Work and Kinetic Energy
Lecture 35

ME 231: Dynamics
Question of the Day

A 50-kg crate is given an initial velocity of 4 m/s down the chute at A. The coefficient of kinetic friction is 0.30.

Determine the velocity \( v \) of the crate when it reaches the bottom of the chute at B.
Outline for Today

• Question of the day
• From $F=ma$ to work and energy
• Definition of work
• Units of work
• Calculation of work
• Examples of work
• Principle of work and kinetic energy
• Advantages of the work-energy method
• Answer your questions!
Recall: Possible Solutions to Kinetics Problems

- Direct application of *Newton’s 2nd Law*
  - force-mass-acceleration method
  - *Chapters 3 and 7*
- Use of *impulse* and *momentum* methods
  - *Chapters 5 and 8*
- Use of *work* and *energy* principles
  - *Chapters 4 and 8*
From \( F = ma \) to Work and Energy

- **Integrate** equations of motion with respect to **displacement**
- **Work** \((U_{1-2})\) on \( m \) equals change in **kinetic energy** \((\Delta T)\) of \( m \)
- Facilitates the **solution** of problems where **forces** act over specified **displacement** interval

\[
\Sigma F = ma \\
\int_{s_1}^{s_2} \Sigma F \cdot dr = \int_{v_1}^{v_2} ma \cdot dv \\
\int_{s_1}^{s_2} F_t \cdot ds = \int_{v_1}^{v_2} mv \cdot dv \\
\int_{s_1}^{s_2} F_t \cdot ds = \frac{1}{2} m (\Delta v^2) \\
U_{1-2} = \Delta T
\]
Definition of Work

- Particle of **mass** $m$ is located by **position vector** $r$
- **Displacement vector** $dr$ is tangent to its path
- Work done by **force** $F$ during **displacement** $dr$ is the **dot product** of $F$ and $dr$

$$dU = F \cdot dr$$
$$dU = F \, ds \, \cos \alpha$$
$$dU = F_t \, ds$$
Units of Work

- SI units are \textbf{force (N)} times \textbf{displacement (m)}
- Special unit named \textbf{joule (J)} equal to 1 N acting over 1 m
- Not to be confused with the unit for \textbf{moment of force} or \textbf{torque (Nm)}

\[
dU = \mathbf{F} \cdot d\mathbf{r}
\]
\[
dU = F \, ds \, \cos \alpha
\]
\[
dU = F_t \, ds
\]
Calculation of Work

During a finite movement, the **force** does an amount of **work** equal to:

\[
U = \int_{s_1}^{s_2} F \cdot dr
\]

\[
U = \int_{1}^{2} \left( F_x dx + F_y dy + F_z dz \right)
\]

\[
U = \int_{s_1}^{s_2} F_t ds
\]

\[
dU = F \cdot dr
\]

\[
dU = F_t ds \cos \alpha
\]

\[
dU = F_t ds
\]
Examples of Work: Constant Force

- **Constant force** $P$ applied to the body as it moves from **position 1** to **2**

- **Work** interpreted as **force** $P \cos \alpha$ times the **distance** $L$ traveled

\[
U = \int_{x_1}^{x_2} P \cos \alpha \, dx = P \cos \alpha (x_2 - x_1) = PL \cos \alpha
\]
**Examples of Work: Spring Force**

- **Linear spring** of stiffness $k$
- **Force** to stretch or compress is proportional to $x$
- **Spring force** exerted on body is $F = -kx$ i

\[
U = \int_{1}^{2} F \cdot dr = \int_{1}^{2} (-kx \textbf{i}) \cdot dx \textbf{i}
\]

\[
U = -\int_{x_1}^{x_2} kx \, dx = \frac{1}{2}k(x_1^2 - x_2^2)
\]
Examples of Work:

- **Acceleration of gravity** $g$ is constant
- **Work** is done by the **weight** $mg$ over an altitude change $(y_2 - y_1)$

\[
U = \int_{1}^{2} \mathbf{F} \cdot d\mathbf{r} = \int_{1}^{2} (-mg \, \mathbf{j}) \cdot (dx \, \mathbf{i} + dy \, \mathbf{j})
\]

\[
U = -mg \int_{y_1}^{y_2} dy = -mg(y_2 - y_1)
\]
Principle of Work and Kinetic Energy

- The **kinetic energy** $T$ of a particle is
  
  \[ T = \frac{1}{2} mv^2 \]

- **Work** done to bring it a particle from velocity $v_1$ to a velocity $v_2$

  \[ U_{1-2} = \frac{1}{2} m (v_2^2 - v_1^2) \]

  \[ U_{1-2} = T_2 - T_1 = \Delta T \quad \text{(work-energy eq.)} \]

  \[ T_1 + U_{1-2} = T_2 \]
Advantages of the Work-Energy Method

- **Avoids** the need for computing *accelerations*
- Leads directly to *velocity changes* as *functions* of *forces* doing *work*
- Involves *only* those *forces which do work*
- Enables *analysis* of a *system* of particles *rigidly connected* without isolating individual particles

\[
T = \frac{1}{2} m v^2
\]

\[
T_1 + U_{1-2} = T_2
\]
Under the action of force $P$, the cart moves from initial position $x_1 = -6\text{ in}$ to the final position $x_2 = 3\text{ in}$.

Determine the work done on the cart by (a) the spring and (b) the weight.
The design of a spring bumper for a 3500-lb car must stop the car from a speed of 5 mph in a distance of 6 in of spring deformation.

Determine the stiffness $k$ for each of two springs behind the bumper.
The 300-lb carriage has an initial *velocity* of *9 ft/s* down the incline at *A*, when a constant *force* of *110 lb* is applied to the cable.

Determine the *velocity* of the carriage when it reaches *B*. 
Outline for Today

• Question of the day
• From $F=ma$ to work and energy
• Definition of work
• Units of work
• Calculation of work
• Examples of work
• Principle of work and kinetic energy
• Advantages of the work-energy method
• Answer your questions!
For Next Time...

- Begin Homework #12 due on **Monday (11/26)**, note date change
- All **grades** (Exam 2a&b, HW 12, projected “**final**” course grade) on **Wednesday (11/28)**
- Final **Review** and first opportunity to choose **Final Exam Weighting** on **Monday (12/3)**
- Read Chapter 4, Sections 4.2 & 4.3