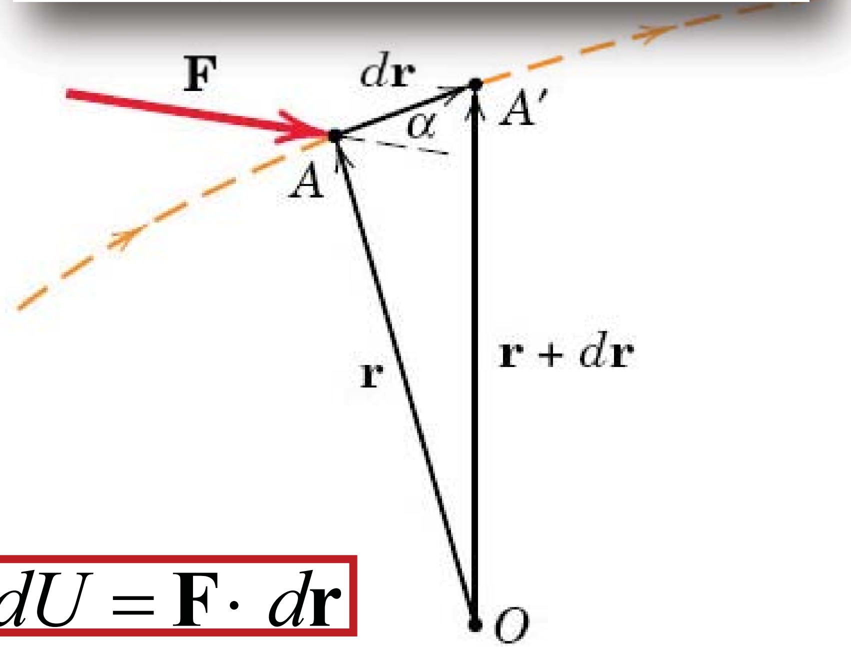
$$\int_{s_1}^{s_2} F_t \, ds = \frac{1}{2} m\left(\Delta v^2\right)$$

$$U_{1-2} = \Delta T$$

Definition of Work



- Particle of ***mass m*** is located by ***position vector*** **r**
- ***Displacement vector*** $d\mathbf{r}$ is tangent to its path
- Work done by ***force*** **F** during ***displacement*** $d\mathbf{r}$ is the ***dot product*** of **F** and $d\mathbf{r}$



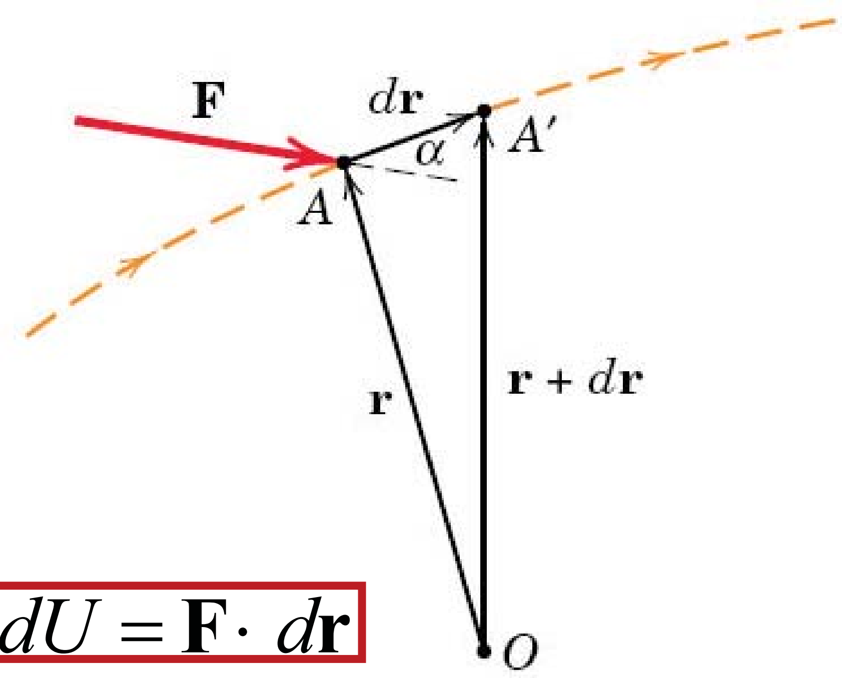
$$dU = \mathbf{F} \cdot d\mathbf{r}$$

$$dU = F\,ds\,\cos\alpha$$

$$dU = F_t\,ds$$

Units of Work

- SI units are ***force (N)*** times ***displacement (m)***
- Special unit named ***joule (J)*** equal to 1 N acting over 1 m
- Not to be confused with the unit for ***moment of force*** or ***torque (Nm)***



$$dU = \mathbf{F} \cdot d\mathbf{r}$$

$$dU = F\,ds\,\cos\alpha$$

$$dU = F_t\,ds$$

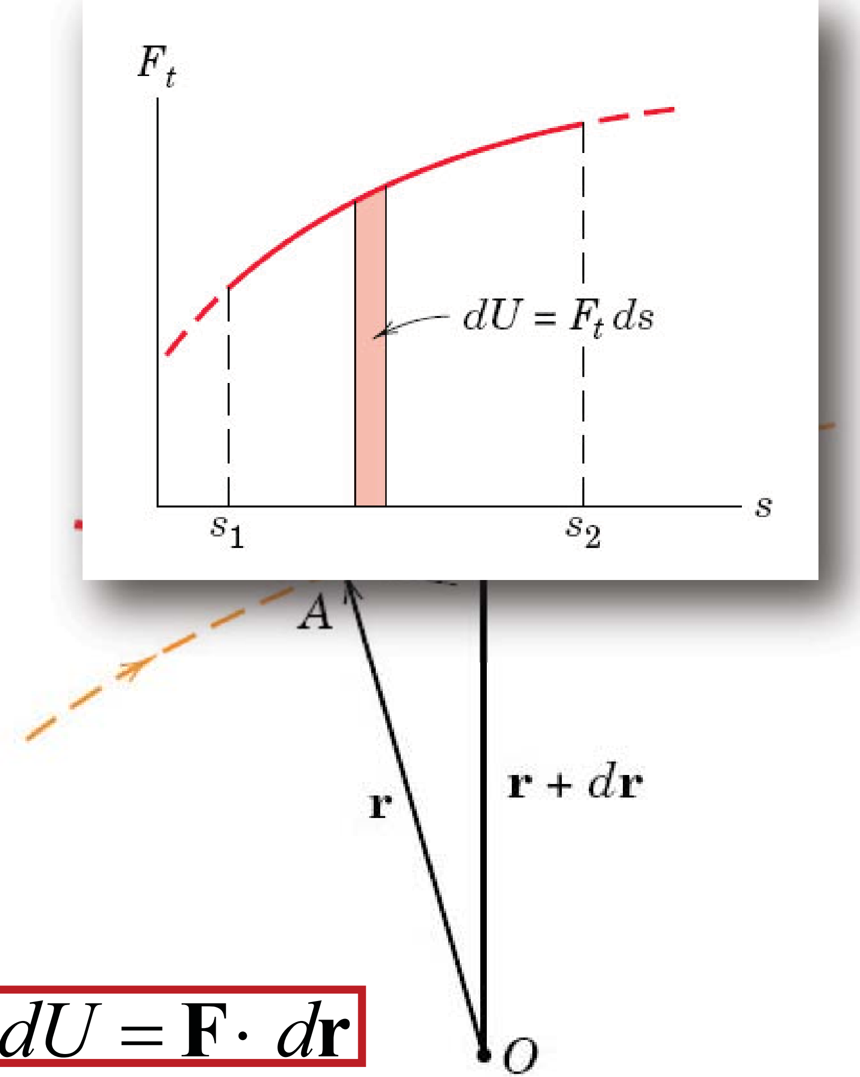
Calculation of Work

During a finite movement, the ***force*** does an amount of ***work*** equal to:

$$U = \int_1^2 \mathbf{F} \cdot d\mathbf{r}$$

$$U = \int_1^2 \left(F_x dx + F_y dy + F_z dz \right)$$

$$\boxed{U = \int_{s_1}^{s_2} F_t \, ds}$$

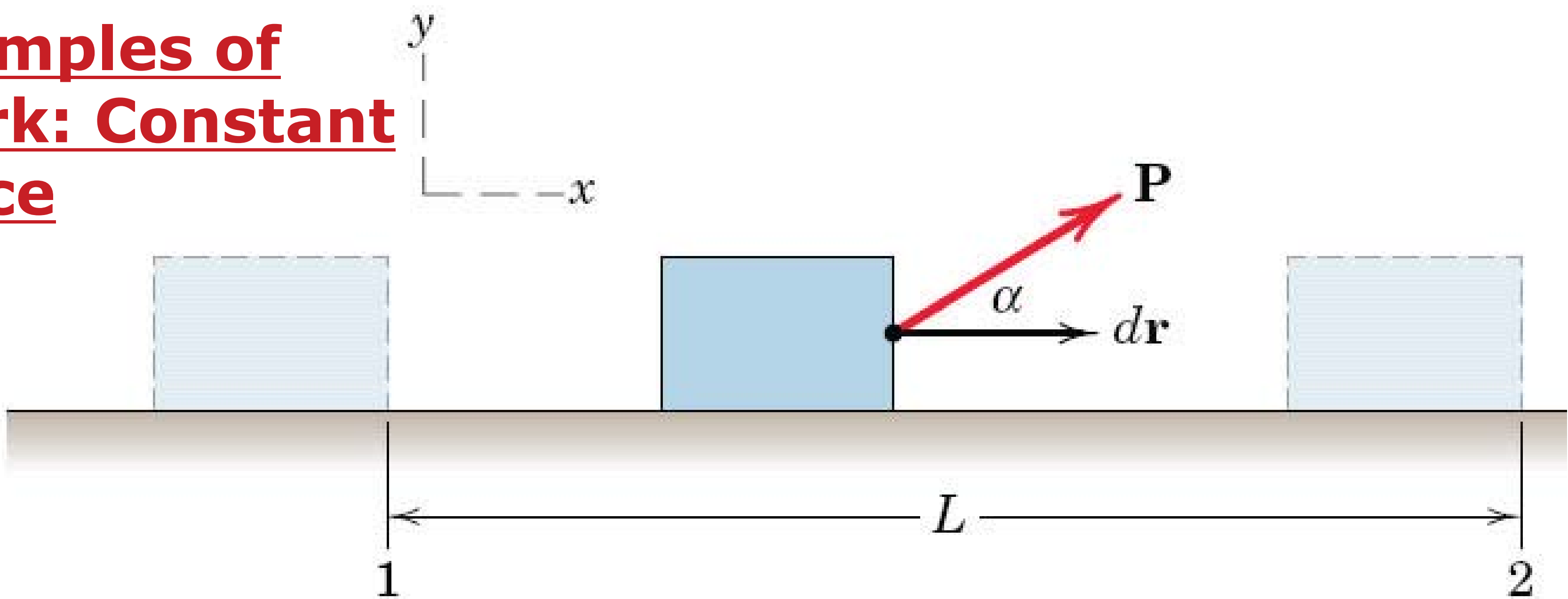


$$\boxed{dU = \mathbf{F} \cdot d\mathbf{r}}$$

$$dU = F \, ds \, \cos\alpha$$

$$\boxed{dU = F_t \, ds}$$

Examples of Work: Constant Force

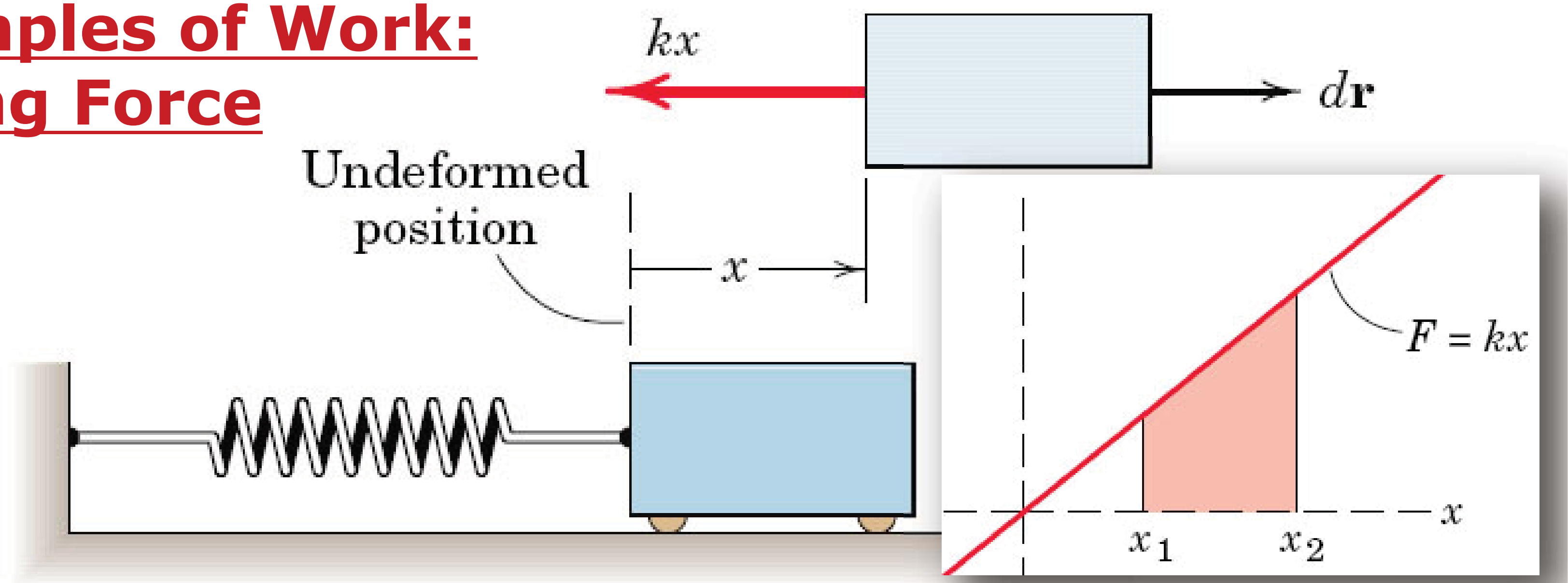


- ***Constant force*** P applied to the body as it moves from ***position 1*** to ***2***
- ***Work*** interpreted as ***force*** P ***cos*** α times the ***distance*** L traveled

$$U = \int_1^2 \mathbf{F} \cdot d\mathbf{r} = \int_1^2 \left[(P\cos\alpha)\,\mathbf{i} + (P\sin\alpha)\,\mathbf{j} \right] \cdot dx\,\mathbf{i}$$

$$U = \int_{x_1}^{x_2} P\cos\alpha\, dx = P\cos\alpha\left(x_2 - x_1\right) = PL\cos\alpha$$

Examples of Work: Spring Force

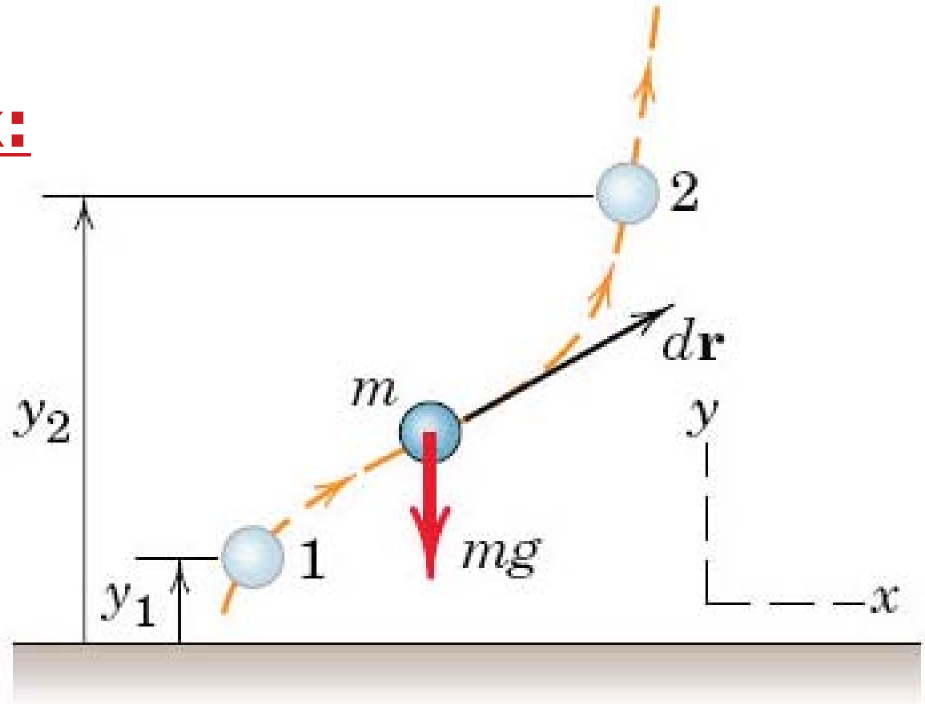


- ***Linear spring*** of stiffness k
- ***Force*** to stretch or compress is proportional to x
- ***Spring force*** exerted on body is $\mathbf{F}=-kx\,\mathbf{i}$

$$U = \int_1^2 \mathbf{F}\cdot d\mathbf{r} = \int_1^2 (-kx\,\mathbf{i})\cdot dx\,\mathbf{i}$$

$$U = -\int_{x_1}^{x_2} kx\,dx = \tfrac{1}{2}\mathrm{k}\left(x_1^2 - x_2^2\right)$$

Examples of Work: Weight



- ***Acceleration of gravity*** g is constant
- ***Work*** is done by the ***weight*** mg over an ***altitude change*** $(y_2 - y_1)$

$$U = \int_1^2 \mathbf{F} \cdot d\mathbf{r} = \int_1^2 (-mg\,\mathbf{j}) \cdot (dx\,\mathbf{i} + dy\,\mathbf{j})$$

$$U = -mg \int_{y_1}^{y_2} dy = -mg(y_2 - y_1)$$

Principle of Work and Kinetic Energy

- The ***kinetic energy*** T of a particle is $T = \frac{1}{2}mv^2$
- ***Work*** done to bring it a particle from ***velocity*** v_1 to a ***velocity*** v_2

$$U_{1-2} = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$$

$$U_{1-2} = T_2 - T_1 = \Delta T$$ ***(work-energy eq.)***

$$T_1 + U_{1-2} = T_2$$

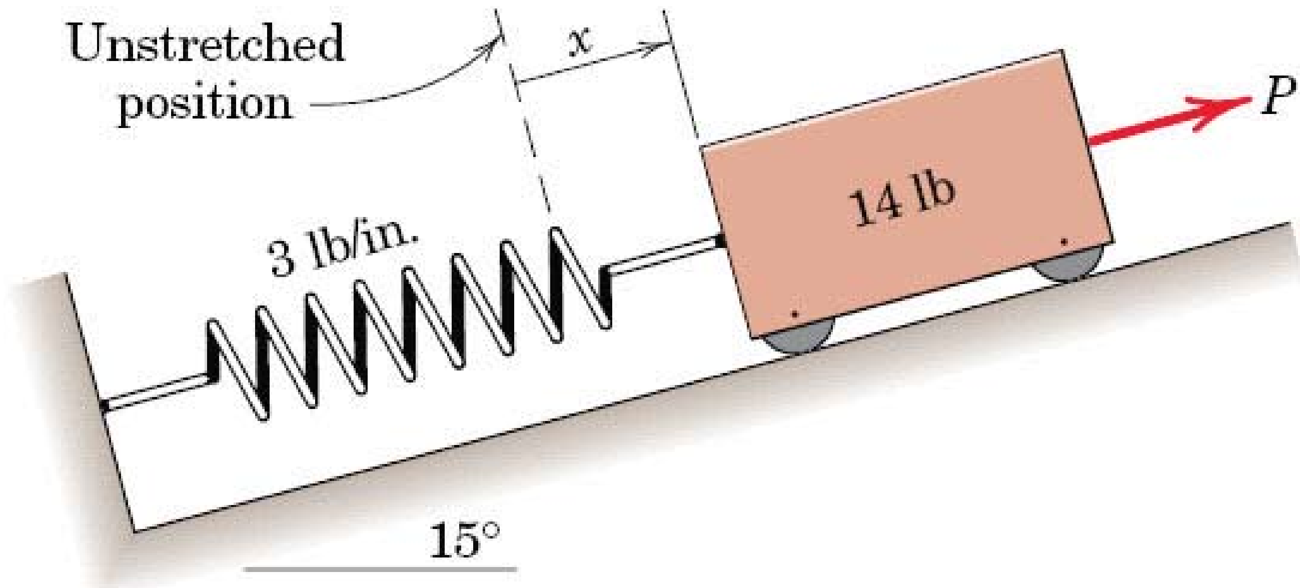
Advantages of the Work-Energy Method

$$T = \frac{1}{2}mv^2$$

$$T_1 + U_{1-2} = T_2$$

- ***Avoids*** the need for computing ***accelerations***
- Leads directly to ***velocity changes*** as ***functions*** of ***forces*** doing ***work***
- Involves ***only*** those ***forces which do work***
- Enables ***analysis*** of a ***system*** of particles ***rigidly connected*** without isolating individual particles

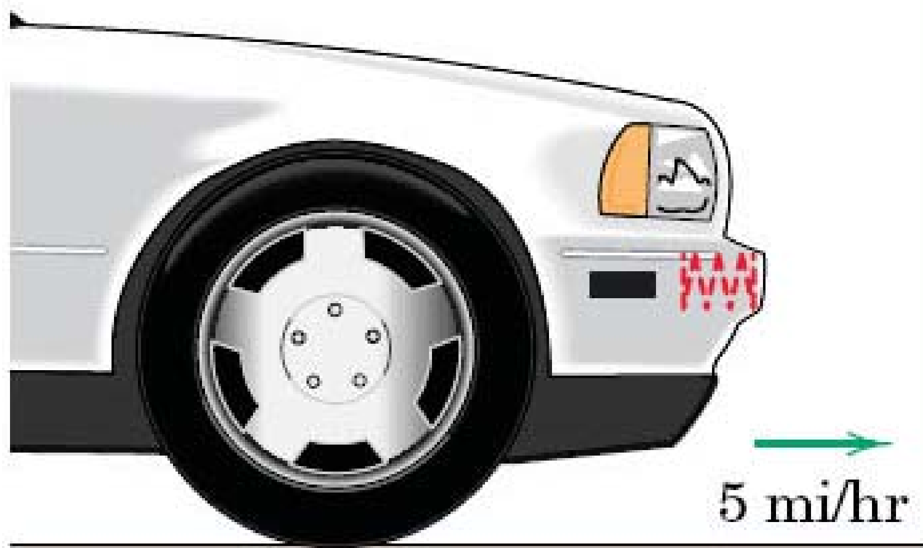
Work and Kinetic Energy: Exercise 1



Under the action of ***force P***, the cart moves from initial ***position x_1 = -6 in*** to the final ***position x_2 = 3 in***.

Determine the ***work*** done on the cart by (a) the ***spring*** and (b) the ***weight***.

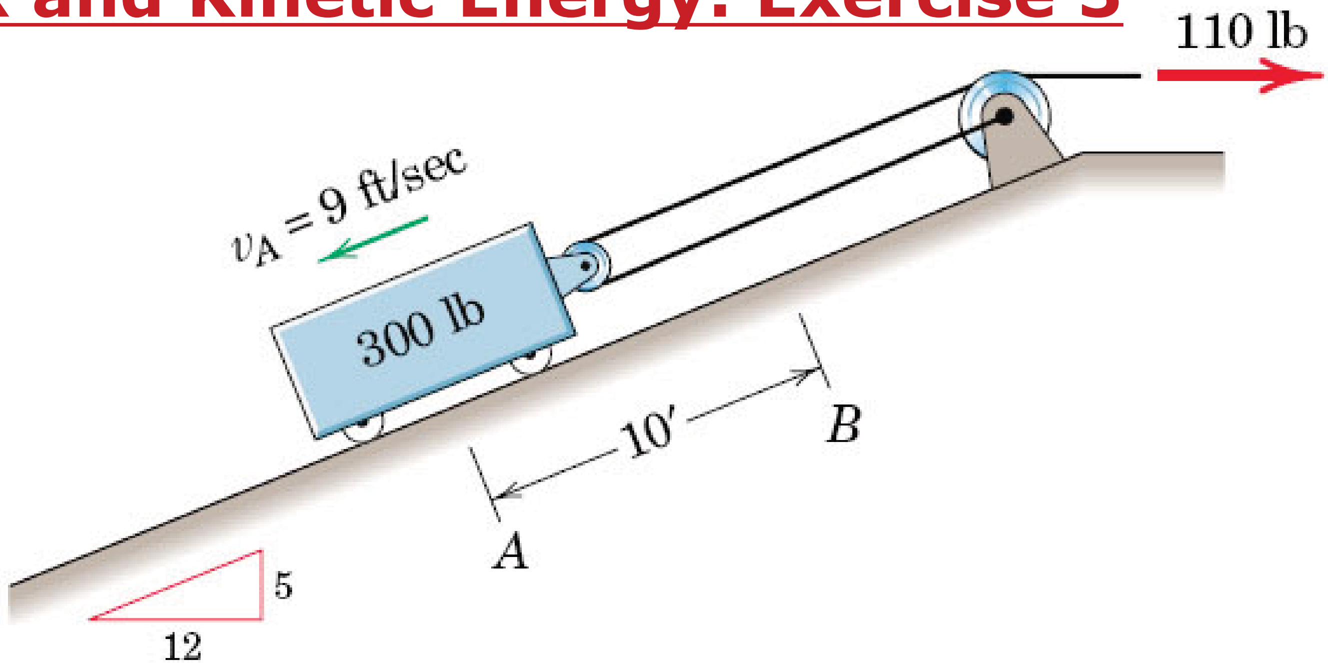
Work and Kinetic Energy: Exercise 2



The design of a spring bumper for a ***3500-lb*** car must stop the car from a ***speed*** of ***5 mph*** in a ***distance*** of ***6 in*** of ***spring deformation***.

Determine the ***stiffness*** k for each of ***two springs*** behind the bumper.

Work and Kinetic Energy: Exercise 3



The 300-lb carriage has an initial ***velocity*** of ***9 ft/s*** down the incline at ***A***, when a constant ***force*** of ***110 lb*** is applied to the cable.

Determine the ***velocity*** of the carriage when it reaches ***B***.

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For Next Time…

- Begin Homework #12 due on ***Monday (11/26), note date change***
- All ***grades*** (Exam 2a&b, HW 12, projected ***"final"*** course grade) on ***Wednesday (11/28)***
- Final ***Review*** and first opportunity to choose ***Final Exam Weighting*** on ***Monday (12/3)***
- Read Chapter 4, Sections 4.2 & 4.3