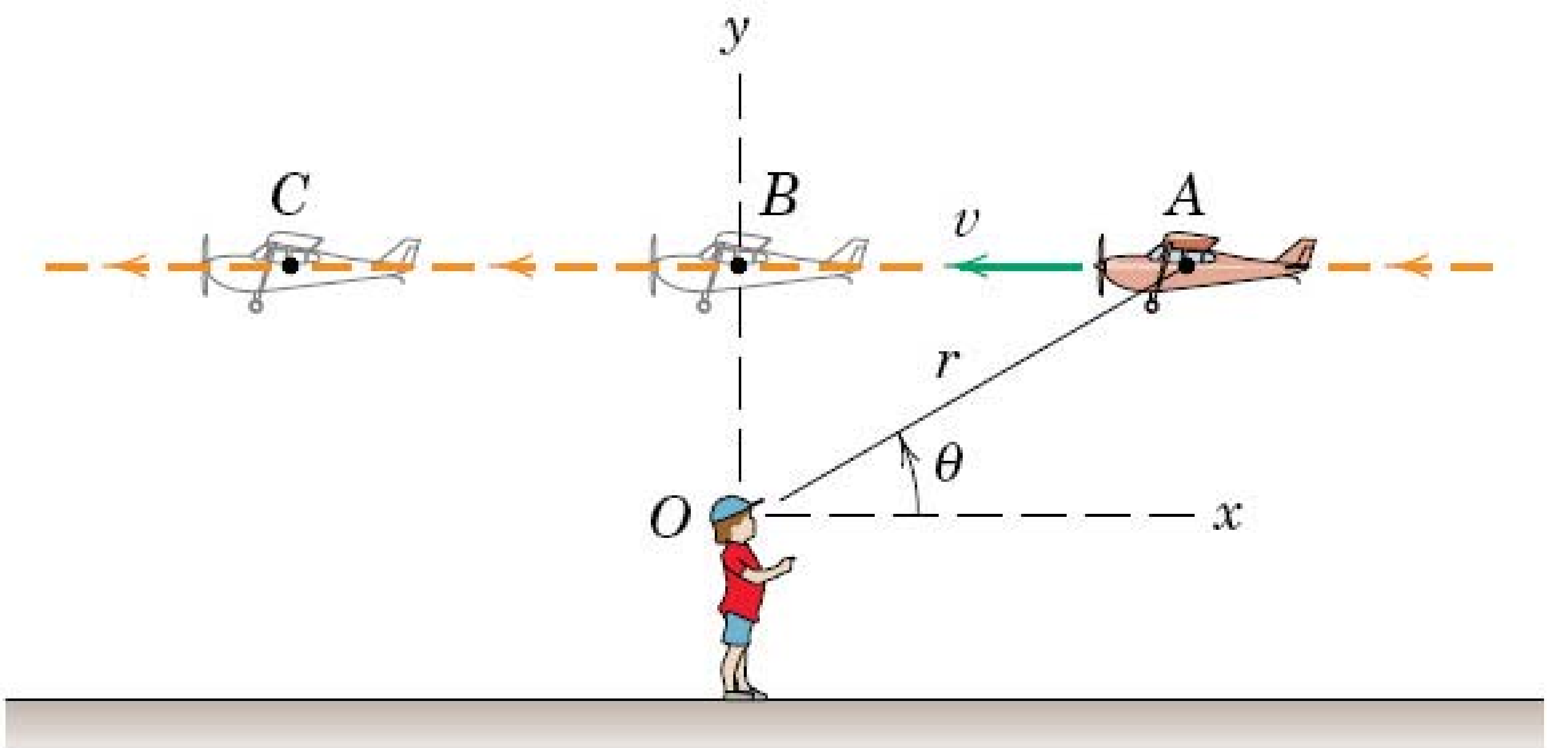
Polar (r-θ) Coordinates

Lecture 5

ME 231: Dynamics

Question of the Day

A model airplane flies over an observer ***O*** with constant speed. Determine the signs (+, -, or 0) for $r, \dot{r}, \ddot{r}, \theta, \dot{\theta}, \text{and } \ddot{\theta}$ at each position ***A***, ***B***, and ***C***.

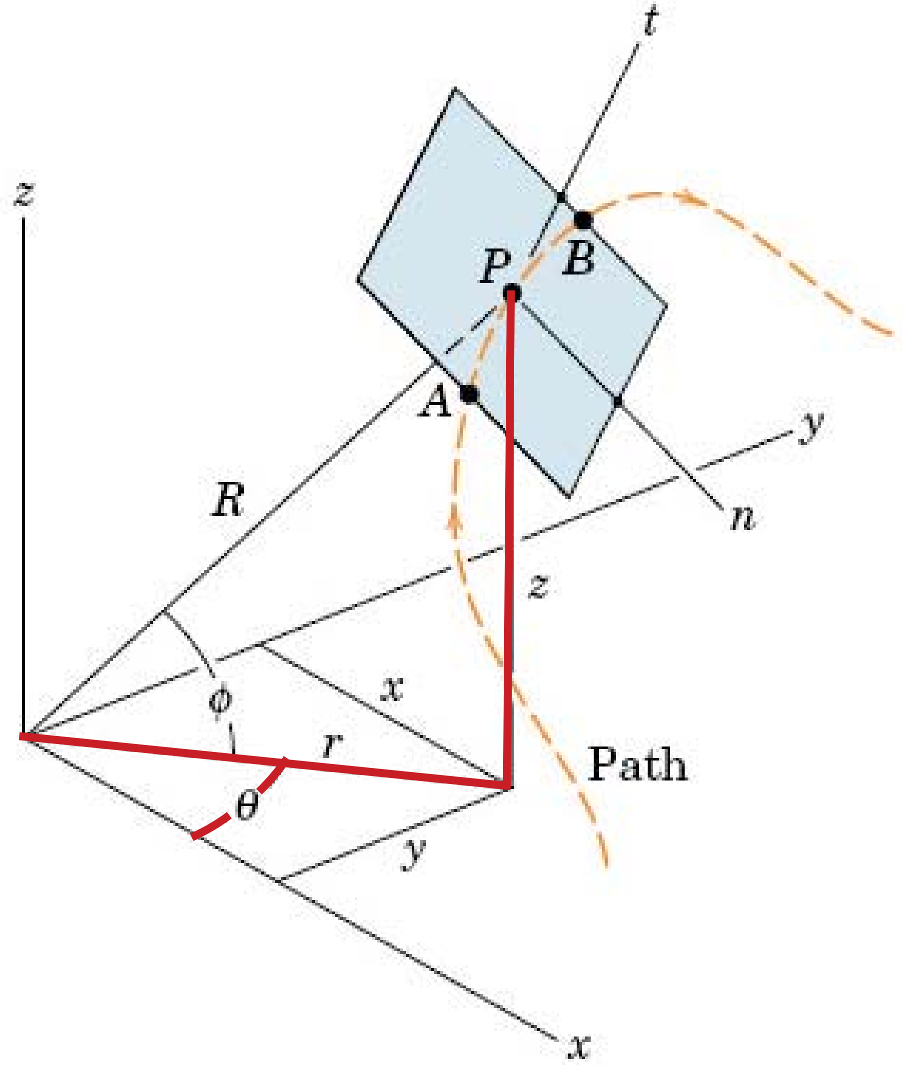


Outline for Today

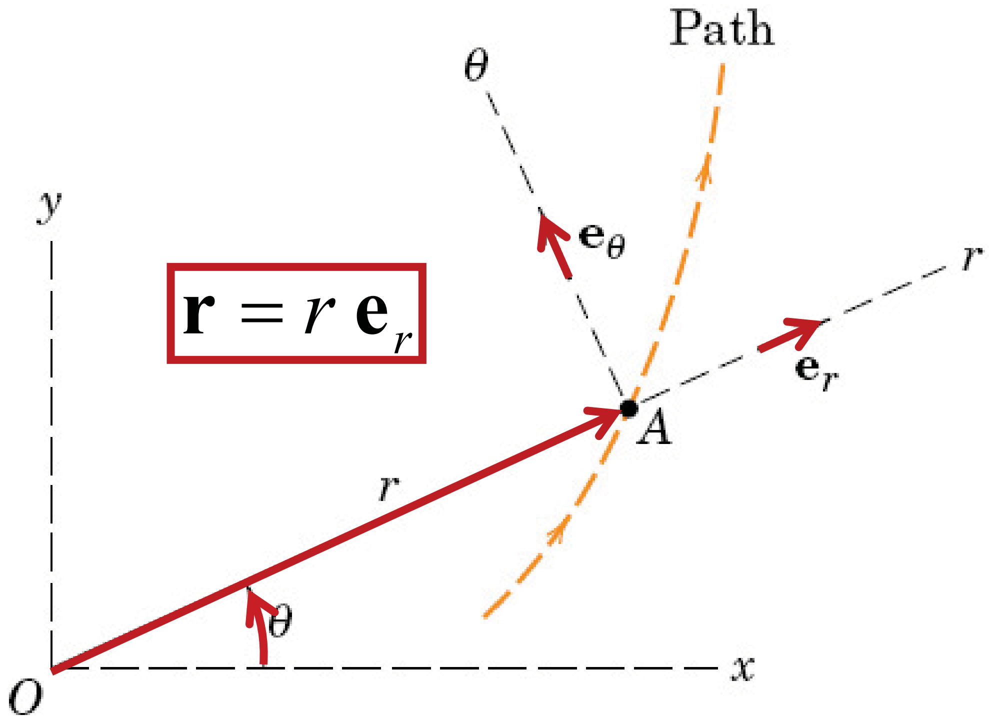
- Question of the day
- Vector representation
- Time derivative of unit vectors
- Velocity and acceleration
- Geometric interpretation
- Circular motion
- Answer your questions!

Recall: Possible Coordinate Systems

- Rectangular (x, y, z)
- **Polar (r, θ, z)**
- Spherical (R, θ, ϕ)
- Normal and Tangential (n, t)



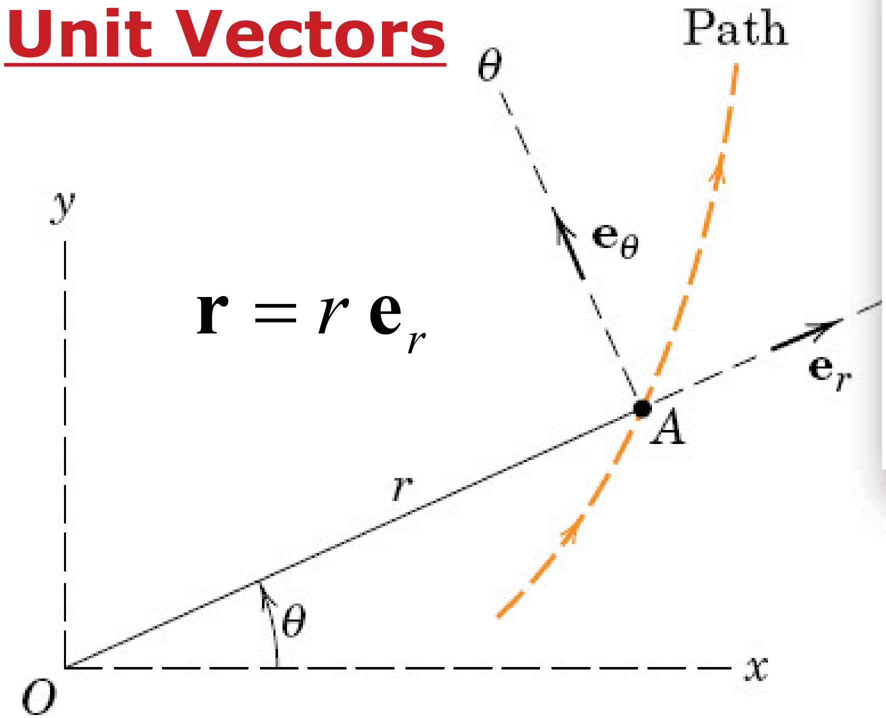
Vector Representation



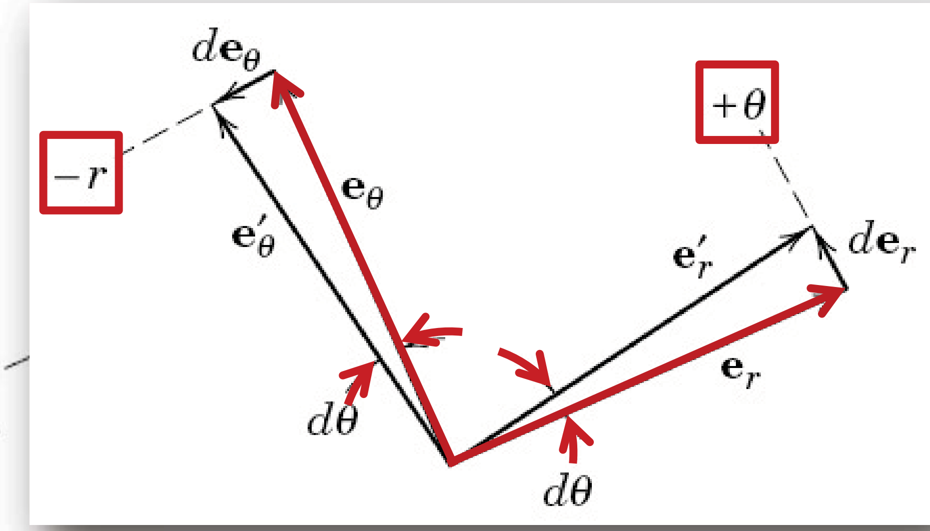
$$\mathbf{r} = r\,\mathbf{e}_r$$

- Useful when motion is measured by a ***radial distance*** (r) and an ***angular position*** (θ)
- $\mathbf{e}_r$ is the unit vector in the r-direction
- $\mathbf{e}_\theta$ is the unit vector in the θ-direction

Time Derivative of Unit Vectors



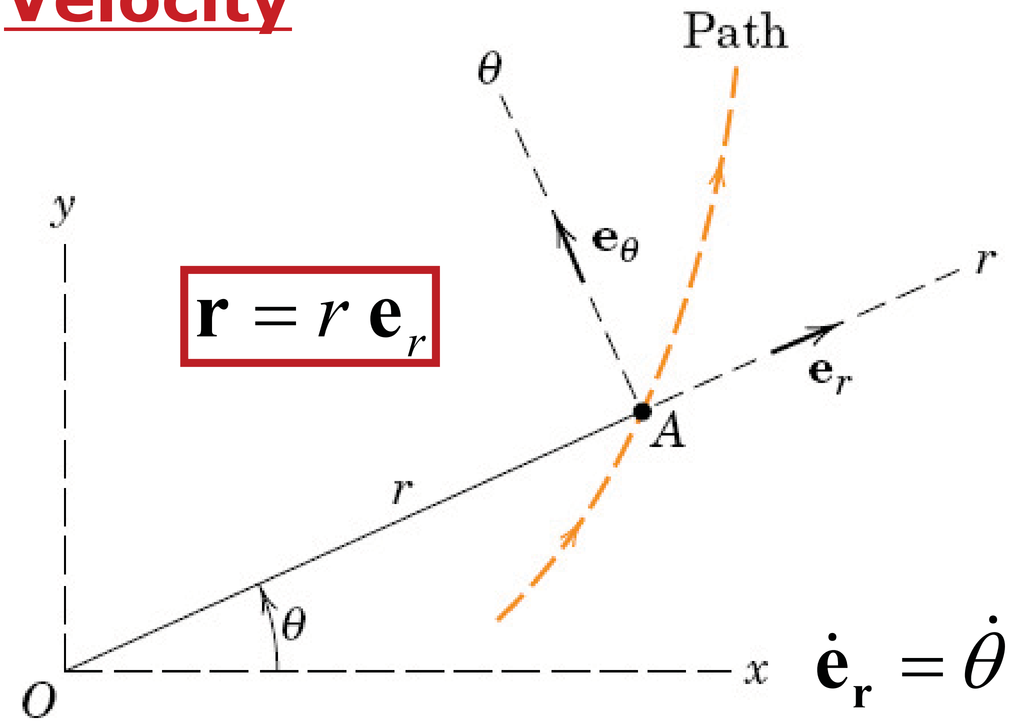
$$\mathbf{r} = r\,\mathbf{e}_r$$



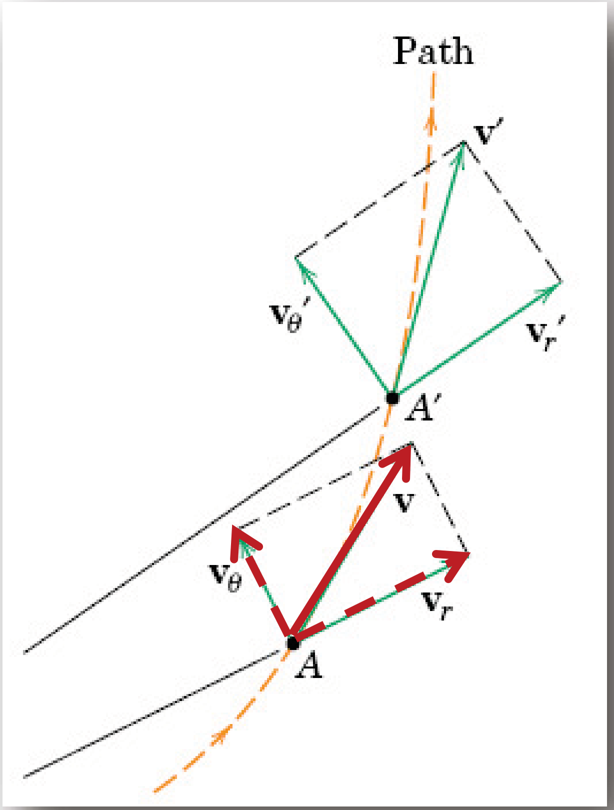
$$\dot{\mathbf{e}}_r = \frac{d\,\mathbf{e}_r}{dt} = \left(\frac{d\theta}{dt}\right)\mathbf{e}_\theta = \dot{\theta}\,\mathbf{e}_\theta$$

$$\dot{\mathbf{e}}_\theta = \frac{d\,\mathbf{e}_\theta}{dt} = -\left(\frac{d\theta}{dt}\right)\mathbf{e}_r = -\dot{\theta}\,\mathbf{e}_r$$

Velocity



$$\mathbf{r} = r\,\mathbf{e}_r$$



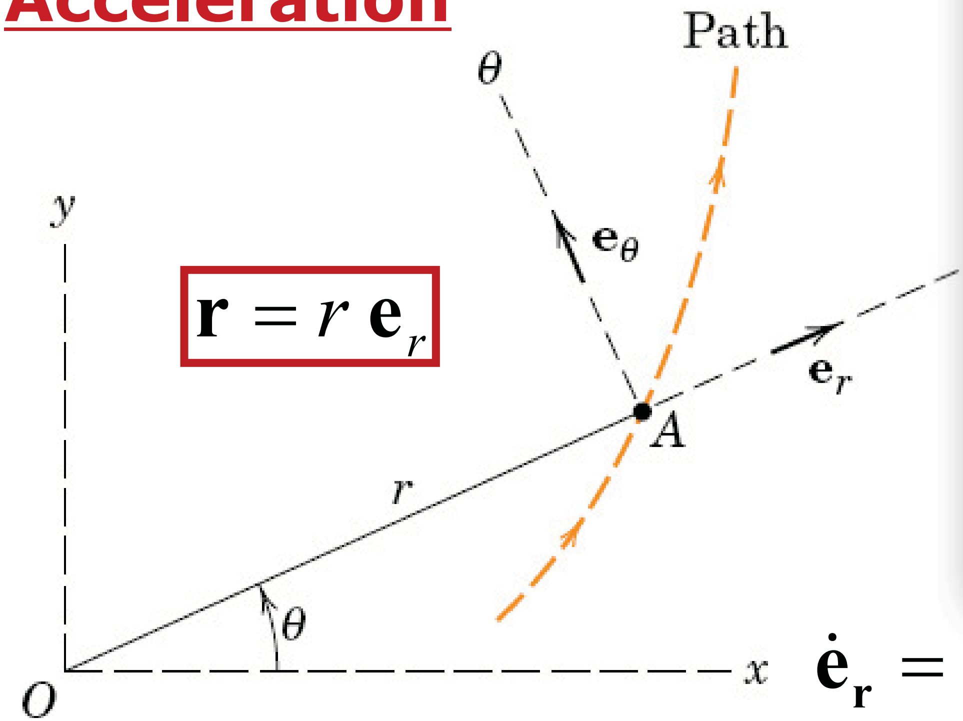
$$\dot{\mathbf{e}}_{\mathbf{r}} = \dot{\theta}\,\mathbf{e}_\theta$$

$$\mathbf{v} = \dot{\mathbf{r}} = \frac{d(r\,\mathbf{e}_{\mathbf{r}})}{dt} = \dot{r}\,\mathbf{e}_{\mathbf{r}} + r\,\dot{\mathbf{e}}_{\mathbf{r}}$$

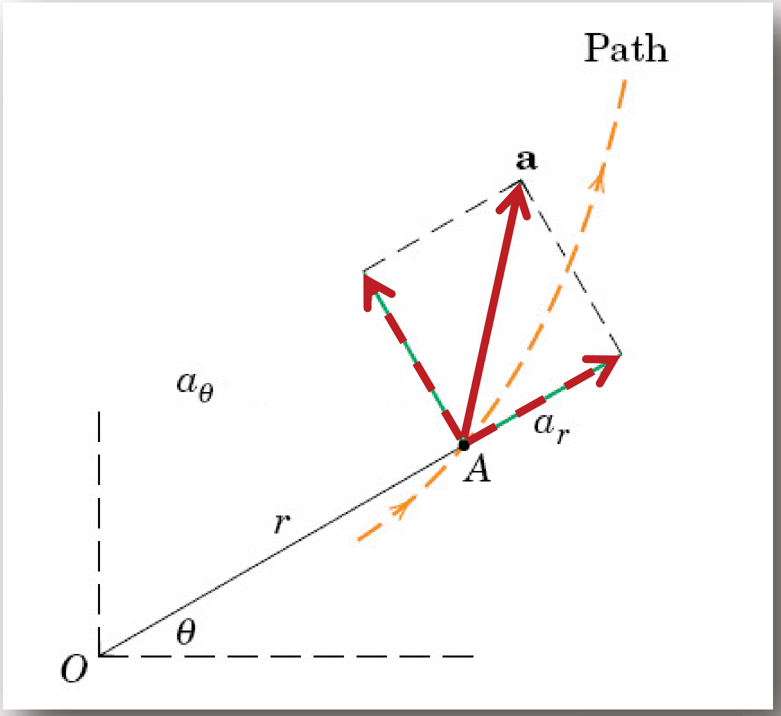
$$\mathbf{v} = \dot{r}\,\mathbf{e}_{\mathbf{r}} + r\dot{\theta}\,\mathbf{e}_\theta$$

- The r-component of $\mathbf{v}$ is the rate at which $\mathbf{r}$ stretches
- The θ-component of $\mathbf{v}$ is due to the rotation θ

Acceleration



$$\mathbf{r} = r\,\mathbf{e}_r$$



$$\dot{\mathbf{e}}_\mathbf{r} = \dot{\theta}\,\mathbf{e}_\theta$$

$$\mathbf{v} = \dot{r}\,\mathbf{e}_\mathbf{r} + r\dot{\theta}\,\mathbf{e}_\theta$$

$$\mathbf{a} = \dot{\mathbf{v}} = \left(\ddot{r}\,\mathbf{e}_\mathbf{r} + \dot{r}\,\dot{\mathbf{e}}_\mathbf{r}\right) + \left(\dot{r}\dot{\theta}\,\mathbf{e}_\theta + r\ddot{\theta}\,\mathbf{e}_\theta + r\dot{\theta}\,\dot{\mathbf{e}}_\theta\right)$$

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e}_\mathbf{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta$$

$$\dot{\mathbf{e}}_\theta = -\dot{\theta}\,\mathbf{e}_r$$

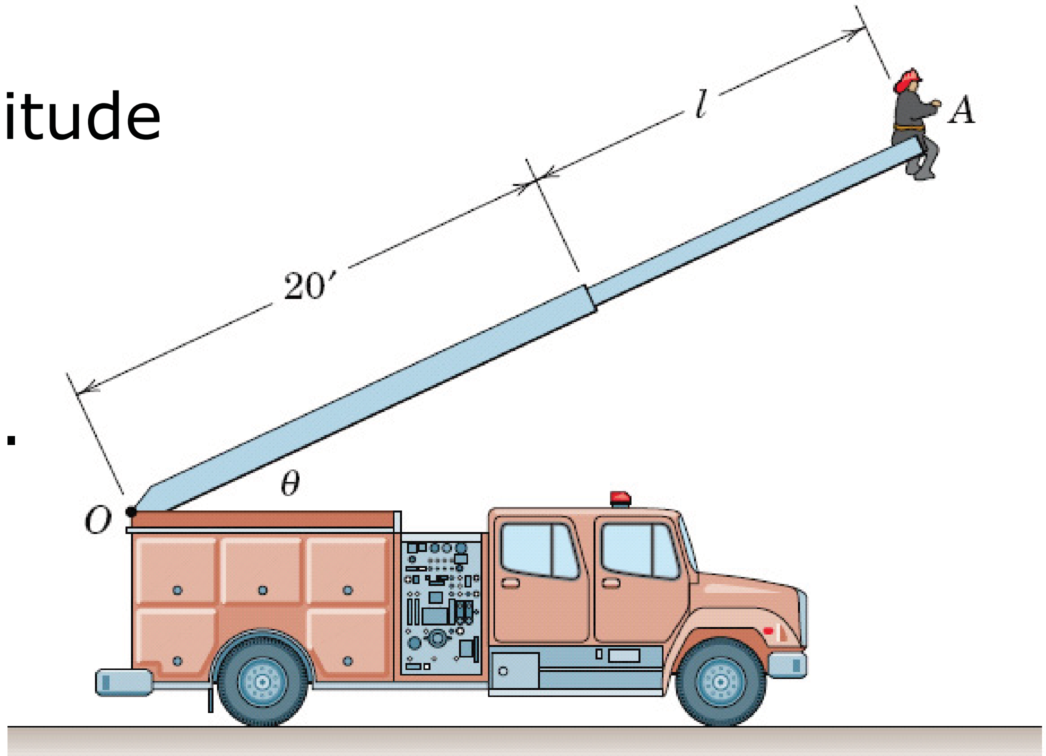
Velocity and Acceleration: Exercise

$$\mathbf{v} = \dot{r}\,\mathbf{e_r} + r\dot{\theta}\,\mathbf{e}_\theta$$

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e_r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta$$

A fire truck ladder ***extends*** at a constant rate of 6 in/s and ***elevates*** at a constant rate of 2 º/s.

Determine the magnitude of ***velocity*** and ***acceleration*** of the fireman at A when $\theta = 50º$ and $l = 15$ ft.



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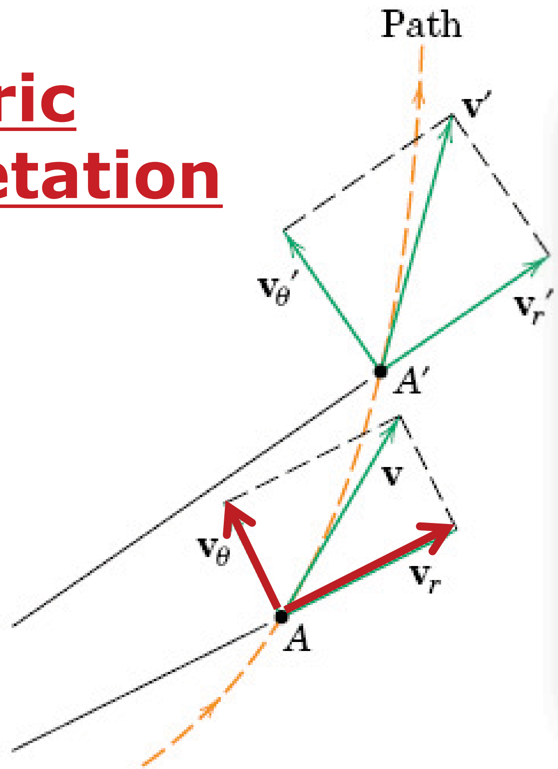
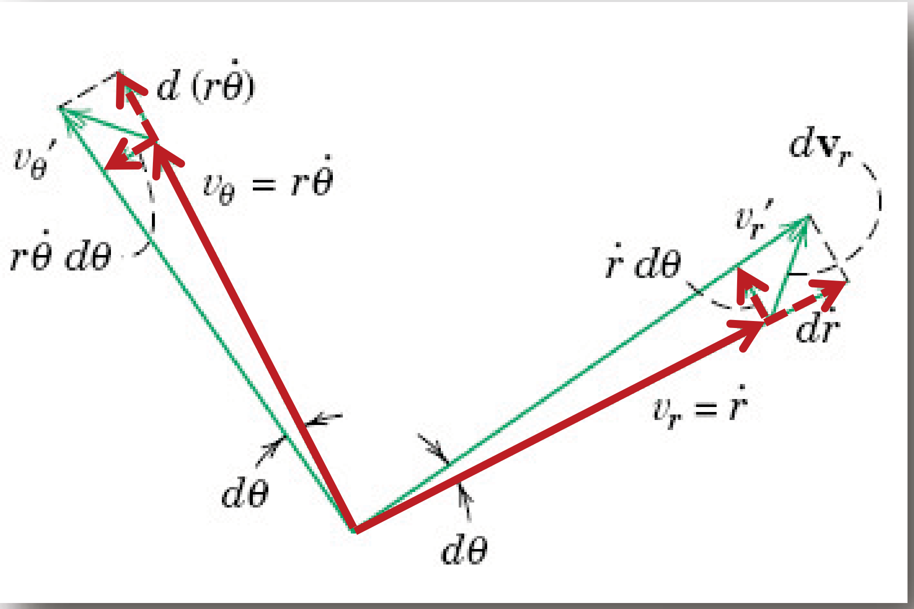
Geometric Interpretation



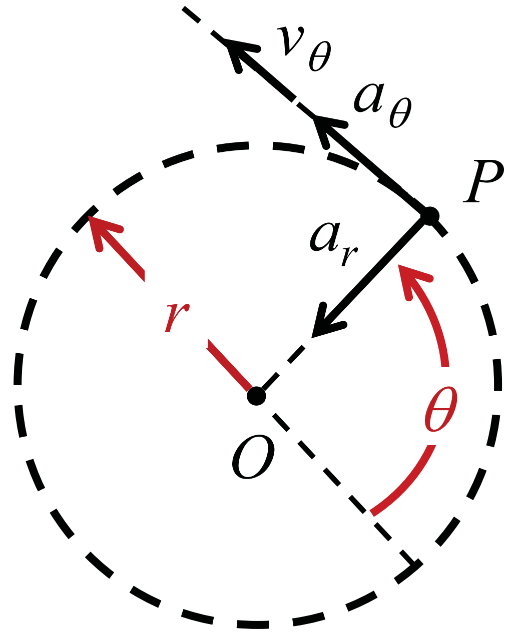


- ***Magnitude*** change of $\mathbf{v}_r$ is $d\dot{r}$
- ***Direction*** change of $\mathbf{v}_r$ is $\dot{r}\,d\theta$
- ***Magnitude*** change of $\mathbf{v}_\theta$ is $d\left(r\dot{\theta}\right)$
- ***Direction*** change of $\mathbf{v}_\theta$ is $r\dot{\theta}\,d\theta$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\,\dot{r}\dot{\theta}$$

Circular Motion



The ***radial distance*** (r) becomes constant

$$v_r = \dot{r} \longrightarrow v_r = 0$$

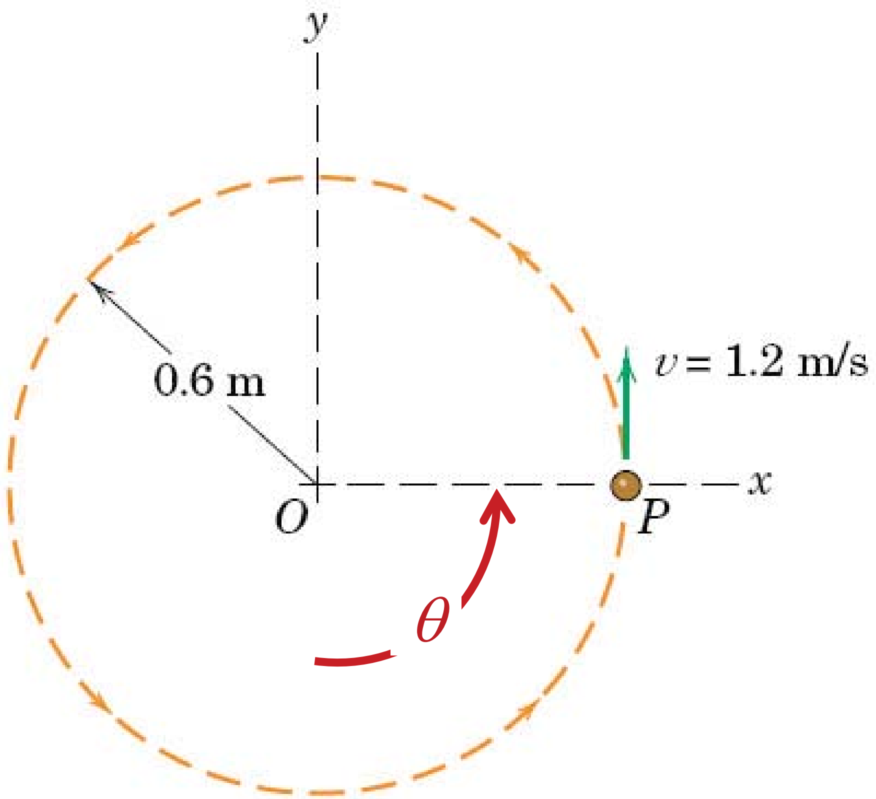
$$v_\theta = r\dot{\theta} \longrightarrow v_\theta = r\dot{\theta}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \longrightarrow a_r = -r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \longrightarrow a_\theta = r\ddot{\theta}$$

Circular Motion: Exercise

Particle *P* moves in a circular path shown.



Determine the magnitude of ***acceleration*** for:

(a) constant ***velocity*** 1.2 m/s
(b) ***velocity*** 1.2 m/s and increasing 2.4 m/s each second
(c) ***velocity*** 1.2 m/s and decreasing 4.8 m/s each second

$$v_\theta = r\dot{\theta}$$

$$a_r = -r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta}$$

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For Next Time…

- Continue Homework #2 due next Wednesday (9/5) at the ***beginning of class***
- Read Chapter 2, Section 2.8

- *Have a great Labor Day Holiday!*