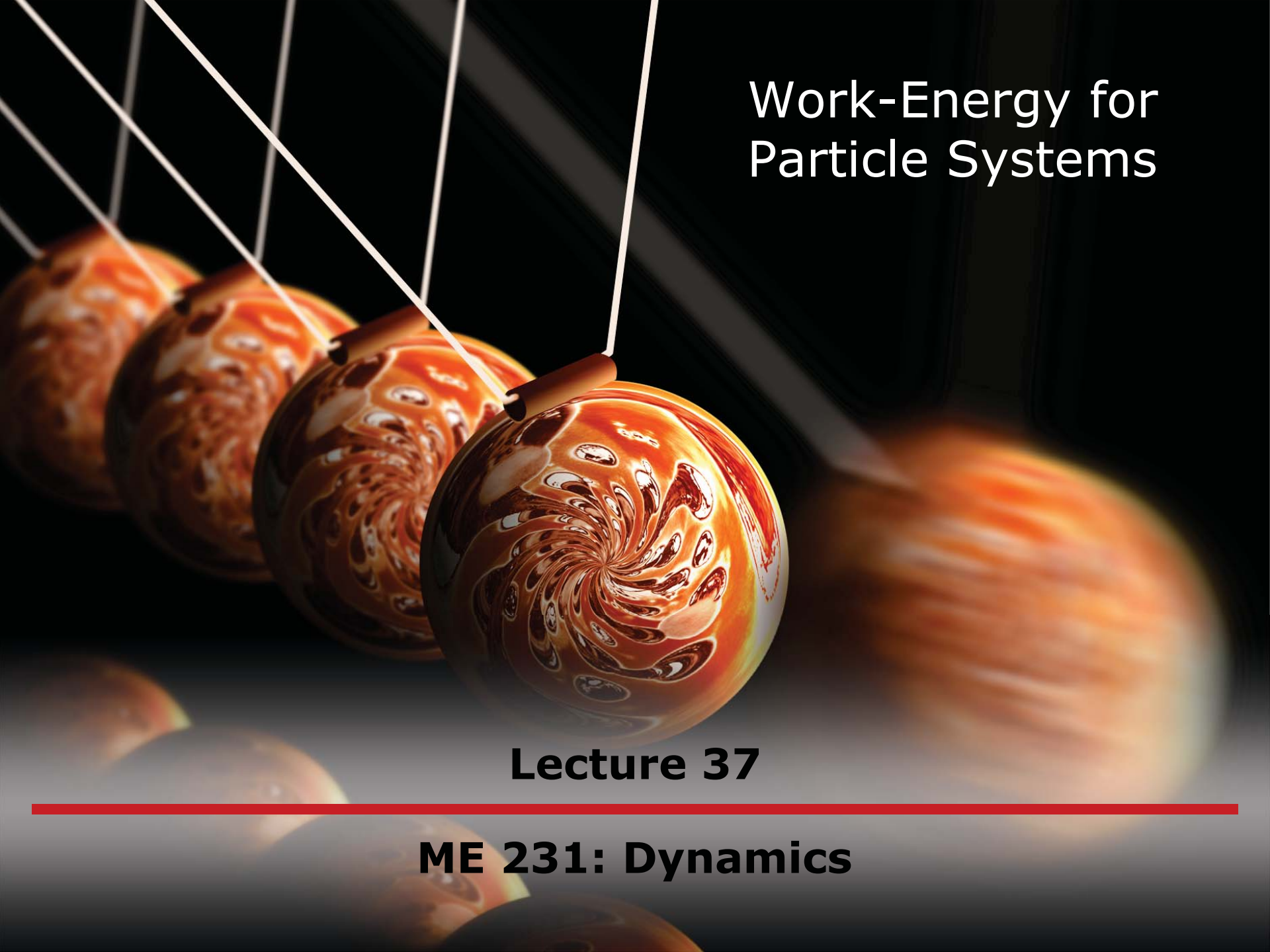


Work-Energy for Particle Systems

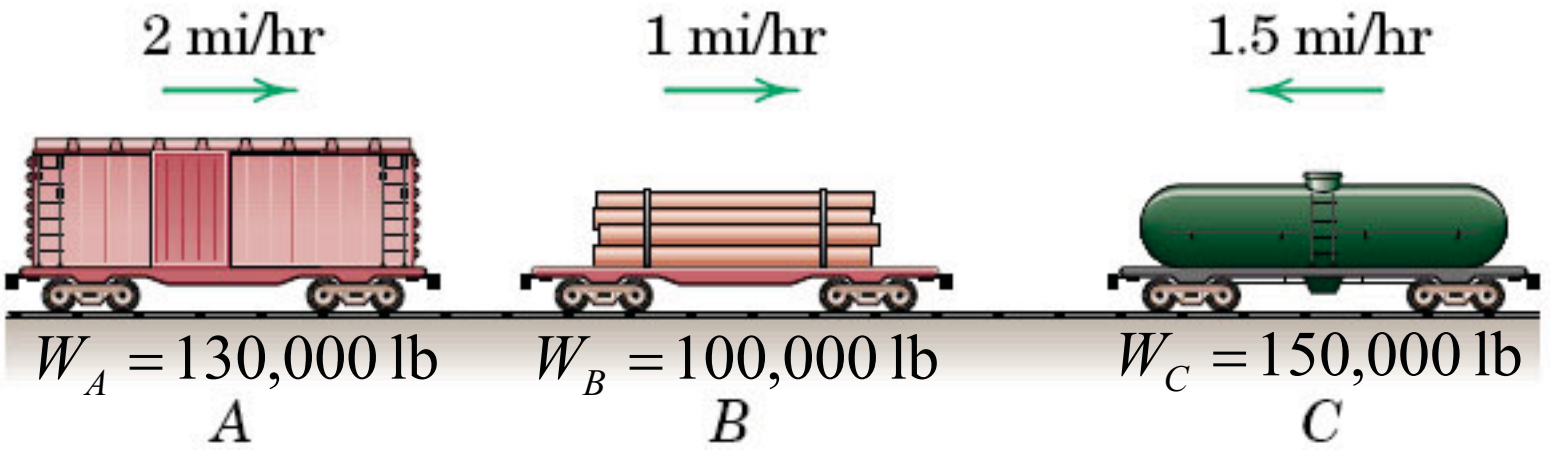
Lecture 37

ME 231: Dynamics

Question of the Day

Three freight cars with ***weights*** and ***velocities*** shown impact each other and become coupled together with a common ***velocity v = 0.355 mph***.

Determine the percentage loss of ***energy*** due to coupling.



Outline for Today

- Question of the day
- Work-energy relation for systems
- Kinetic energy expression for systems
- Conservation of energy
- Answer your questions!

Recall: Principle of Work and Kinetic Energy

- The ***kinetic energy T*** of a particle is $\boxed{T = \frac{1}{2}mv^2}$

- ***Work*** done to bring a particle from ***velocity*** v_1 to a ***velocity*** v_2

$$U_{1-2} = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$$

$$U_{1-2} = T_2 - T_1 = \Delta T \quad \textbf{(work-energy eq.)}$$

$$\boxed{T_1 + U_{1-2} = T_2}$$

Work-Energy Relation for Particle Systems: Σi

All gravitational and elastic forces are doing work!

- The ***kinetic energy*** T of a particle system is

$$\Sigma T_i = \Sigma \frac{1}{2} m_i v_i^{\,2}$$

- ***Work*** done to bring a particle system from ***kinetic energy*** T_1 to a ***kinetic energy*** T_2

$$\Sigma \left(U_{1-2} \right)_i = \Sigma \Delta T_i$$

$$\Sigma \left(T_1 \right)_i + \Sigma \left(U_{1-2} \right)_i = \Sigma \left(T_2 \right)_i$$

Recall: Work-Energy Equation

- The ***work*** of all external forces *other than* gravitational and spring forces is $\boxed{U'_{1-2}}$

$$\boxed{U'_{1-2} = \Delta T + \Delta V} \quad \textbf{\textit{(work-energy eq.)}}$$

$$T_1 + V_1 + U'_{1-2} = T_2 + V_2$$

Work-Energy Relation for Particle Systems: Σi

System includes gravitational and elastic members!

- The ***work*** of all external forces *other than* gravitational and spring forces is $\boxed{\Sigma\left(U'_{1-2}\right)_i}$

$$\boxed{\Sigma\left(U'_{1-2}\right)_i = \Sigma\Delta T_i + \Sigma\Delta V_i}$$

$$\Sigma\left(T_1\right)_i + \Sigma\left(V_1\right)_i + \Sigma\left(U'_{1-2}\right)_i = \Sigma\left(T_2\right)_i + \Sigma\left(V_2\right)_i$$

Kinetic Energy Expression for Systems

$$\Sigma T_i = \Sigma \frac{1}{2} m_i v_i^{\,2}$$

$$\mathbf{v}_i = \bar{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i$$

$$\Sigma T_i = \Sigma \frac{1}{2} m_i \mathbf{v}_i \cdot \mathbf{v}_i = \Sigma \frac{1}{2} m_i \left(\bar{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i\right) \cdot \left(\bar{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i\right)$$

$$\Sigma T_i = \Sigma \frac{1}{2} m_i \bar{v}^2 + \Sigma \frac{1}{2} m_i \left|\dot{\rho}_i\right|^2 + \Sigma m_i \bar{\mathbf{v}} \cdot \dot{\boldsymbol{\rho}}_i$$

$$\Sigma T_i = \frac{1}{2} m \bar{v}^2 + \Sigma \frac{1}{2} m_i \left|\dot{\rho}_i\right|^2$$

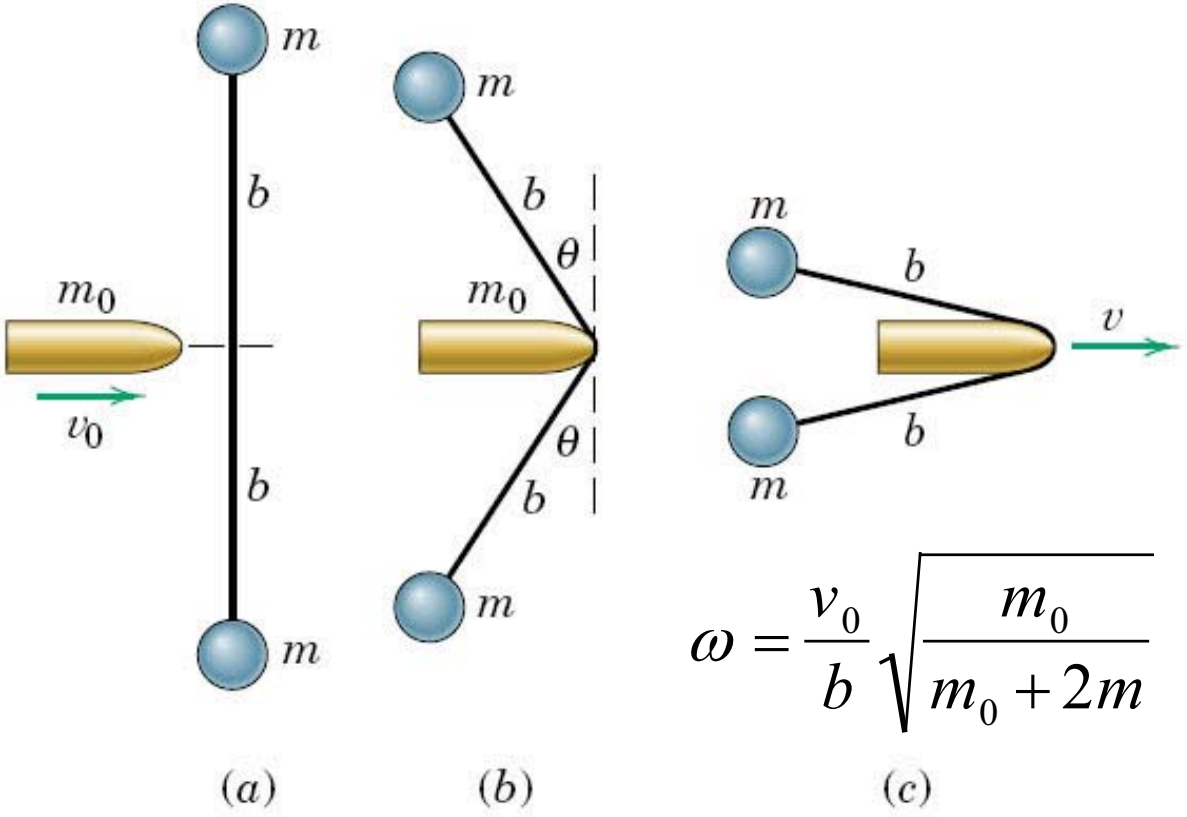
Conservation of Energy for Particle Systems

$$\Sigma \Delta T_i + \Sigma \Delta V_i = 0$$

$$\Sigma \left(T_1\right)_i + \Sigma \left(V_1\right)_i = \Sigma \left(T_2\right)_i + \Sigma \left(V_2\right)_i$$

- A ***conservative system*** does not lose energy by virtue of ***internal friction*** or ***inelastic members*** which dissipate energy
- If ***no work*** is done on a ***conservative system***, the total ***energy*** is ***constant*** (***law of conservation of dynamical energy***)

Work-Energy: Exercise 1



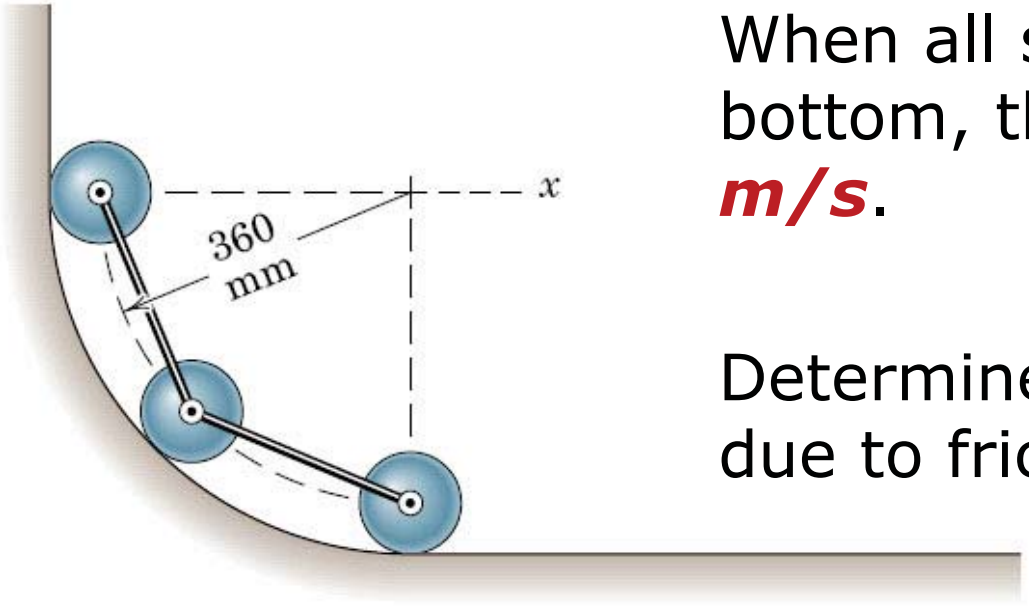
$$\omega = \frac{v_0}{b}\sqrt{\frac{m_0}{m_0 + 2m}}$$

Two spheres connected by a cord are initially at rest on a horizontal surface and a projectile hits the middle of the cord.

Determine the ***velocity v*** when θ approaches ***90°***.

Work-Energy: Exercise 2

Three steel balls, each of ***mass 2.75 kg***, are connected by hinged links of negligible mass. They are released from rest in the position shown and slide down the quarter-circular guide. When all spheres reach the bottom, their ***velocity*** is ***1.56 m/s***.



Determine the ***energy loss*** due to friction.

For Next Time…

- Read Chapter 4, Section 4.3
- Read Chapter 8, Section 8.1

- SAIS response rate = 89% (58 of 65)