Space Curvilinear (*Three-Dimensional*) Motion

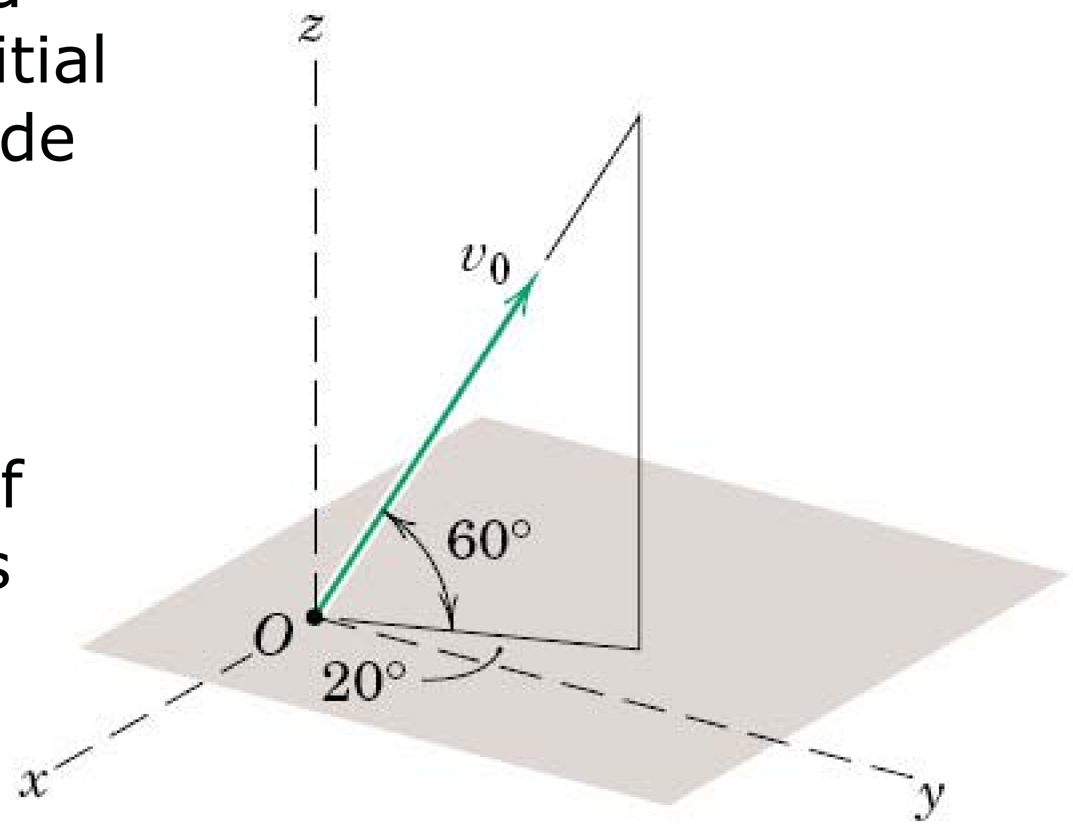
Lecture 6

ME 231: Dynamics

Question of the Day

A projectile launched from point O with initial ***velocity*** of magnitude of v_0 = 600 ft/s.

Compute the x-, y-, and z-components of ***velocity*** 20 seconds after launch.



Outline for Today

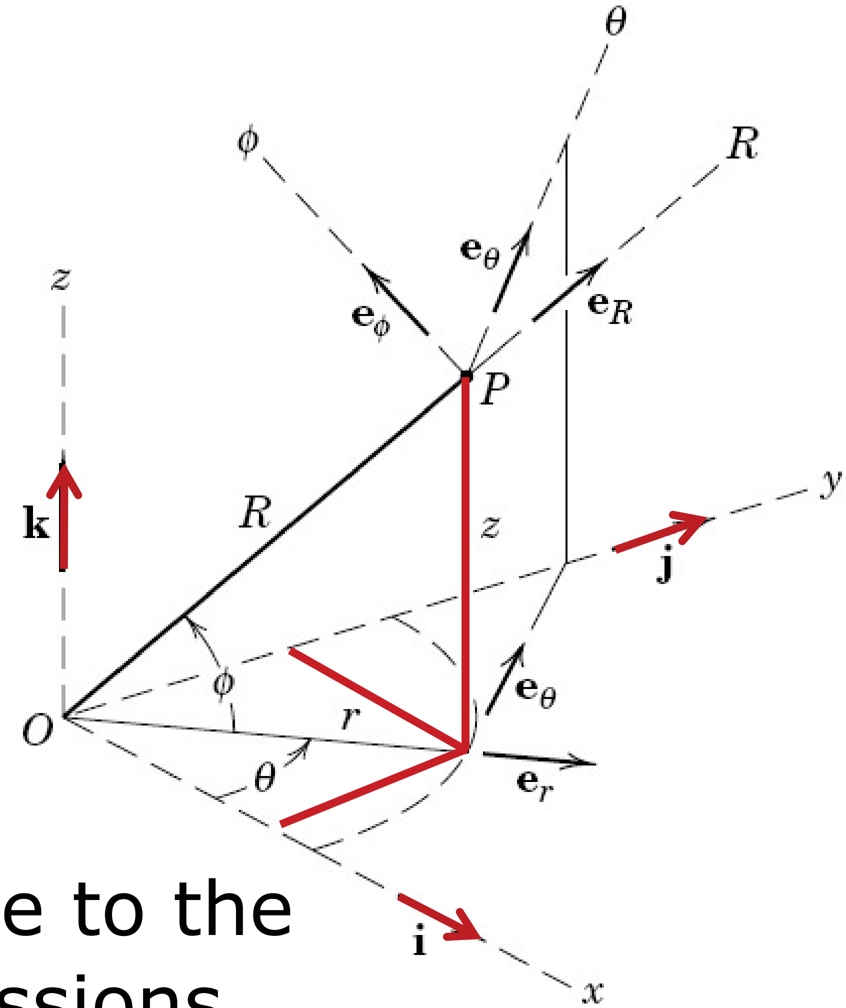
- Question of the day
- Rectangular coordinates (x, y, z)
- Cylindrical coordinates (r, θ, z)
- Spherical coordinates (R, θ, ϕ)
- Answer your questions!

Rectangular coordinates (x, y, z)

$$\mathbf{R} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$$

$$\mathbf{v} = \dot{\mathbf{R}} = \dot{x}\,\mathbf{i} + \dot{y}\,\mathbf{j} + \dot{z}\,\mathbf{k}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{R}} = \ddot{x}\,\mathbf{i} + \ddot{y}\,\mathbf{j} + \ddot{z}\,\mathbf{k}$$



Simply add z-coordinate to the two-dimensional expressions

Rectangular coordinates (x, y, z): Exercise

A particle moving in three-dimensions has a ***position*** vector ($\mathbf{r}$) as a function of ***time*** (t) with coordinates given by

$$x(t) = 30\cos(2t),\quad y(t) = 40\sin(2t),\quad z(t) = 20t + 3t^2$$

where $\mathbf{r}$ is measured in millimeters and t is in seconds.

Determine the magnitude of the ***velocity*** ($\mathbf{v}$) and the ***acceleration*** ($\mathbf{a}$) at time t =2 s.

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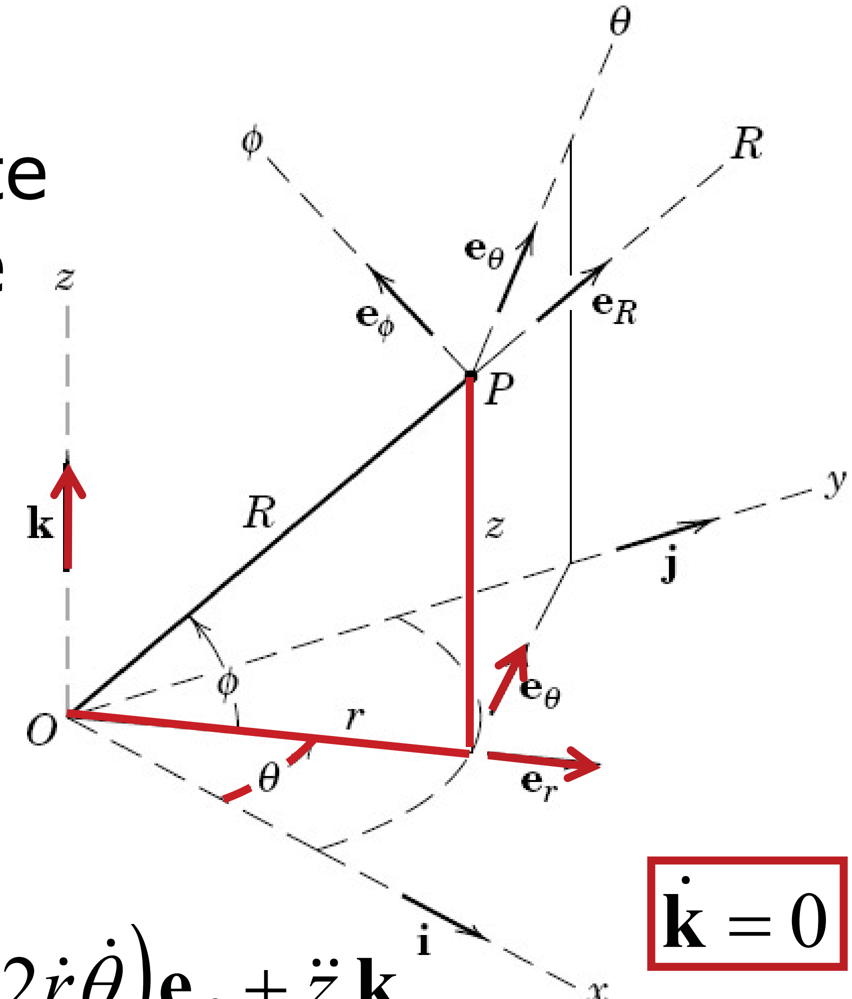
Cylindrical coordinates (r, θ, z)

$$\dot{\mathbf{e}}_{\mathbf{r}} = \dot{\theta}\,\mathbf{e}_{\theta}$$

$$\dot{\mathbf{e}}_{\theta} = -\dot{\theta}\,\mathbf{e}_{r}$$

Simply add z-coordinate to the polar-coordinate expressions



$$\mathbf{R} = r\,\mathbf{e}_{\mathbf{r}} + z\,\mathbf{k}$$

$$\mathbf{v} = \dot{\mathbf{R}} = \dot{r}\,\mathbf{e}_{\mathbf{r}} + r\dot{\theta}\,\mathbf{e}_{\theta} + \dot{z}\,\mathbf{k}$$

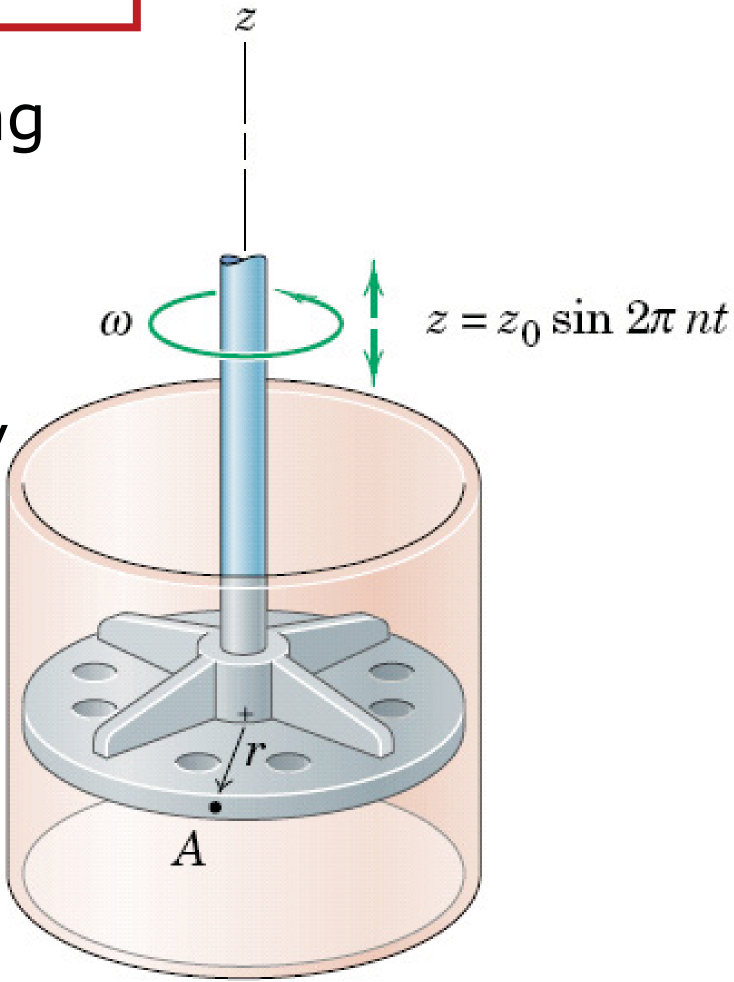
$$\dot{\mathbf{k}} = 0$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{R}} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e}_{\mathbf{r}} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_{\theta} + \ddot{z}\,\mathbf{k}$$

Cylindrical coordinates (r, θ, z): Exercise

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e_r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta + \ddot{z}\,\mathbf{k}$$

The rotating element of a mixing chamber has a periodic ***axial movement*** $z = z_0 \sin(2\pi nt)$ while rotating at the constant ***angular velocity*** ω. Frequency n is constant.



Determine the magnitude of the ***acceleration*** of point A on the rim of radius r.

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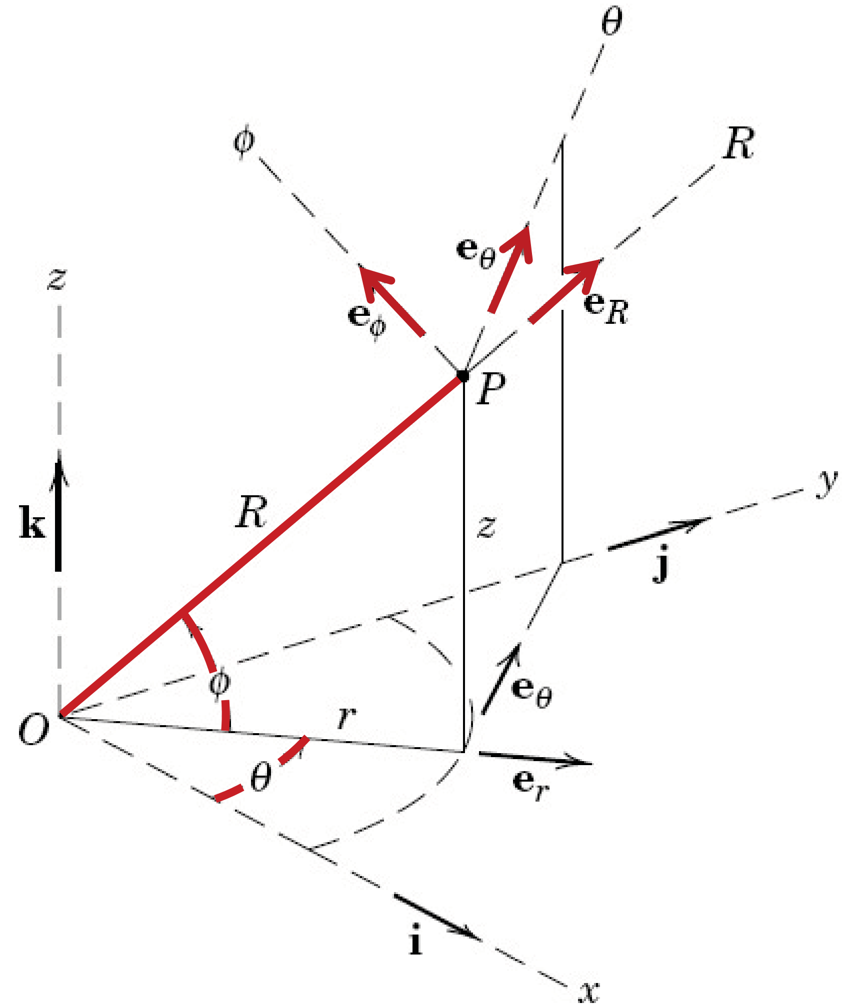
Spherical coordinates (R, θ, ϕ)

$$\mathbf{R} = R\,\mathbf{e}_R$$

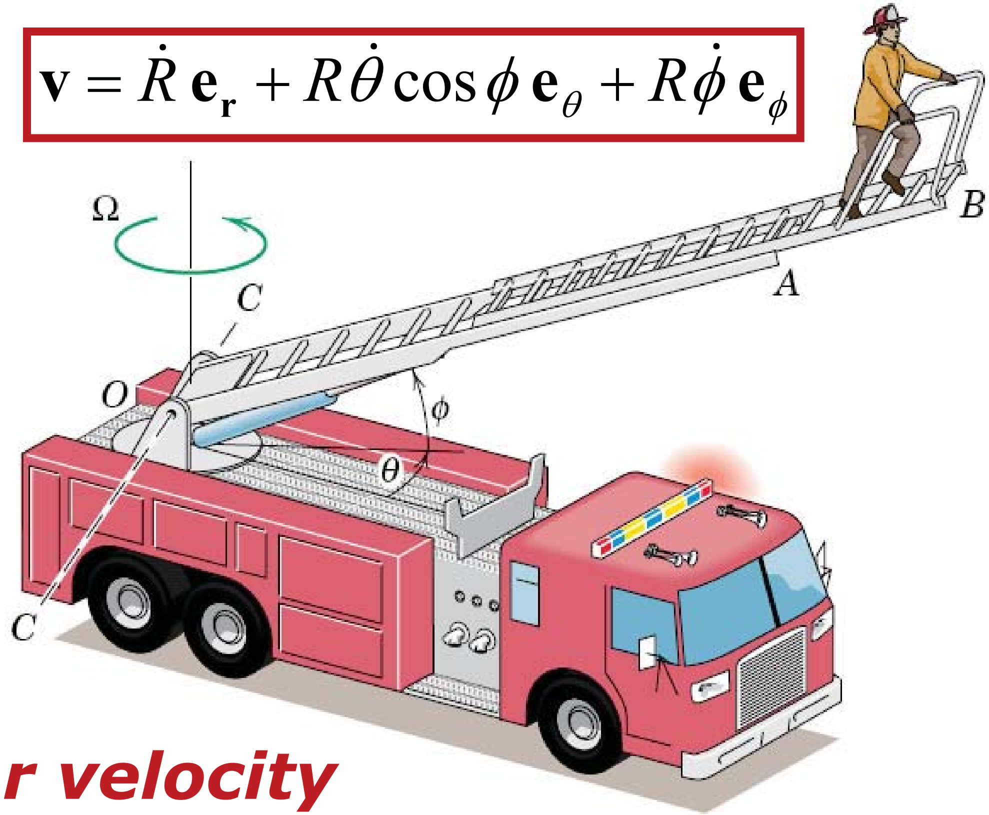
$$\mathbf{v} = \dot{\mathbf{R}} = \dot{R}\,\mathbf{e}_R + R\dot{\theta}\cos\phi\,\mathbf{e}_\theta + R\dot{\phi}\,\mathbf{e}_\phi$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{R}} = \left(\ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi\right)\mathbf{e}_R$$
$$+\left(\frac{\cos\phi}{R}\frac{d\left(R^2\dot{\theta}\right)}{dt} - 2R\dot{\theta}\dot{\phi}\sin\phi\right)\mathbf{e}_\theta$$
$$+\left(\frac{1}{R}\frac{d\left(R^2\dot{\phi}\right)}{dt} + R\dot{\theta}^2\sin\phi\cos\phi\right)\mathbf{e}_\phi$$



Spherical coordinates (R, θ, ϕ): Exercise

$$\mathbf{v} = \dot{R}\,\mathbf{e_r} + R\dot{\theta}\cos\phi\,\mathbf{e}_\theta + R\dot{\phi}\,\mathbf{e}_\phi$$



A firetruck ladder rotates with constant ***angular velocity*** Ω = 10 º/s, elevates with a constant ***angular velocity*** $\dot{\phi}$ = 7 º/s, and ***extends*** with a constant rate 0.5 m/s.

When ϕ = 30º and the ladder is 15 m long, determine the magnitude of the ***velocity*** of the end B of the ladder.

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For Next Time...

- Homework #2 was due ***at the beginning of class***
- Begin working on Homework #3 due Wednesday (9/12) at the beginning of class

- Read Chapter 2, Section 2.7