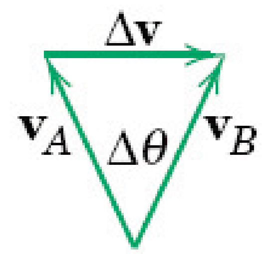
Normal & Tangential (*n*-*t*) Coordinates

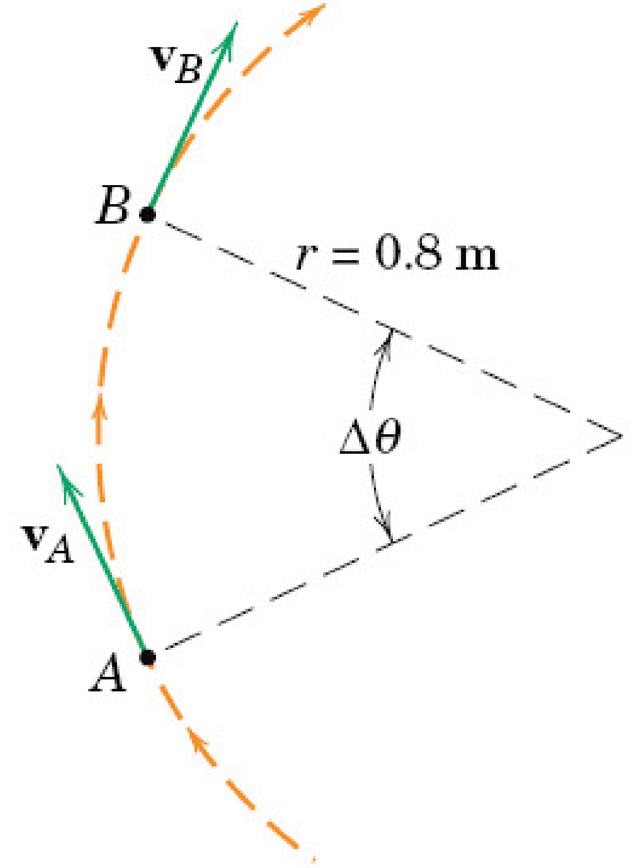
Lecture 4

ME 231: Dynamics

<u>Question of the Day</u>

A particle moves in a circular path of radius r = 0.8 m with constant speed (v) of 2 m/s. The ***velocity*** undergoes a vector change $\Delta\mathbf{v}$ from A to B.





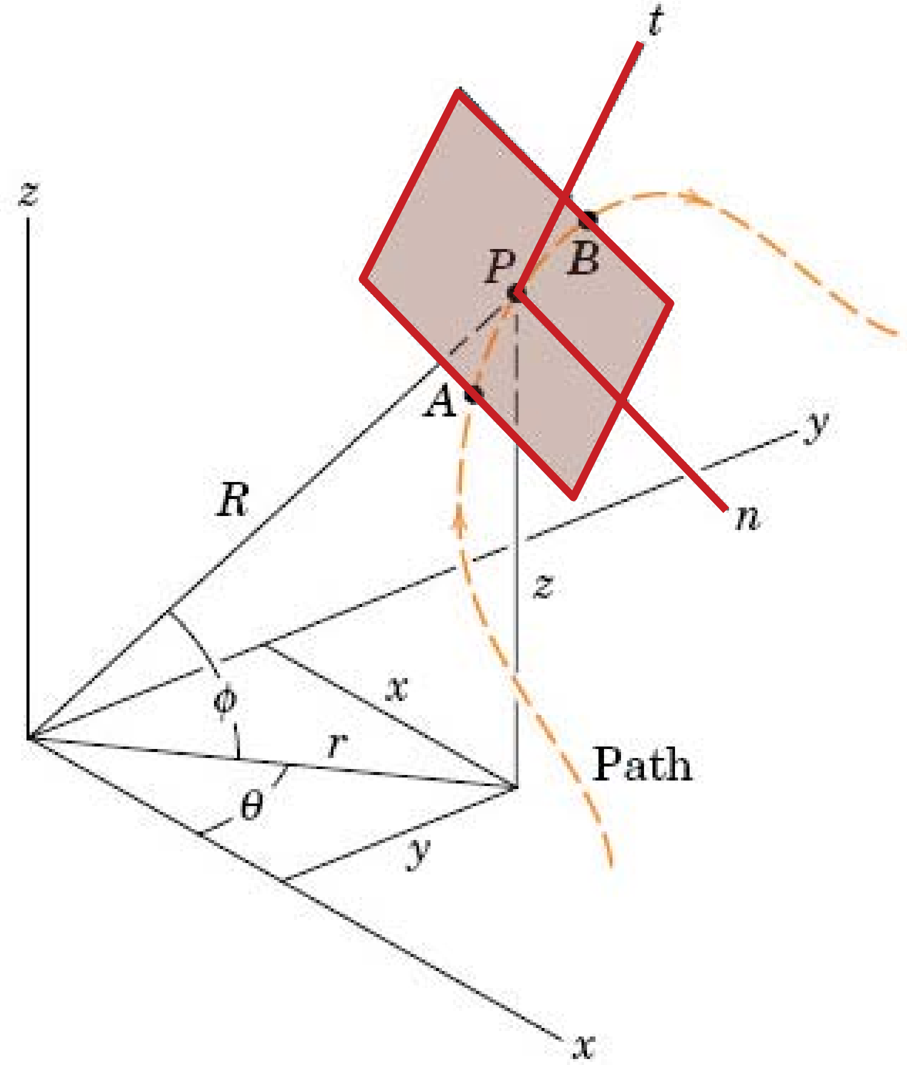
Express the magnitude of $\Delta\mathbf{v}$ in terms of v and $\Delta\theta$. Express the time interval Δt in terms of v, $\Delta\theta$, and r. Obtain the magnitude of average acceleration by computing $\Delta\mathbf{v}/\Delta t$.

Outline for Today

- Question of the day
- N-T vector representation
- Velocity and acceleration
- Geometric interpretation
- Circular motion
- Answer your questions!

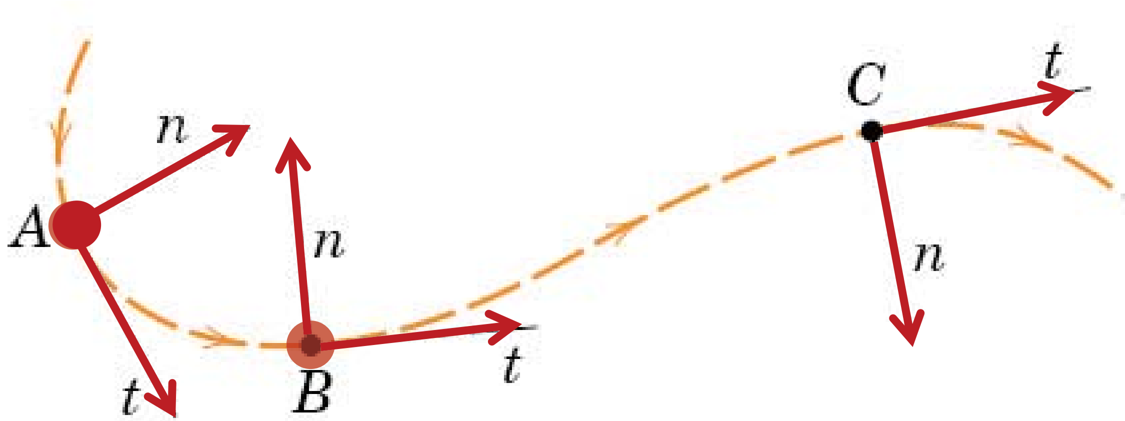
Recall: Possible Coordinate Systems

- Rectangular (x, y, z)
- Polar (r, θ, z)
- Spherical (R, θ, ϕ)
- Normal and Tangential (n, t)



N-T Vector Representation

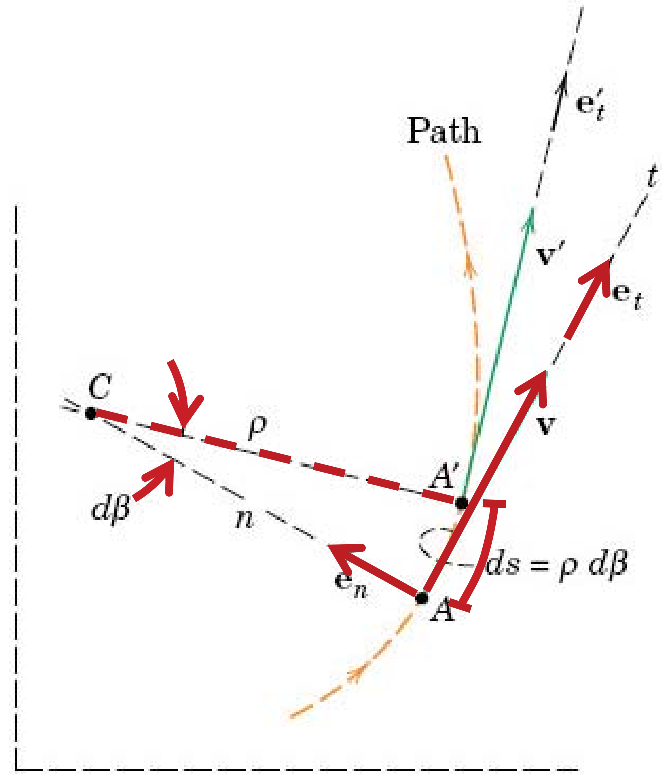
Path variables along the tangent (*t*) and normal (*n*)



- The ***n***- and ***t***-coordinates move along the path with the particle
- ***Tangential*** coordinate is parallel to the ***velocity***
- The positive direction for the ***normal*** coordinate is toward the center of curvature

Velocity

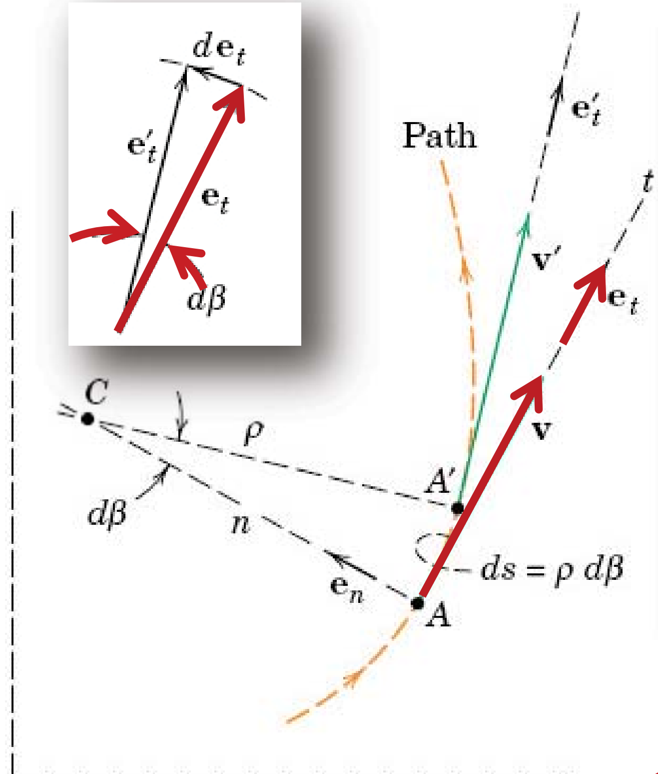
$$ds = \rho \, d\beta$$

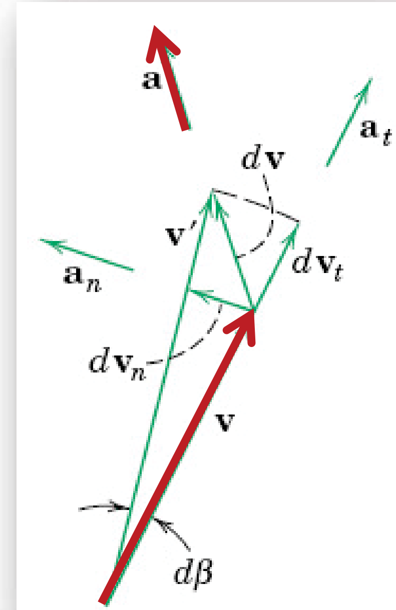


$$\mathbf{v} = \frac{ds}{dt}\mathbf{e_t} = v\,\mathbf{e_t} = \rho\dot{\beta}\,\mathbf{e_t}$$

- ***ds*** is the scalar displacement along the path (A→A′)
- Radius of curvature of the path is ρ and $d\beta$ is the angle change
- $\mathbf{e_n}$ is the unit vector in the ***normal*** direction
- $\mathbf{e_t}$ is the unit vector in the ***tangent*** direction

Acceleration





$$ds = \rho\, d\beta$$

$$\mathbf{v} = \frac{ds}{dt}\mathbf{e}_\mathbf{t} = v\,\mathbf{e}_\mathbf{t} = \rho\dot{\beta}\,\mathbf{e}_\mathbf{t}$$

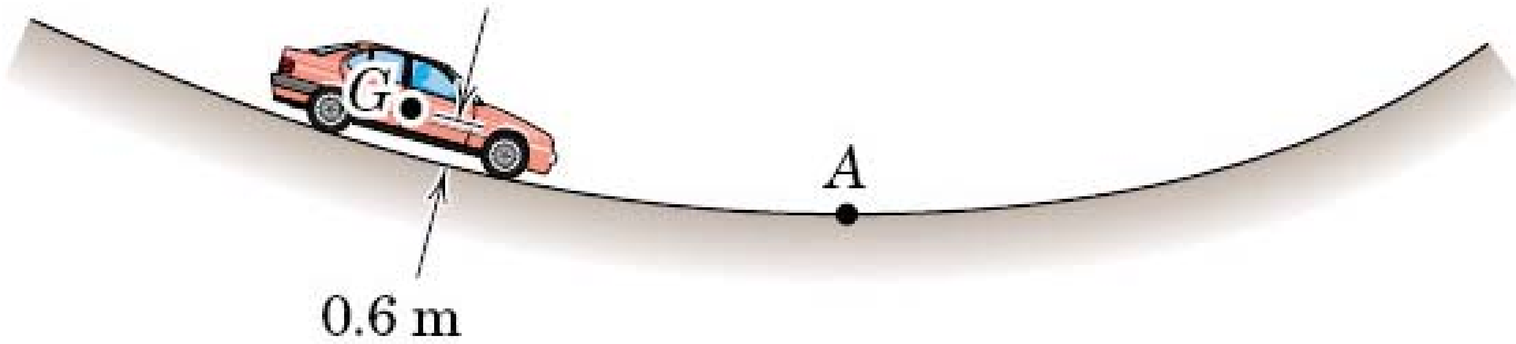
$$\mathbf{v} = v\,\mathbf{e}_\mathbf{t}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d\left(v\,\mathbf{e}_\mathbf{t}\right)}{dt} = v\,\dot{\mathbf{e}}_\mathbf{t} + \dot{v}\,\mathbf{e}_\mathbf{t}$$

$$\dot{\mathbf{e}}_\mathbf{t} = \frac{d\,\mathbf{e}_\mathbf{t}}{dt} = \left(\frac{d\beta}{dt}\right)\mathbf{e}_\mathbf{n} = \dot{\beta}\,\mathbf{e}_\mathbf{n} = \frac{v}{\rho}\mathbf{e}_\mathbf{n}$$

$$\mathbf{a} = \frac{v^2}{\rho}\mathbf{e}_\mathbf{n} + \dot{v}\,\mathbf{e}_\mathbf{t}$$

Velocity and Acceleration: Exercise



A car passes through a dip in the road at A with constant ***speed*** (v) giving it an ***acceleration*** (a) equal to 0.5g. The ***radius of curvature*** (ρ) at A is 100 m and the distance from the road to the mass center G of the car is 0.6 m.

Determine the ***speed*** (v) of the car.

$$\mathbf{a} = \frac{v^2}{\rho}\mathbf{e_n} + \dot{v}\,\mathbf{e_t}$$

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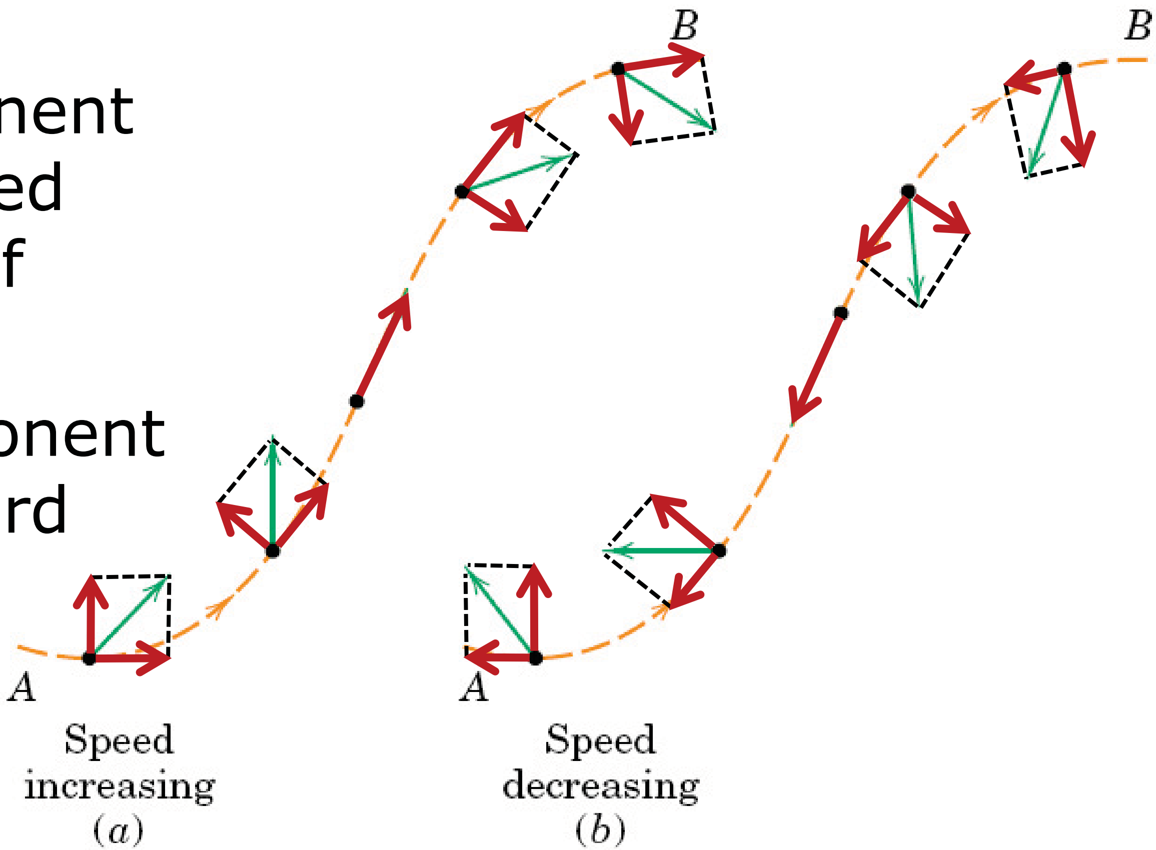
Geometric Interpretation

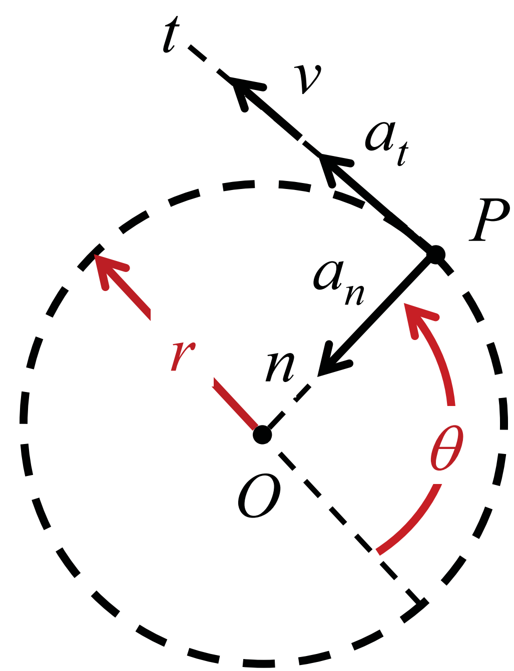
$$\mathbf{a} = \frac{v^2}{\rho}\mathbf{e_n} + \dot{v}\,\mathbf{e_t}$$

- ***Normal*** component is always directed toward center of curvature
- ***Tangent*** component is directed toward $+t$ (or $-t$) direction if speed is increasing (or decreasing)



Acceleration vectors for particle moving from A to B

Circular Motion



- Radius of curvature ρ becomes constant r
- Angle β is replaced by angle θ

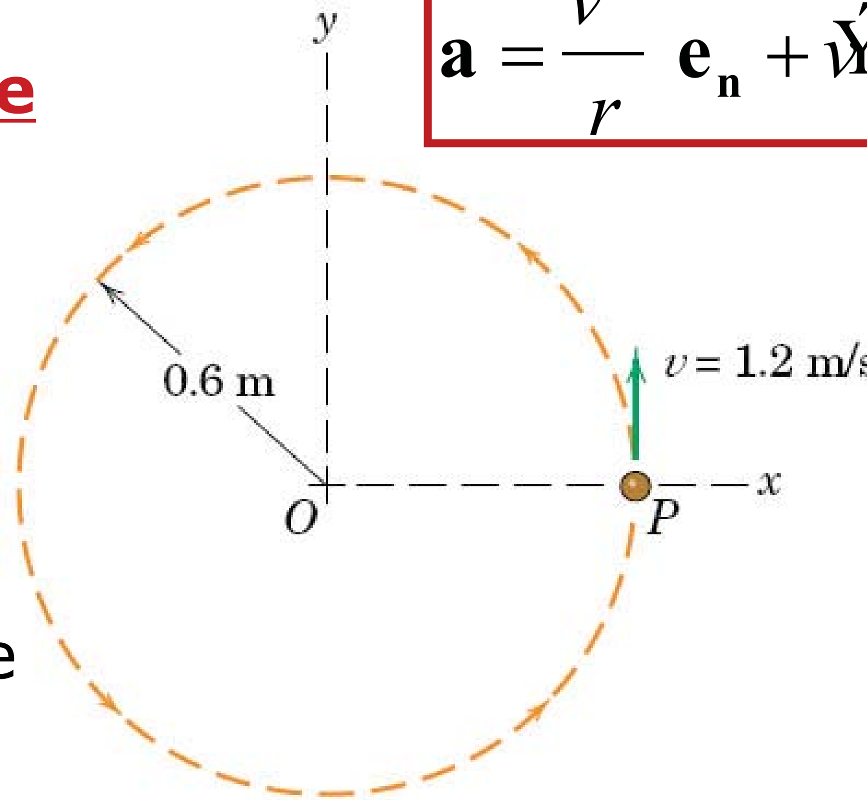
$$v = \rho\dot{\beta} \longrightarrow v = r\dot{\theta}$$

$$a_n = \frac{v^2}{\rho} \longrightarrow a_n = \frac{v^2}{r} = r\dot{\theta}^2 = v\dot{\theta}$$

$$a_t = \dot{v} = \rho\ddot{\beta} \longrightarrow a_t = \dot{v} = r\ddot{\theta}$$

$$\mathbf{a} = \frac{v^2}{r}\mathbf{e}_n + \dot{v}\mathbf{e}_t$$

Circular Motion: Exercise



Particle *P* moves in a circular path shown.

Determine the magnitude of ***acceleration*** for:

(a) constant ***velocity*** 1.2 m/s

(b) ***velocity*** 1.2 m/s and increasing 2.4 m/s each second

(c) ***velocity*** 1.2 m/s and decreasing 4.8 m/s each second

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For Next Time…

- Begin Homework #2 due next week (9/5)
- Read Chapter 2, Section 2.6