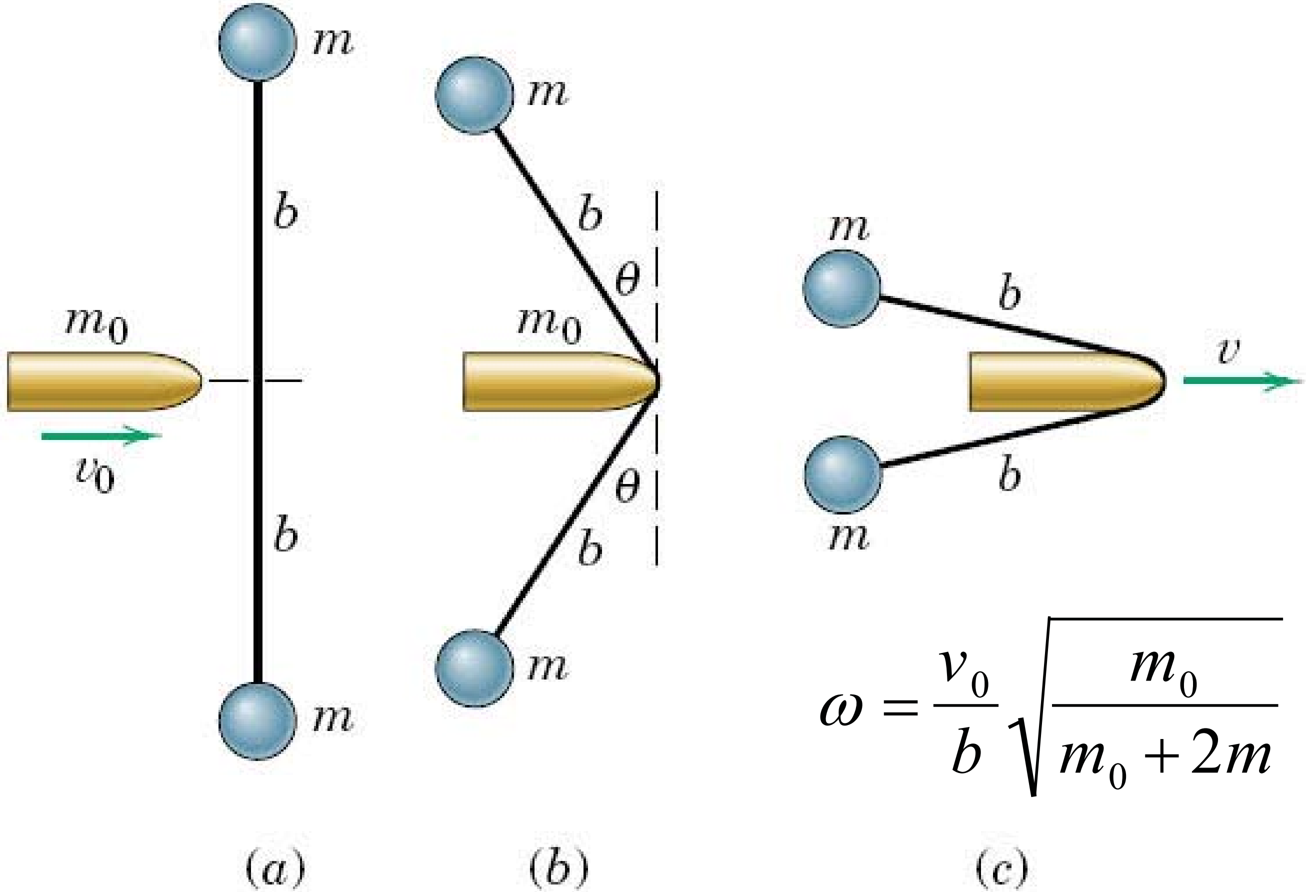
Work-Energy for Rigid Bodies

Lecture 38

ME 231: Dynamics

Question of the Day



$$\omega = \frac{v_0}{b}\sqrt{\frac{m_0}{m_0 + 2m}}$$

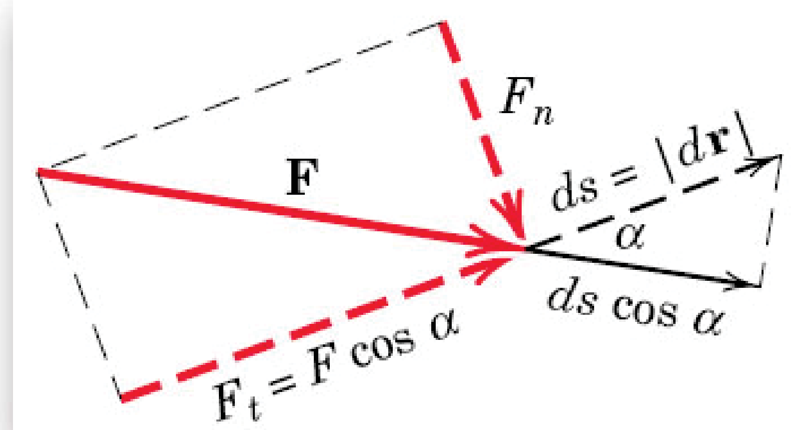
Two spheres connected by a cord are initially at rest on a horizontal surface and a projectile hits the middle of the cord.

Determine the ***velocity*** v when θ approaches ***90º***.

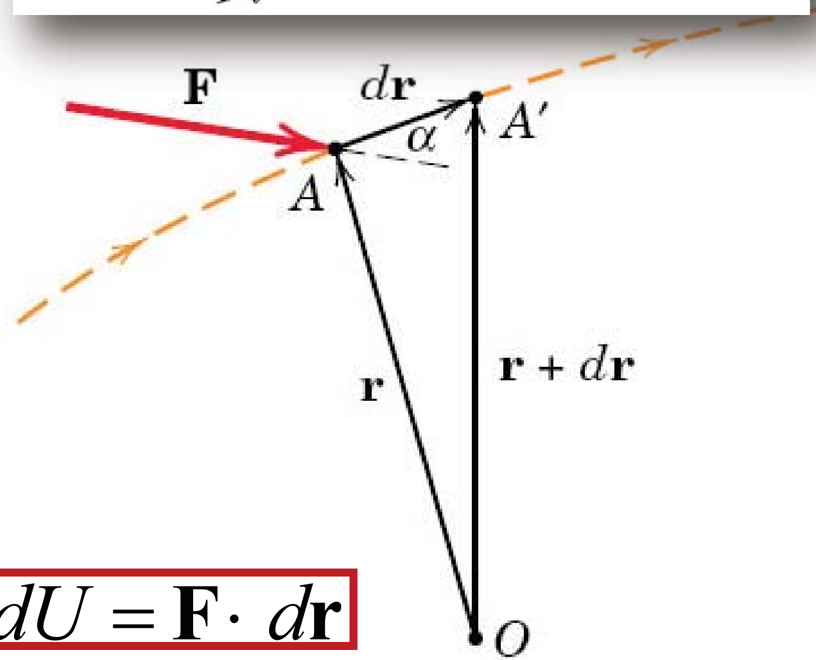
Outline for Today

- Question of the day
- Work of forces and couples
- Kinetic energy for rigid bodies
- Work-energy equation for rigid bodies
- Final exam addendum
- Answer your questions!

Recall: Definition of Work



- Particle of ***mass** m* is located by ***position vector*** $\mathbf{r}$
- ***Displacement vector*** $d\mathbf{r}$ is tangent to its path
- Work done by ***force*** $\mathbf{F}$ during ***displacement*** $d\mathbf{r}$ is the ***dot product*** of $\mathbf{F}$ and $d\mathbf{r}$

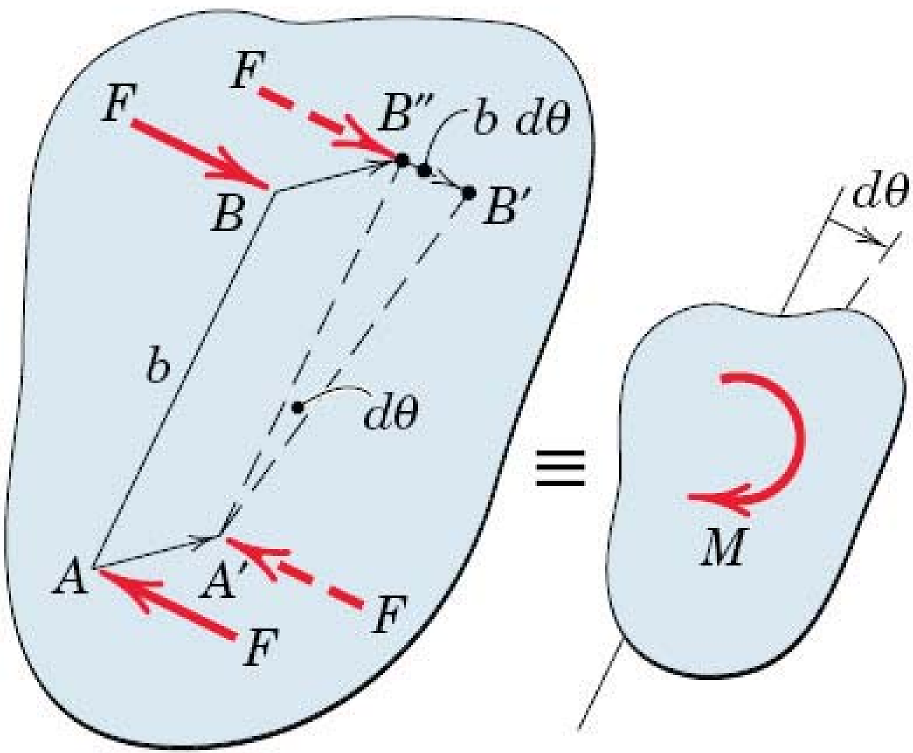


$$dU = \mathbf{F} \cdot d\mathbf{r}$$

$$dU = F\, ds\, \cos\alpha$$

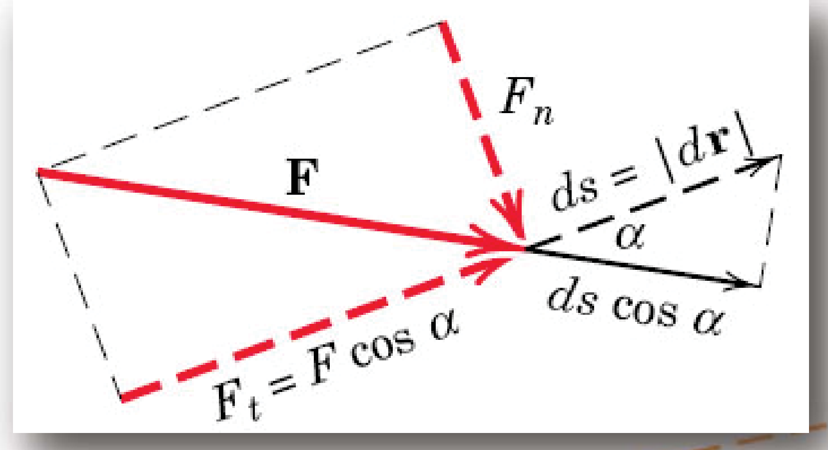
$$dU = F_t\, ds$$

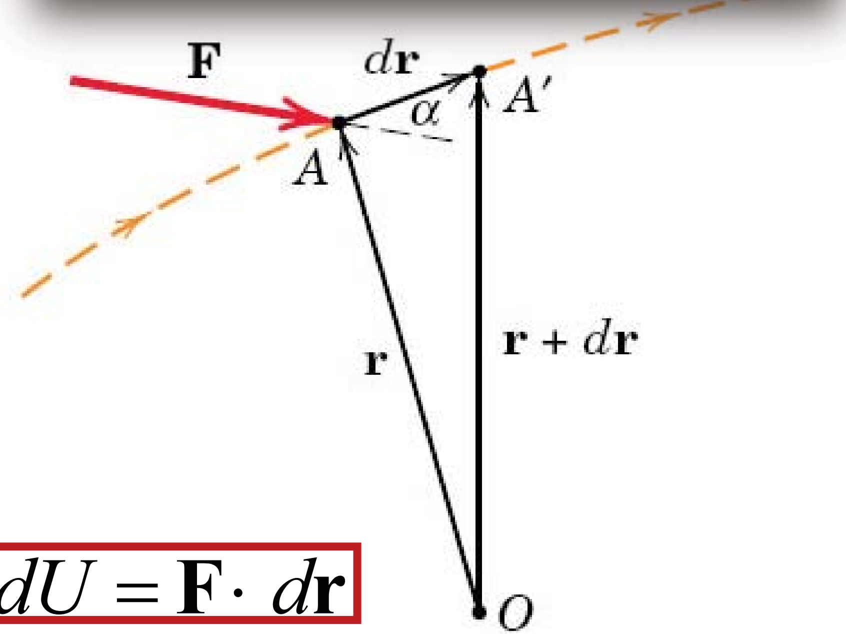
Work of Forces and Couples



$$dU = F\,(b\,d\theta)$$

$$dU = M\,d\theta$$





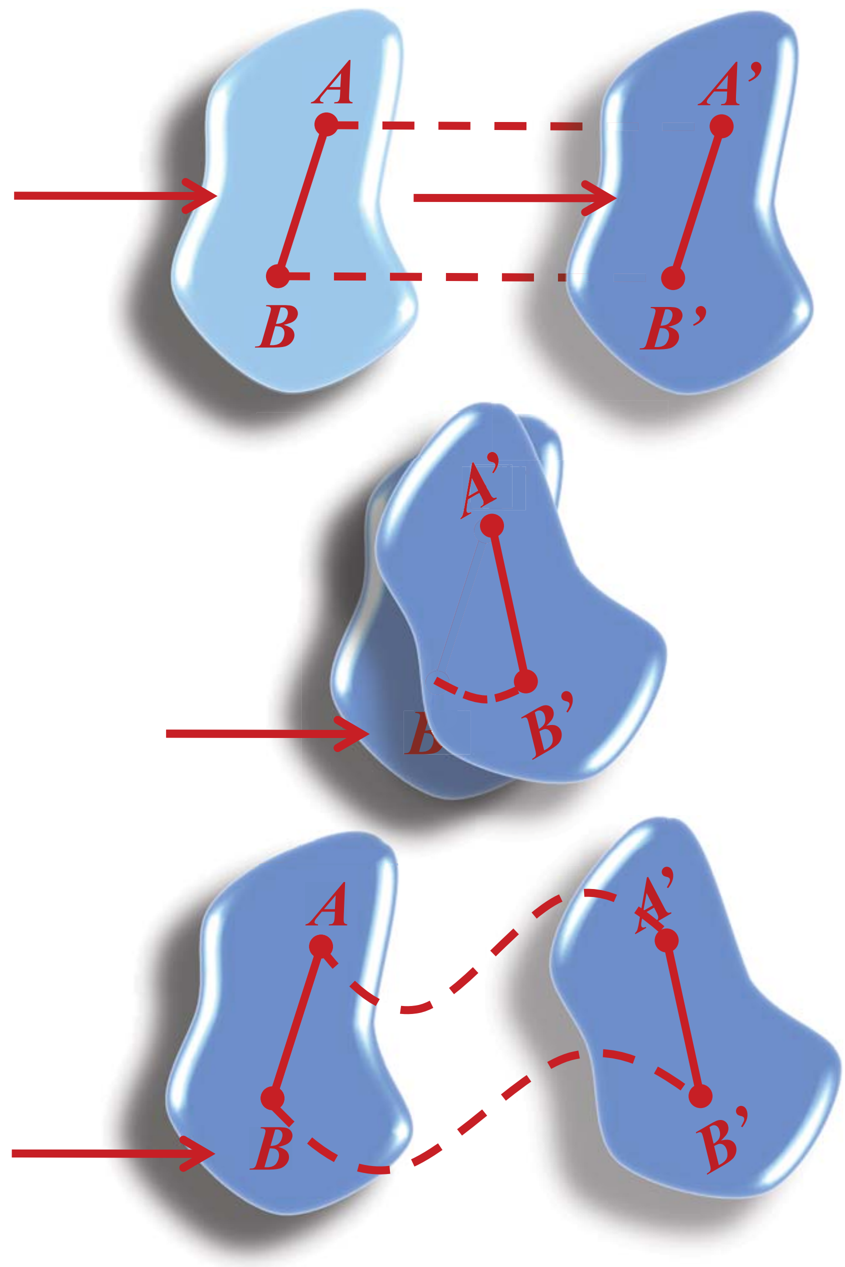
$$dU = \mathbf{F}\cdot d\mathbf{r}$$

$$dU = F\,ds\,\cos\alpha$$

$$dU = F_t\,ds$$

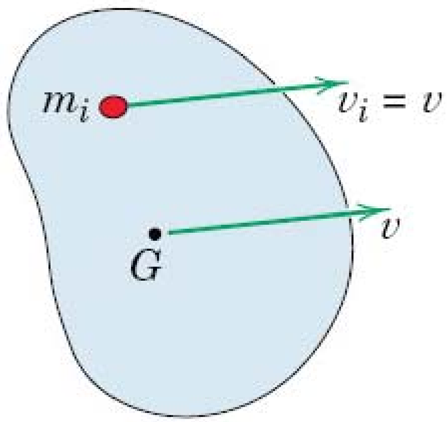
Recall: Plane Motion Types for Rigid Bodies

- Translation
- Fixed-axis rotation
- General plane motion



Kinetic Energy for Rigid Bodies

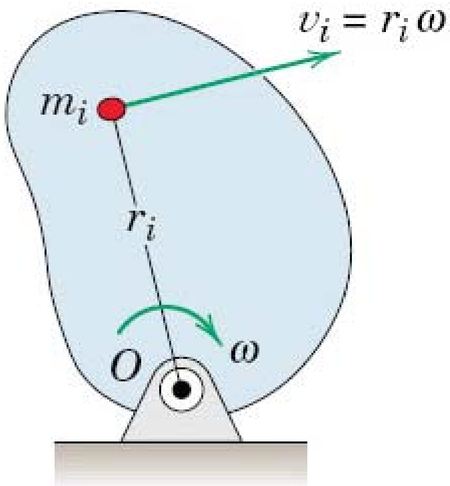
translation



$$T = \Sigma \frac{1}{2} m_i v^2$$

$$T = \frac{1}{2} m v^2$$

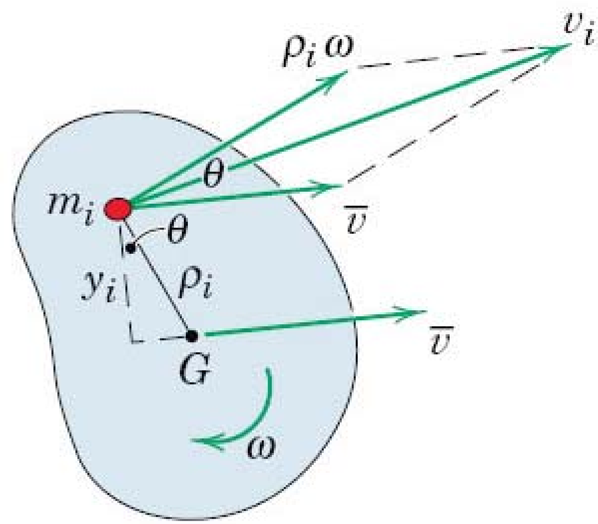
fixed-axis rotation



$$T = \Sigma \frac{1}{2} m_i (r_i \omega)^2$$

$$T = \frac{1}{2} I_O \omega^2$$

general plane motion



$$T = \Sigma \frac{1}{2} m_i \left(v_G^2 + \rho_i^2 \omega^2 \right)$$

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

Work-Energy Equation for Rigid Bodies

Express weight and springs as doing work

- ***Work*** done to bring a rigid body from ***kinetic energy*** T_1 to a ***kinetic energy*** T_2

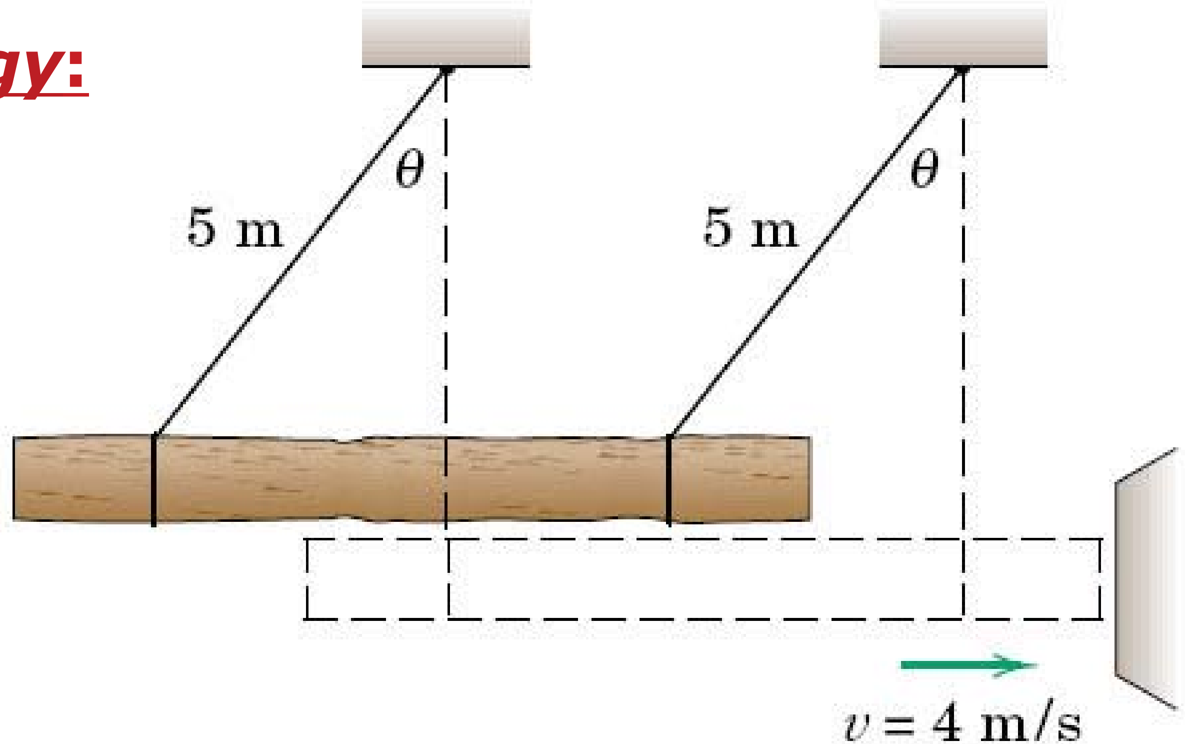
$$T_1 + U_{1-2} = T_2$$

Express weight and springs by means of potential energy

- The ***work*** of all external forces *other than* gravitational and spring forces is

$$T_1 + V_1 + U'_{1-2} = T_2 + V_2$$

Work-*Energy*: Exercise 1

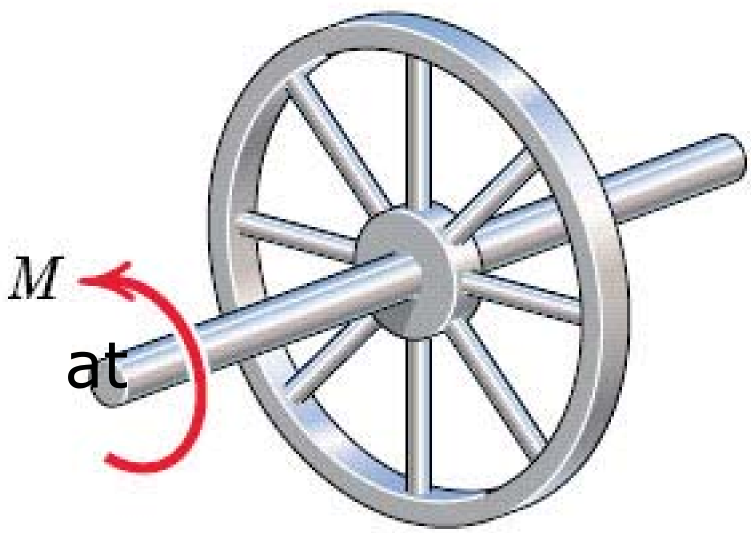


The log is suspended by two parallel 5-m cables and used as a battering ram.

Determine the ***angle*** θ for the log to be released from rest in order to strike the object to be smashed with a ***velocity*** of ***4 m/s***.

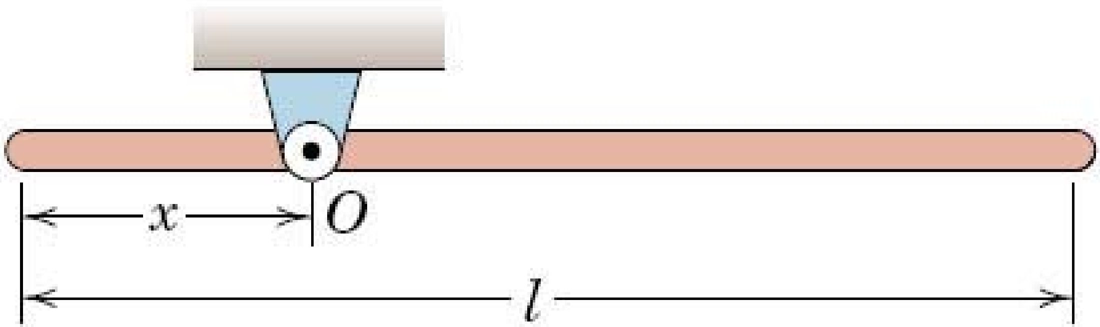
Work-Energy: Exercise 2

The ***50-kg*** flywheel has a ***radius of gyration*** of ***0.4 m*** about its shaft axis and is subjected to the ***torque*** $M = 2(1-e^{-0.1\theta})$ **Nm**, where θ is in radians.



Determine its ***angular velocity*** after ***5 revolutions*** if it starts at rest when $\theta = 0$.

Work-Energy: Exercise 3



Determine the ***distance*** x for which the ***angular velocity*** of the pivoted slender rod will be ***maximum*** as the bar passes the ***vertical position*** after being released from rest in the position shown.

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For Next Time…

- ***Final Review*** on Monday, December 3rd
- ***Final Exam*** on Monday, December 10th from 8:00am to 10:00am in Min Kao 524

- SAIS ***response rate = 91%*** (59 of 65)