Normal & Tangential \((n-t)\) Coordinates

Lecture 4

ME 231: Dynamics
A particle moves in a circular path of radius \( r = 0.8 \text{ m} \) with constant speed \( v \) of 2 m/s. The *velocity* undergoes a vector change \( \Delta v \) from \( A \) to \( B \).

Express the magnitude of \( \Delta v \) in terms of \( v \) and \( \Delta \theta \). Express the time interval \( \Delta t \) in terms of \( v \), \( \Delta \theta \), and \( r \). Obtain the magnitude of average acceleration by computing \( \Delta v / \Delta t \).
Outline for Today

- Question of the day
- N-T vector representation
- Velocity and acceleration
- Geometric interpretation
- Circular motion
- Answer your questions!
Recall: Possible Coordinate Systems

- Rectangular \((x, y, z)\)
- Polar \((r, \theta, z)\)
- Spherical \((R, \theta, \phi)\)
- Normal and Tangential \((n, t)\)
N-T Vector Representation

Path variables along the tangent \((t)\) and normal \((n)\)

- The \(n\)- and \(t\)-coordinates move along the path with the particle
- **Tangential** coordinate is parallel to the **velocity**
- The positive direction for the **normal** coordinate is toward the center of curvature
**Velocity**

\[ ds = \rho \, d\beta \]

\[ \mathbf{v} = \frac{ds}{dt} \mathbf{e}_t = v \mathbf{e}_t = \rho \dot{\beta} \mathbf{e}_t \]

- \( ds \) is the scalar displacement along the path (A→A’)
- Radius of curvature of the path is \( \rho \) and \( d\beta \) is the angle change
- \( \mathbf{e}_n \) is the unit vector in the *normal* direction
- \( \mathbf{e}_t \) is the unit vector in the *tangent* direction
Acceleration

\[ ds = \rho \, d\beta \]

\[ \mathbf{v} = \frac{ds}{dt} \, \mathbf{e}_t = \nu \, \mathbf{e}_t = \rho \hat{\beta} \, \mathbf{e}_t \]

\[ \mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d(\nu \, \mathbf{e}_t)}{dt} = \nu \, \dot{\mathbf{e}}_t + \dot{\nu} \, \mathbf{e}_t \]

\[ \dot{\mathbf{e}}_t = \frac{d \mathbf{e}_t}{dt} = \left( \frac{d\beta}{dt} \right) \mathbf{e}_n = \hat{\beta} \, \mathbf{e}_n = \frac{\nu}{\rho} \, \mathbf{e}_n \]

\[ \mathbf{a} = \frac{\nu^2}{\rho} \, \mathbf{e}_n + \dot{\nu} \, \mathbf{e}_t \]
Velocity and Acceleration: Exercise

A car passes through a dip in the road at \( A \) with constant speed \( (v) \) giving it an acceleration \( (a) \) equal to \( 0.5g \). The radius of curvature \( (\rho) \) at \( A \) is 100 m and the distance from the road to the mass center \( G \) of the car is 0.6 m.

Determine the speed \( (v) \) of the car.

\[
a = \frac{v^2}{\rho} \mathbf{e}_n + \dot{v} \mathbf{e}_t
\]
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Geometric Interpretation

- **Normal** component is always directed toward center of curvature
- **Tangent** component is directed toward $+t$ (or $-t$) direction if speed is increasing (or decreasing)

\[ a = \frac{v^2}{\rho} \mathbf{e}_n + \dot{v} \mathbf{e}_t \]

Acceleration vectors for particle moving from $A$ to $B$
Circular Motion

- Radius of curvature $\rho$ becomes constant $r$
- Angle $\beta$ is replaced by angle $\theta$

\[
\begin{align*}
v &= \rho \dot{\beta} \\
a_n &= \frac{v^2}{\rho} \\
a_t &= \ddot{r} = \rho \dot{\beta}
\end{align*}
\]

\[
\begin{align*}
v &= r \dot{\theta} \\
a_n &= \frac{v^2}{r} = r \dot{\theta}^2 = v \dot{\theta} \\
a_t &= \dot{v} = r \ddot{\theta}
\end{align*}
\]
Circular Motion: Exercise

Particle $P$ moves in a circular path shown.

Determine the magnitude of acceleration for:

(a) constant velocity $1.2$ m/s
(b) velocity $1.2$ m/s and increasing $2.4$ m/s each second
(c) velocity $1.2$ m/s and decreasing $4.8$ m/s each second
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For Next Time...

- Begin Homework #2 due next week (9/5)
- Read Chapter 2, Section 2.6