Final Review
Lecture 39

ME 231: Dynamics

Questions of the Day

What is the most important concept in Chapter 2? **Time Derivative of a Vector**

What is the most important concept in Chapter 6? **Transformation of a Time Derivative**

What is the most important concept in mechanics? **Free Body Diagram**

What is the most important concept in dynamics? **Equations of Motion**

Outline for Today

- Questions of the day
- Concept and calculation of dynamics?
- Velocity and acceleration of a point
- Absolute- and relative-motion analysis
- Locating the instantaneous center
- Velocity and acceleration of a body

Kinematics (Exam 1)

- Inverse vs. forward dynamics
- Kinetics: cause of motion
- Direct application of Newton's 2nd Law
- Impulse-momentum principles
- Equations, equations, equations…

Kinetics (Exam 2)

- Principle of work and kinetic energy
- Work vs. potential energy and work-energy equation
- Work of forces and couples & energy of rigid bodies

Work-Energy (new stuff)

- Answer your questions!

Concept: What is dynamics?

Chapters 1, 2, 6 | **Chapters 3, 4, 5, 7, 8**

Relationship among ***position***, ***velocity***, and ***acceleration***

Relationship among ***forces*** and ***acceleration***

Calculation: How Do We Use Dynamics?

Newton's 2nd Law

Force. A push or pull exerted on a body, characterized by:

- magnitude
- direction
- point of application

Mass. Measure of the resistance of a body to linear acceleration.

$$F = m\,a$$

Acceleration. Velocity rate of change with respect to time

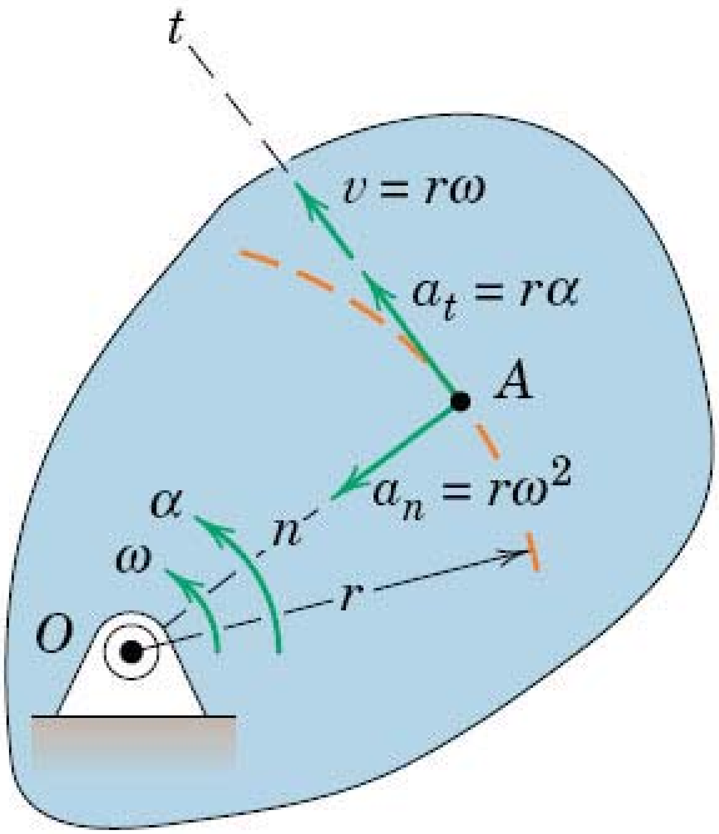
Velocity and Acceleration of a Point

Lecture	Velocity	Acceleration
2. Rectilinear	$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$	$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$
3. Curvilinear	$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\,\mathbf{i} + \dot{y}\,\mathbf{j}$	$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \ddot{x}\,\mathbf{i} + \ddot{y}\,\mathbf{j}$
4. Normal & Tangential	$\mathbf{v} = v\,\mathbf{e_t}$	$\mathbf{a} = \frac{v^2}{\rho}\mathbf{e_n} + \dot{v}\,\mathbf{e_t}$
5. Polar Coordinates	$\mathbf{v} = \dot{r}\,\mathbf{e_r} + r\dot{\theta}\,\mathbf{e}_\theta$	$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e_r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta$
7. Relative Motion	$\mathbf{v}_A = \dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \dot{\mathbf{r}}_{A/B}$	$\mathbf{a}_A = \dot{\mathbf{v}}_A = \ddot{\mathbf{r}}_A = \ddot{\mathbf{r}}_B + \ddot{\mathbf{r}}_{A/B}$

Absolute-Motion Analysis

- The method relates the ***position*** of a ***point***, A, on a rigid body to the ***angular position***, θ, of a ***line*** contained in the body

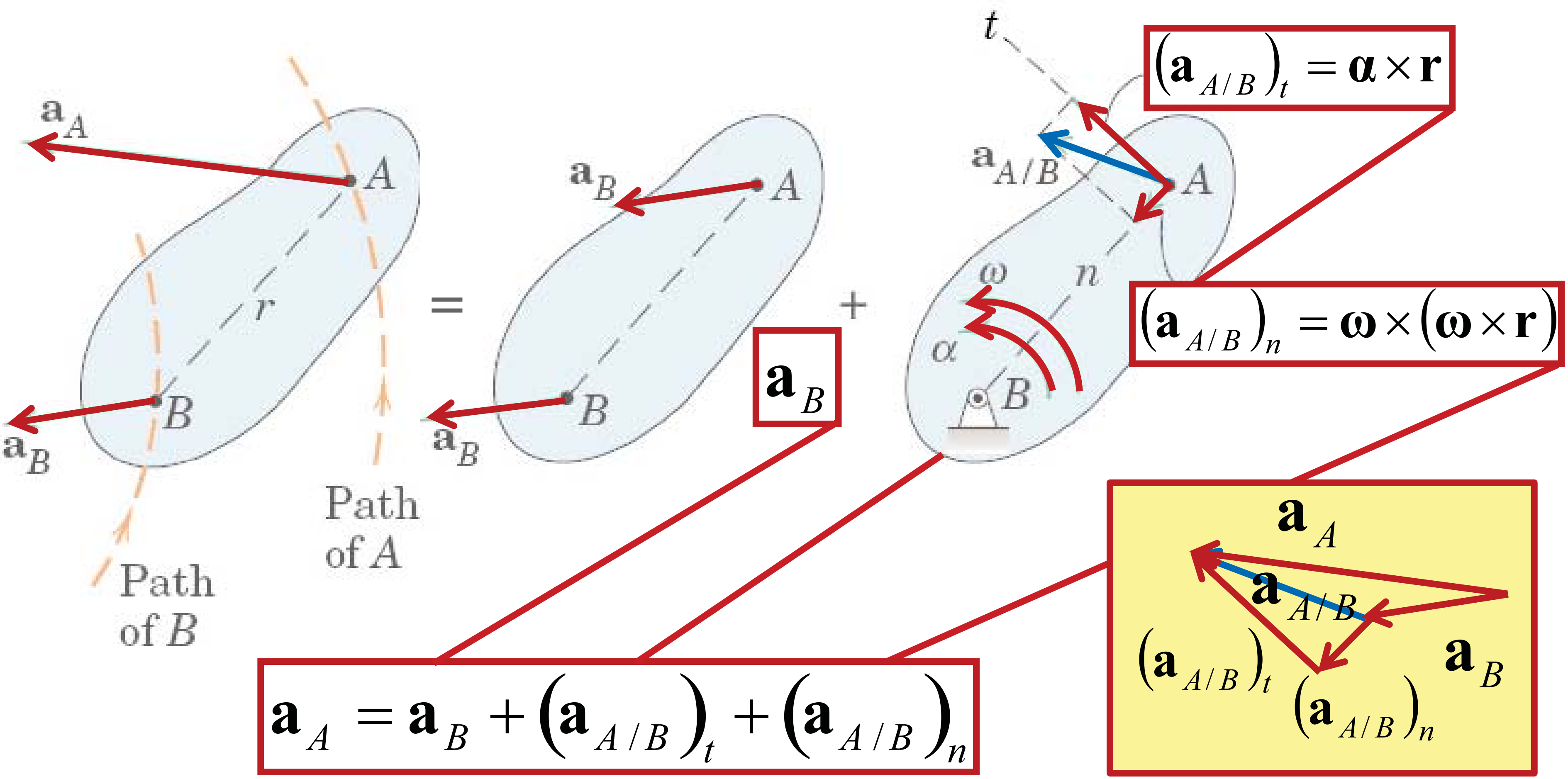
- The ***velocity*** and ***acceleration*** of ***point*** A are obtained in terms of the ***angular velocity***, ω, and ***angular acceleration***, α, of the rigid ***body***



Relative-Motion Analysis: Acceleration

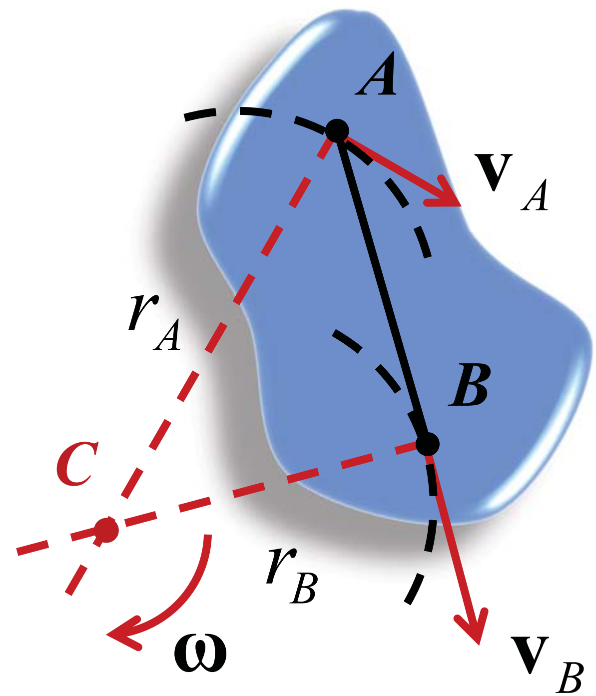
Translational portion

Rotational portion



Locating the Instantaneous Center: Case #1

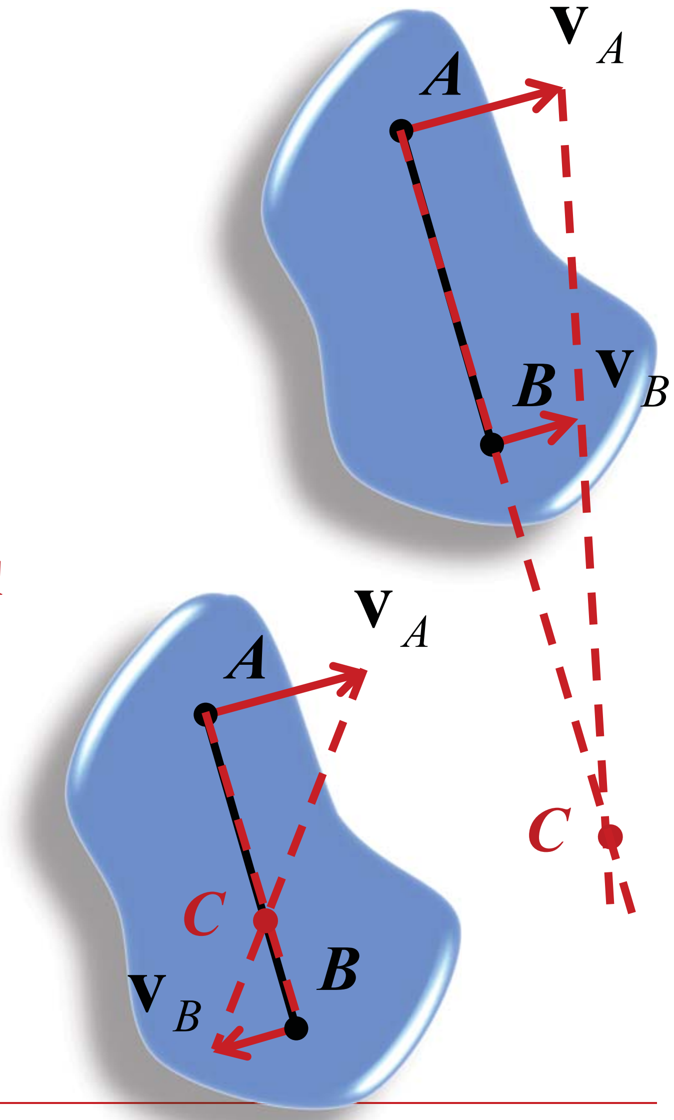
- Directions of absolute ***velocities*** for A and B are known (*and not parallel*)
- ***Point A*** has circular motion about some point on the ***line perpendicular*** to ***velocity*** $\mathbf{v}_A$
- ***Point B*** has a ***similar*** motion
- ***Point C*** is the ***instantaneous center*** of zero velocity (*may lie on or off the body*)



$$\omega = \frac{v_A}{r_A} = \frac{v_B}{r_B}$$

Locating the Instantaneous Center: Case #2

- Directions of absolute ***velocities*** for A and B are known AND ***parallel***
- The ***line*** joining the points is ***perpendicular*** to ***velocity*** $\mathbf{v}_A$ and $\mathbf{v}_B$
- ***Instantaneous center*** found by ***direct proportions***



Velocity and Acceleration of a Body

Lecture	Velocity	Acceleration
9. Rotation	$\omega = \dot{\theta}$ $v = r\omega$ $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$	$\alpha = \dot{\omega} = \ddot{\theta}$ $a_t = r\alpha \qquad a_n = r\omega^2$ $\mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r} \qquad \mathbf{a}_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$
10. Absolute Motion	$\omega = \dot{\theta}$ $v = r\omega$ $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$	$\alpha = \dot{\omega} = \ddot{\theta}$ $a_t = r\alpha \qquad a_n = r\omega^2$ $\mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r} \qquad \mathbf{a}_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$
11. Relative Velocity	$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$ $\mathbf{v}_{A/B} = \boldsymbol{\omega} \times \mathbf{r}$	
13. Relative Acceleration		$\mathbf{a}_A = \mathbf{a}_B + (\mathbf{a}_{A/B})_t + (\mathbf{a}_{A/B})_n$ $(\mathbf{a}_{A/B})_t = \boldsymbol{\alpha} \times \mathbf{r} \quad (\mathbf{a}_{A/B})_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$
14. Rotating Axes	$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}$	$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ $+ 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$

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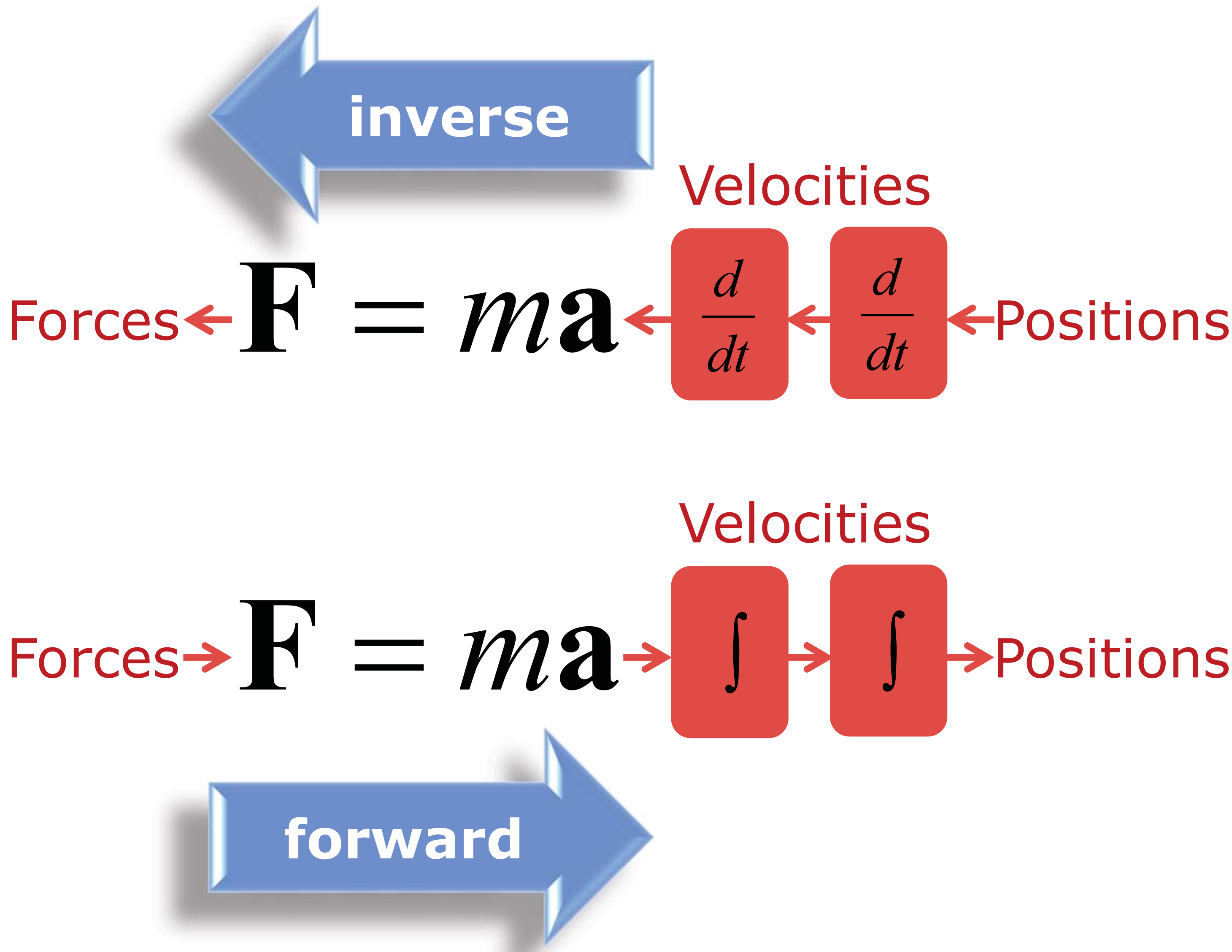
Kinematics (Exam 1)

Kinetics (Exam 2)

Work-Energy (new stuff)

Inverse vs. Forward Dynamics



Kinetics: Cause of Motion?

Concept: What is kinetics?

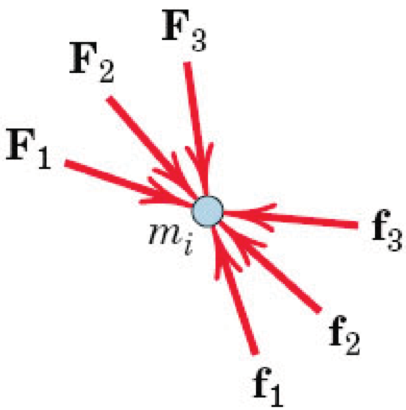
ME 202 | **Chapters 1, 2, 6** | **Chapters 3, 4, 5, 7, 8**

Relationship among ***forces (and moments)*** and ***equilibrium***

Relationship among ***position***, ***velocity***, and ***acceleration***

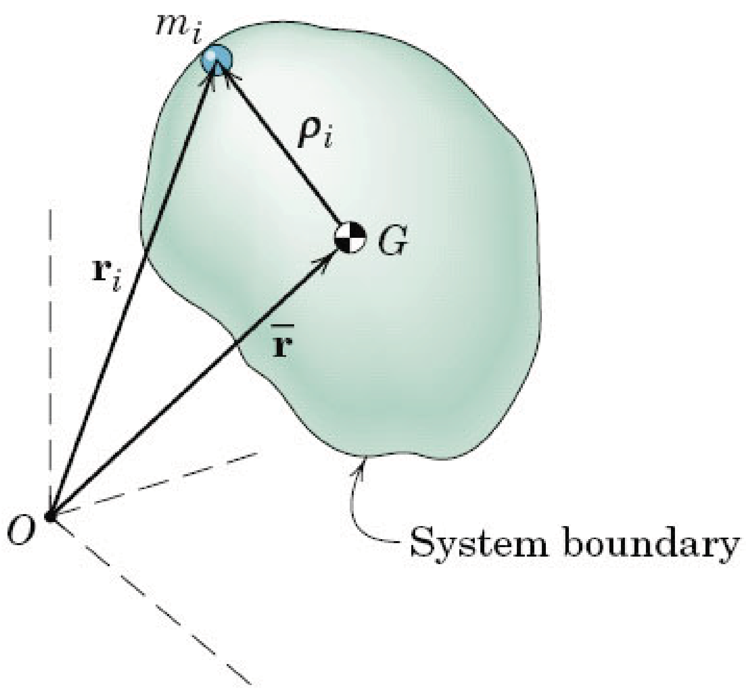
Relationship among ***forces (and moments)*** and ***acceleration***

Direct Application of Newton's 2nd Law



$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots + \mathbf{f}_1 + \mathbf{f}_2 + \mathbf{f}_3 + \cdots = m_i \ddot{\mathbf{r}}_i$$

$$\sum \mathbf{F} + \sum \mathbf{f} = \sum m_i \ddot{\mathbf{r}}_i$$



$$\boxed{\sum \mathbf{F} = m\ddot{\bar{\mathbf{r}}}} \quad \text{or} \quad \boxed{\sum \mathbf{F} = m\bar{\mathbf{a}}}$$

$$\sum F_x = m\bar{a}_x$$

$$\sum F_y = m\bar{a}_y$$

$$\sum F_z = m\bar{a}_z$$

Impulse-Momentum Principle: Linear

$$\mathbf{G} = m\mathbf{v}$$

$$\Sigma\mathbf{F} = \dot{\mathbf{G}}$$

$$\int_{t_1}^{t_2} \Sigma\mathbf{F}\, dt = \int_{t_1}^{t_2} \dot{\mathbf{G}}\, dt$$

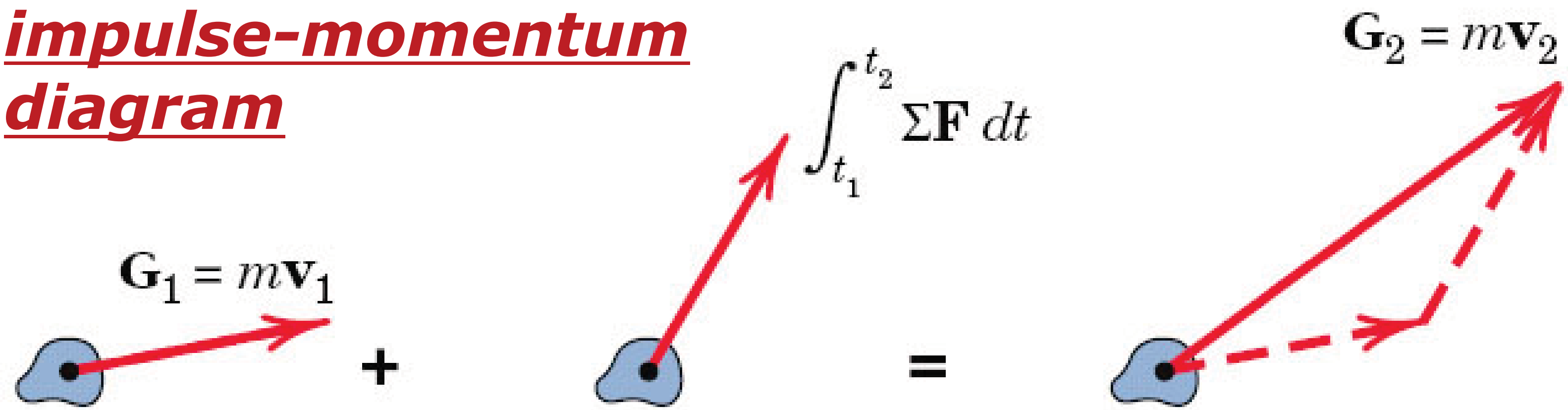
$$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma\mathbf{F}\, dt = \mathbf{G}_2$$

$$m(v_1)_x + \int_{t_1}^{t_2} \Sigma F_x\, dt = m(v_2)_x$$

$$m(v_1)_y + \int_{t_1}^{t_2} \Sigma F_y\, dt = m(v_2)_y$$

$$m(v_1)_z + \int_{t_1}^{t_2} \Sigma F_z\, dt = m(v_2)_z$$

impulse-momentum diagram



- ***Integrate*** to describe the effect of the ***resultant force*** $\Sigma\mathbf{F}$ on ***linear momentum*** over a finite period of ***time***

Impulse-Momentum Principle: Angular

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$$

$$\sum \mathbf{M}_O = \dot{\mathbf{H}}_O$$

$$\int_{t_1}^{t_2} \Sigma \mathbf{M}_O \, dt = \int_{t_1}^{t_2} \dot{\mathbf{H}}_O \, dt$$

$$\left(\mathbf{H}_O\right)_1 + \int_{t_1}^{t_2} \Sigma \mathbf{M}_O \, dt = \left(\mathbf{H}_O\right)_2$$

- ***Integrate*** to describe the effect of the ***angular impulse*** $\Sigma M_O * t$ on ***angular momentum*** H_O ***of*** m ***about*** O over a finite period of ***time***

$$m(v_z y - v_y z)_1 + \int_{t_1}^{t_2} \Sigma(\mathbf{M}_O)_x \, dt = m(v_z y - v_y z)_2$$

$$m(v_x z - v_z x)_1 + \int_{t_1}^{t_2} \Sigma(\mathbf{M}_O)_y \, dt = m(v_x z - v_z x)_2$$

$$m(v_y x - v_x y)_1 + \int_{t_1}^{t_2} \Sigma(\mathbf{M}_O)_z \, dt = m(v_y x - v_x y)_2$$

Equations, Equations, Equations...

Lecture	Equations	
18. Newton 2nd Law 19. Eqs. of Motion 20. Rectilinear	$\sum \mathbf{F} = m\mathbf{a}$ $\sum F_x = ma_x = m\ddot{x}$	$\sum F_y = ma_y = m\ddot{y}$ $\sum F_z = ma_z = m\ddot{z}$
21. Curvilinear	$\sum F_r = ma_r$ $\sum F_\theta = ma_\theta$	$\sum F_n = ma_n$ $\sum F_t = ma_t$
27. Lin. Imp. Mom.	$\mathbf{G} = m\mathbf{v}$ $\Sigma\mathbf{F} = \dot{\mathbf{G}}$	$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma\mathbf{F}\, dt = \mathbf{G}_2$ $\Delta\mathbf{G} = \mathbf{0}$
28. Ang. Imp. Mom.	$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$ $\sum \mathbf{M}_O = \dot{\mathbf{H}}_O$	$\left(\mathbf{H}_O\right)_1 + \int_{t_1}^{t_2} \Sigma\mathbf{M}_O\, dt = \left(\mathbf{H}_O\right)_2$ $\Delta\mathbf{H}_O = \mathbf{0}$
29. Sys. Imp. Mom.	$\mathbf{G} = m\overline{\mathbf{v}}$ $\quad \mathbf{H}_G = \sum\left(\boldsymbol{\rho}_i \times m_i\dot{\boldsymbol{\rho}}_i\right)$ $\mathbf{H}_O = \sum\left(\mathbf{r}_i \times m_i\mathbf{v}_i\right)$ $\quad \sum\mathbf{M}_G = \dot{\mathbf{H}}_G$	$\mathbf{H}_P = \mathbf{H}_G + \overline{\boldsymbol{\rho}} \times m\overline{\mathbf{v}}$ $\sum\mathbf{M}_P = \dot{\mathbf{H}}_G + \overline{\boldsymbol{\rho}} \times m\overline{\mathbf{a}}$

Equations, Equations, Equations…

Lecture	Equations	
18. Newton 2nd Law 22. Gen. Eqs. Mot. I 23. Gen. Eqs. Mot. II	$\sum \mathbf{M}_G = \dot{\mathbf{H}}_G$ $\dot{\mathbf{H}}_G = \sum \boldsymbol{\rho}_i \times \mathbf{F}_i$	$\sum M_P = I_G \alpha + mad$ $\sum \mathbf{M}_P = I_P \boldsymbol{\alpha} + \boldsymbol{\rho} \times m\mathbf{a}_P$
24. Fixed-Axis Rot.	$\sum \mathbf{F} = m\mathbf{a} \quad \sum \mathbf{M}_G = I_G \boldsymbol{\alpha}$	$\sum \mathbf{M}_O = I_O \boldsymbol{\alpha}$ $I_O = I_G + mr^2 \quad I_O = mk_O^2$
25. Gen. Plane Mot. I	$\sum \mathbf{M}_P = I_P \boldsymbol{\alpha} + \boldsymbol{\rho} \times m\mathbf{a}_P$	$\sum M_P = I_G \alpha + mad$
26. Gen. Plane Mot. II	$\sum \mathbf{F} = m\mathbf{a} \quad \sum \mathbf{M}_G = I_G \boldsymbol{\alpha}$	
31. Body Imp. Mom.	$\mathbf{G} = m\bar{\mathbf{v}} \quad \mathbf{H}_G = I_G \boldsymbol{\omega} \quad H_P = I_G \omega + mvd \quad H_O = I_O \omega$ $\sum \mathbf{F} = \dot{\mathbf{G}} \quad \sum \mathbf{M}_G = \dot{\mathbf{H}}_G \quad \sum \mathbf{M}_P = \dot{\mathbf{H}}_P \quad \sum \mathbf{M}_O = \dot{\mathbf{H}}_O$ $\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \mathbf{G}_2 \quad \left(\mathbf{H}_G\right)_1 + \int_{t_1}^{t_2} \Sigma \mathbf{M}_G\, dt = \left(\mathbf{H}_G\right)_2$	

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Kinematics (Exam 1)

Kinetics (Exam 2)

Work-Energy (new stuff)

Principle of Work and Kinetic Energy

- The ***kinetic energy*** $\boldsymbol{T}$ of a particle is $\boxed{T = \frac{1}{2}mv^2}$

- ***Work*** done to bring it a particle from ***velocity*** $\boldsymbol{v_1}$ to a ***velocity*** $\boldsymbol{v_2}$

$$U_{1-2} = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$$

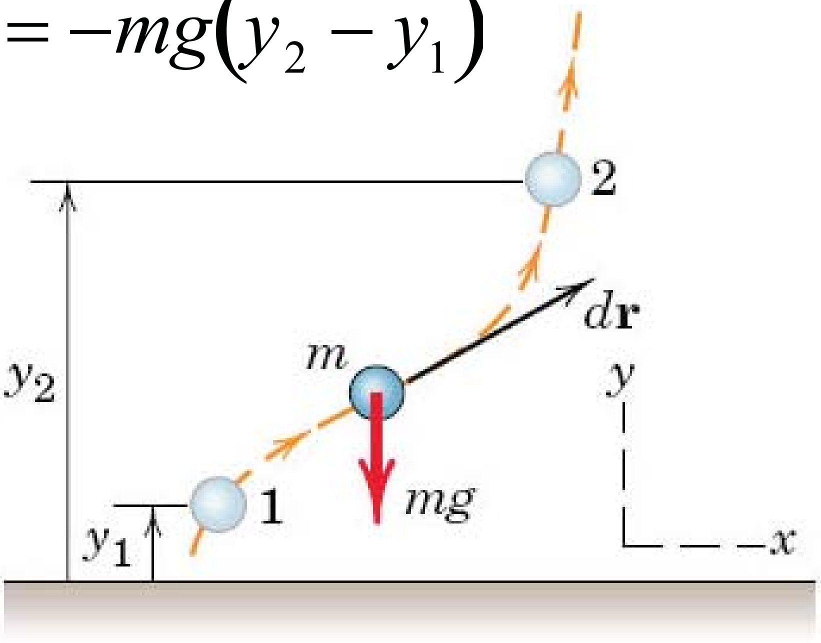
$$U_{1-2} = T_2 - T_1 = \Delta T \quad \textbf{\textit{(work-energy eq.)}}$$

$$\boxed{T_1 + U_{1-2} = T_2}$$

Work vs. Potential Energy

$$U = -mg\int_{y_1}^{y_2} dy = -mg(y_2 - y_1)$$

- Recall: ***Work*** is done by the ***weight*** mg over an ***altitude change*** $(y_2\text{-}y_1)$



- ***Potential energy*** is simply the ***opposite sign (-work)*** because of its ***potential*** to be converted into ***energy***

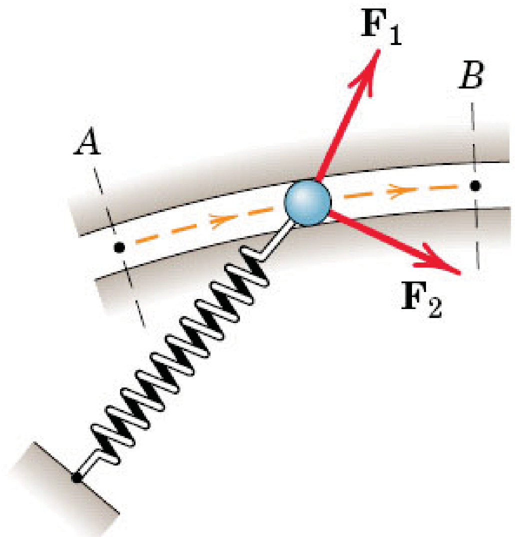
Work-Energy Equation

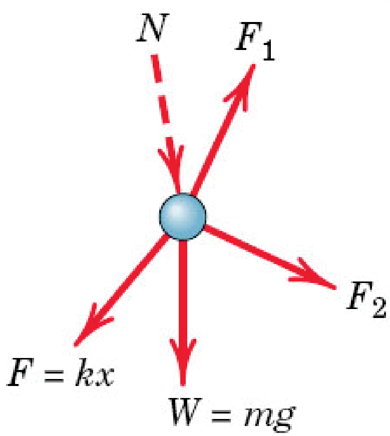
- The ***work*** of all external forces *other than* gravitational and spring forces is U'_{1-2}

$$U'_{1-2} = \Delta T + \Delta V$$ ***(work-energy eq.)***

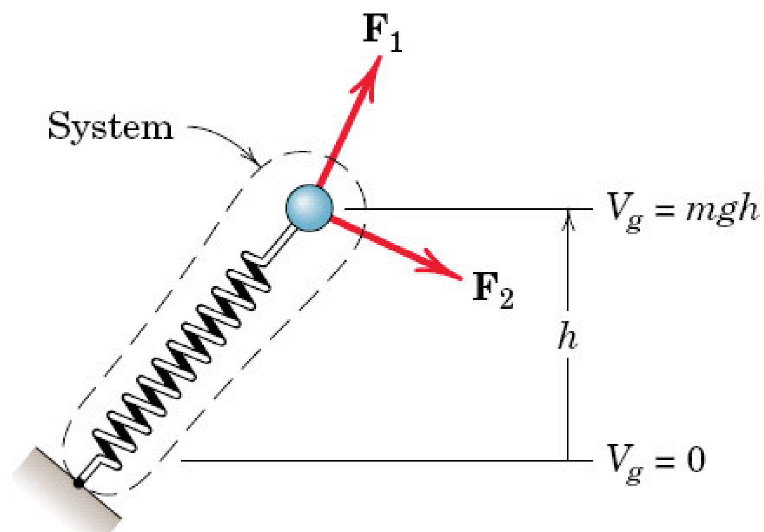
$$T_1 + V_1 + U'_{1-2} = T_2 + V_2$$

Work-Energy Equation: Usage



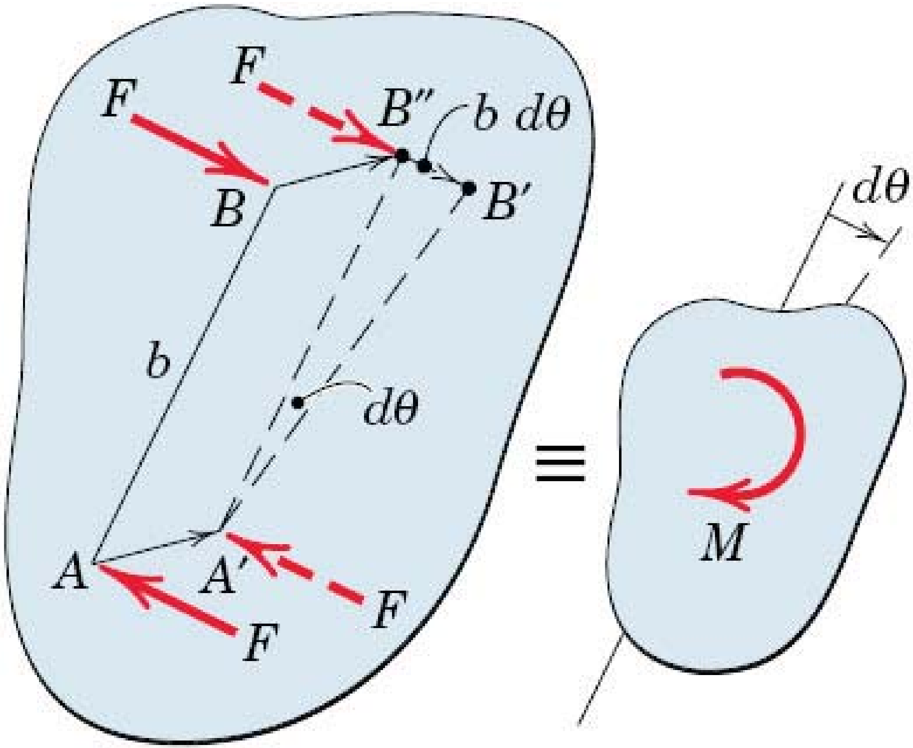


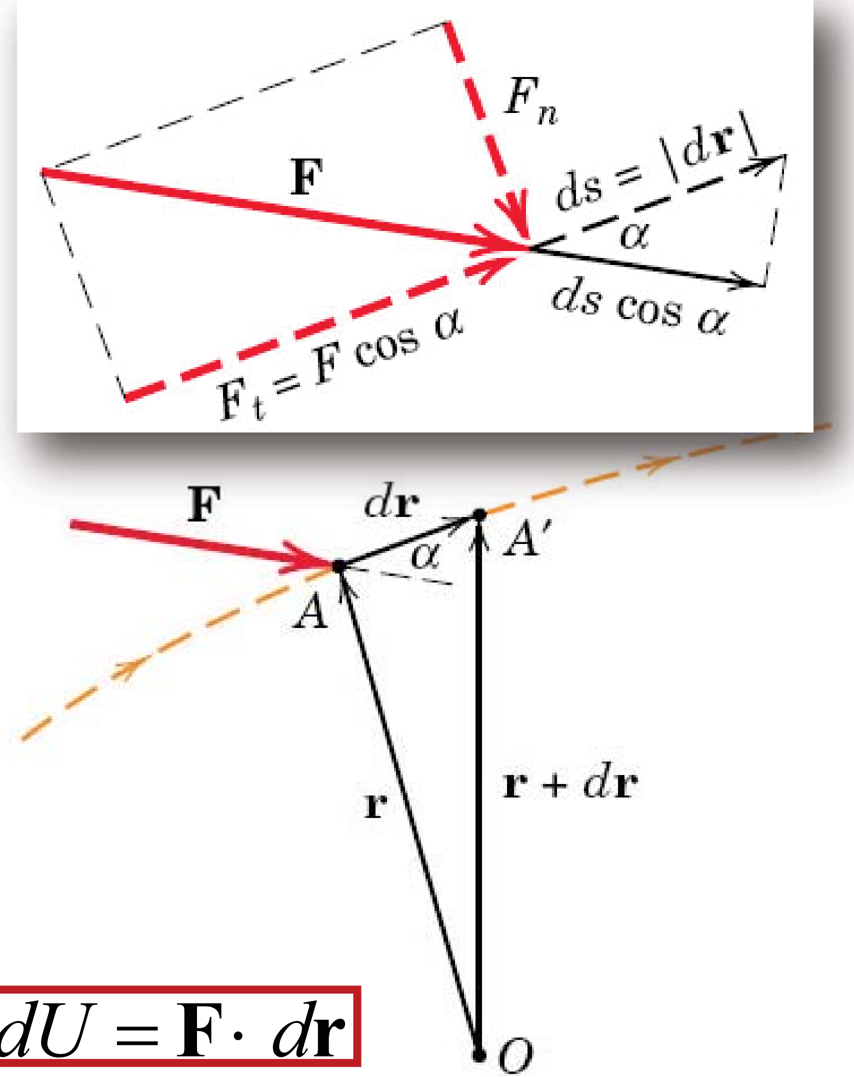
$$U_{1-2} = \Delta T$$



$$U'_{1-2} = \Delta T + \Delta V$$

Work of Forces and Couples





$$dU = \mathbf{F} \cdot d\mathbf{r}$$

$$dU = F\left(b\,d\theta\right)$$

$$dU = F\,ds\,\cos\alpha$$

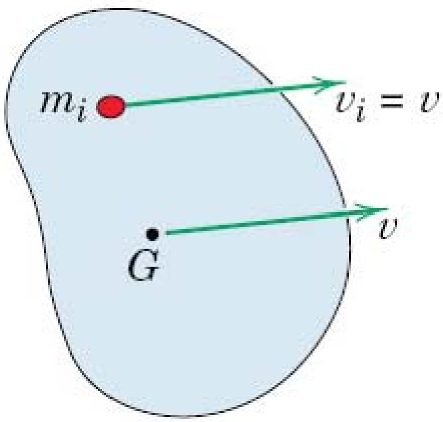
$$dU = M\,d\theta$$

$$dU = F_t\,ds$$

Kinetic Energy for Rigid Bodies

translation	***fixed-axis rotation***	***general plane motion***

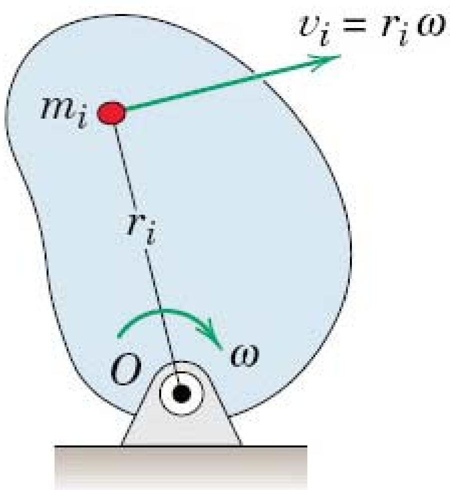
translation



$$T = \Sigma \frac{1}{2} m_i v^2$$

$$\boxed{T = \frac{1}{2} m v^2}$$

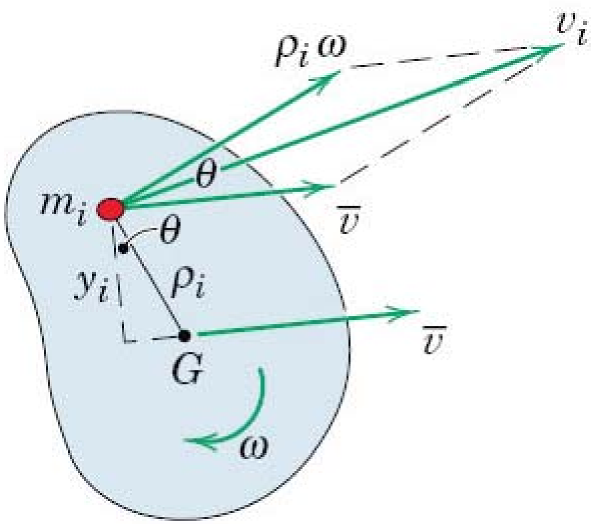
fixed-axis rotation



$$T = \Sigma \frac{1}{2} m_i (r_i \omega)^2$$

$$\boxed{T = \frac{1}{2} I_O \omega^2}$$

general plane motion



$$T = \Sigma \frac{1}{2} m_i \left(v_G^2 + \rho_i^2 \omega^2\right)$$

$$\boxed{T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2}$$

Work-Energy Equation for Rigid Bodies

Express weight and springs as doing work

- ***Work*** done to bring a rigid body from ***kinetic energy*** T_1 to a ***kinetic energy*** T_2

$$T_1 + U_{1-2} = T_2$$

Express weight and springs by means of potential energy

- The ***work*** of all external forces *other than* gravitational and spring forces is

$$T_1 + V_1 + U'_{1-2} = T_2 + V_2$$

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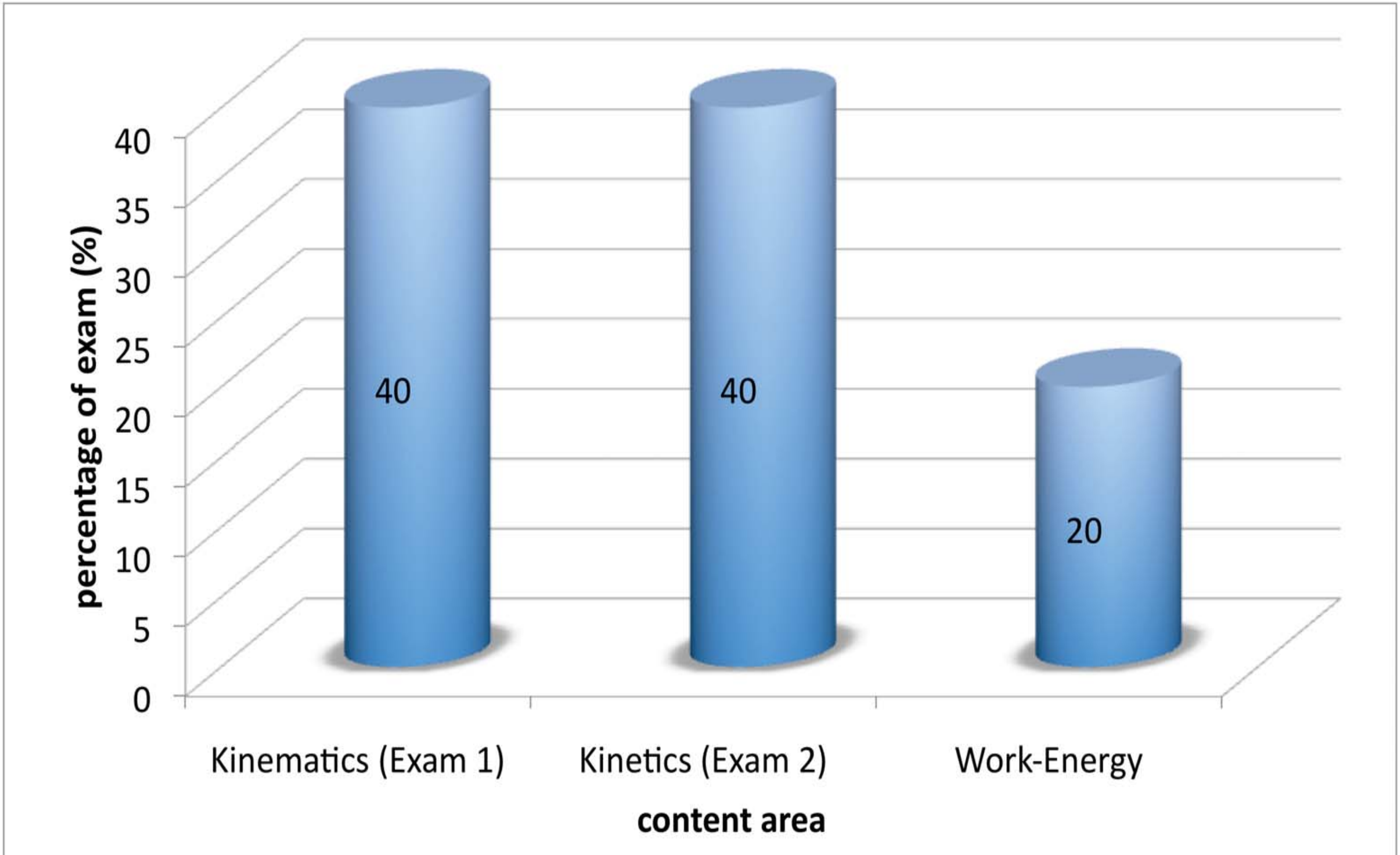
Final Exam General Breakdown



For Next Time…

- Review Chapters 1, 2, 3, 4, 5, 6, 7, & 8
- Review Lectures slides
  - http://rrg.utk.edu/resources/ME231/lectures.html
- Review Examples from class
  - http://rrg.utk.edu/resources/ME231/examples.html

- *Final Exam on Monday (12/10) @ 8:00am here (in Min Kao 524)*