# Rotation

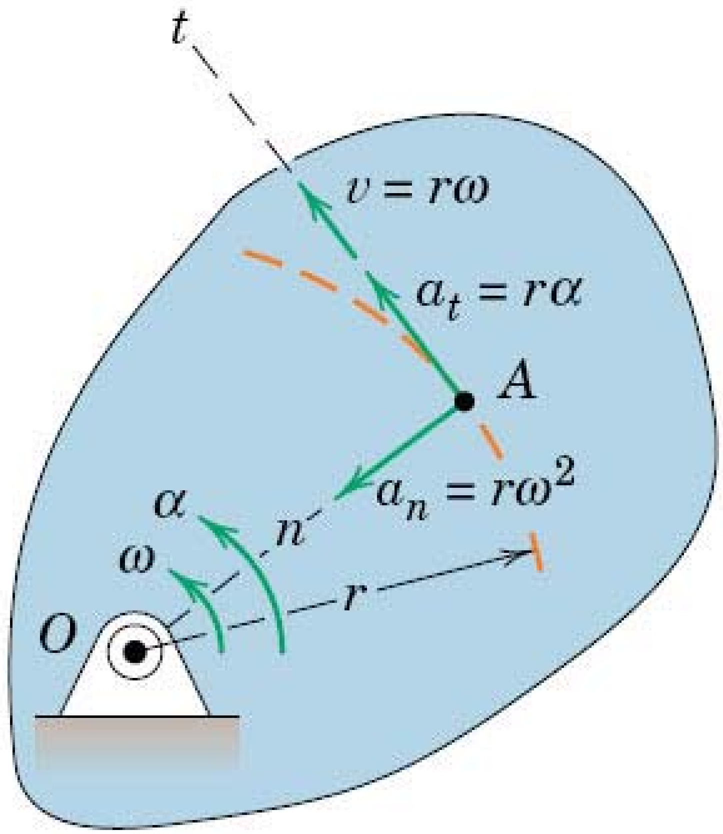
**Lecture 9**

**ME 231: Dynamics**

# Question of the Day

Point $A$ on a rigid body rotating about an axis fixed at $O$ has a ***velocity*** of 4 m/s and ***tangential acceleration*** of 8 m/s$^2$. The radius ($r$) of the point $A$'s path is 2 m.

Determine the ***angular velocity*** ($\omega$) and ***angular acceleration*** ($\alpha$) of the rigid body.



# Admin Details

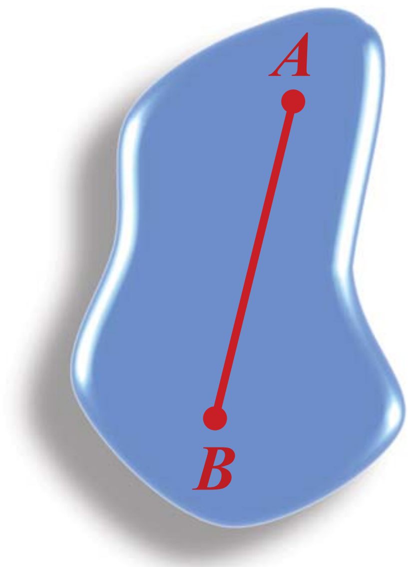
**Before requesting my help, you should help yourself.**

1. Dig beyond the lectures (Google, books, etc.)
2. Be truly resourceful
   - Make a list of 3 possible solution approaches to resolve the problem or understand it better
   - Try each possibility
   - Make detailed notes on what you did/learned (What is the problem exactly? What is known? What is unknown? What information is available or needs to be acquired to solve for the unknowns? Does the solution make sense?)
   - Make a new list of 3 possible solution approaches and repeat the process above
3. Ask your study group for help (How did others do it?)
4. Ask the teaching assistant, Misagh, for help

## Outline for Today

- Question of the day
- Rigid-body assumption
- Plane motion
- Rotation
- Angular-motion relations
- Rotation about a fixed axis
- Vector representation
- Answer your questions!

# Rigid-Body Assumption

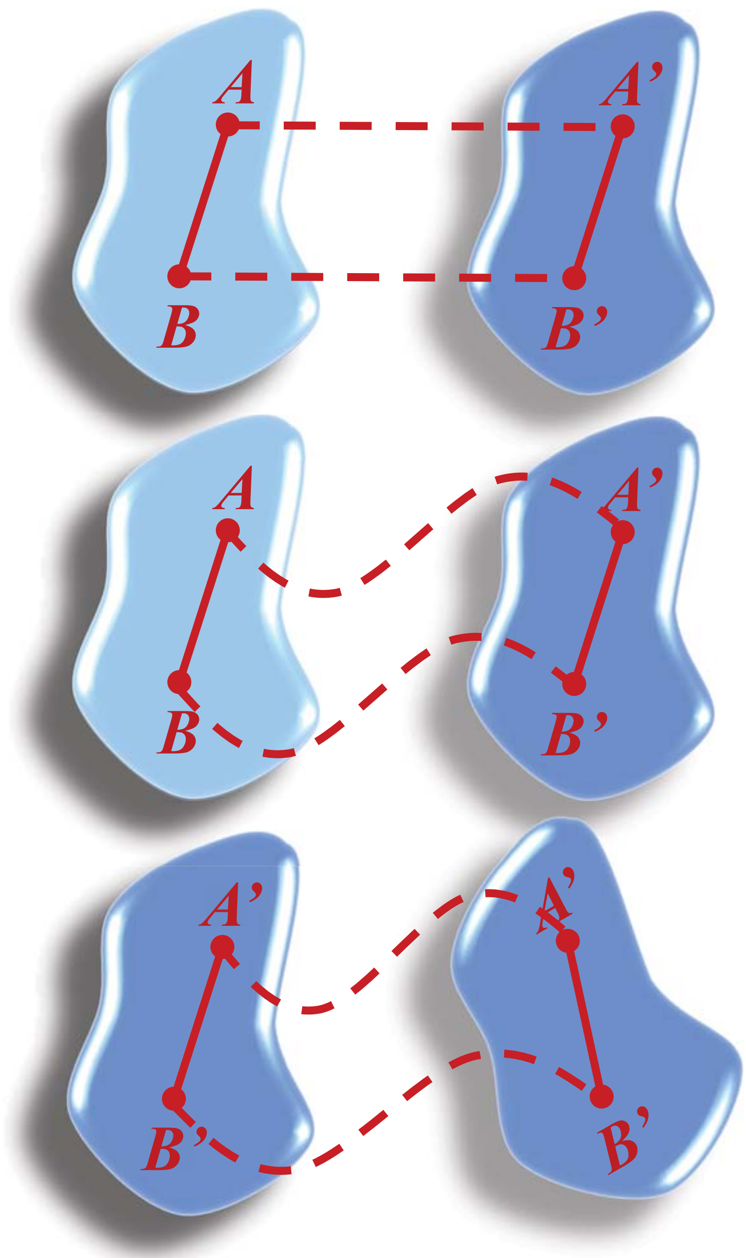


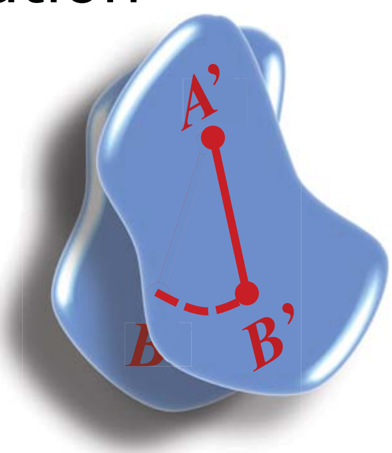

- Rigid body: a ***system of particles*** with ***constant distances between particles***
- ***Movement*** of the body **>>** its ***shape changes***

*Example: overall flight path of a jet is not affected by wing flutter*

# Plane Motion

- Rigid body is a thin slab
- Motion in the plane of slab
- Rectilinear translation
- Curvilinear translation
- Fixed-axis rotation
- General plane motion



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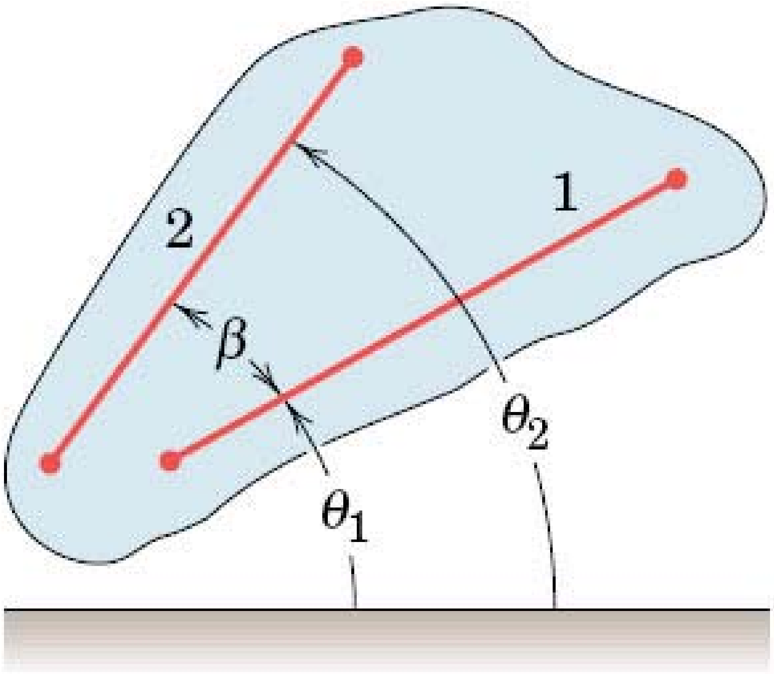
## Rotation

- Rotation of a body is described by its ***angular motion***
- ***Angular positions*** of lines **1** and **2** are specified by $\boldsymbol{\theta_1}$ and $\boldsymbol{\theta_2}$
- The angle $\boldsymbol{\beta}$ is constant

$$\theta_2 = \theta_1 + \beta$$

$$\dot{\theta}_2 = \dot{\theta}_1$$

$$\ddot{\theta}_2 = \ddot{\theta}_1$$



# Angular-Motion Relations

**A key concept in dynamics!**

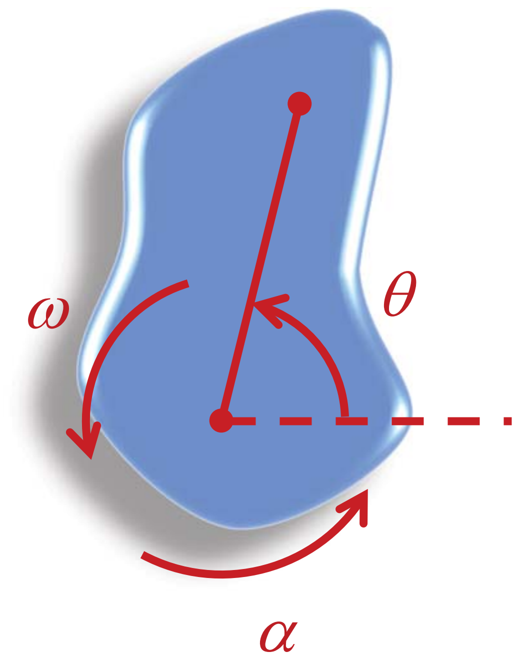
- Angular position ($\theta$)
- Angular velocity ($\omega$)

$$\omega = \frac{d\theta}{dt} = \dot{\theta}$$

- Angular acceleration ($\alpha$)

$$\alpha = \frac{d\omega}{dt} = \dot{\omega} = \frac{d^2\theta}{dt^2} = \ddot{\theta}$$

$$\omega \, d\omega = \alpha \, d\theta$$



# Integrating Constant Angular Acceleration

$$\alpha = \frac{d\omega}{dt} \qquad \int_{\omega_0}^{\omega} d\omega = \alpha \int_0^t dt \qquad \omega = \omega_0 + \alpha t$$

$$\alpha = \omega \frac{d\omega}{d\theta} \qquad \int_{\omega_0}^{\omega} \omega\, d\omega = \alpha \int_{\theta_0}^{\theta} d\theta \qquad \omega^2 = {\omega_0}^2 + 2\alpha\left(\theta - \theta_0\right)$$

$$\omega = \frac{d\theta}{dt} \qquad \int_{\theta_0}^{\theta} d\theta = \int_0^t \left(\omega_0 + \alpha t\right) dt \qquad \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

***CAUTION: Equations only for constant angular acceleration***

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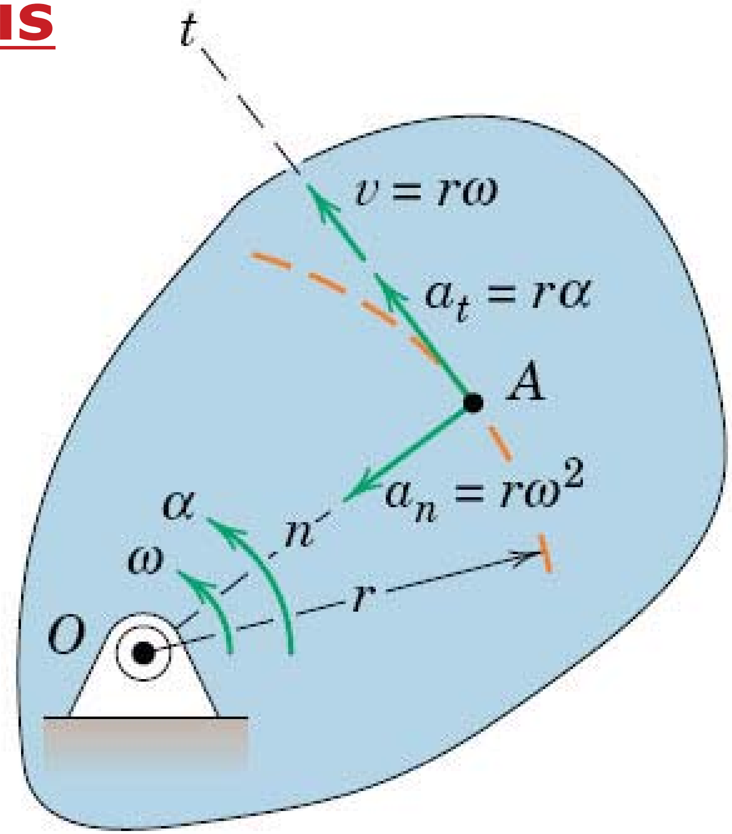
# Rotation About a Fixed Axis

- All ***points*** (*other than those on the axis*) move in ***concentric circles*** about the axis
- ***Point*** $A$ moves in a circle of ***radius*** $r$
- ***Angular motion*** of ***normal line*** is the ***angular motion*** of the ***rigid body***




$$v = r\omega$$

$$a_n = r\omega^2 = \frac{v^2}{r} = v\omega$$

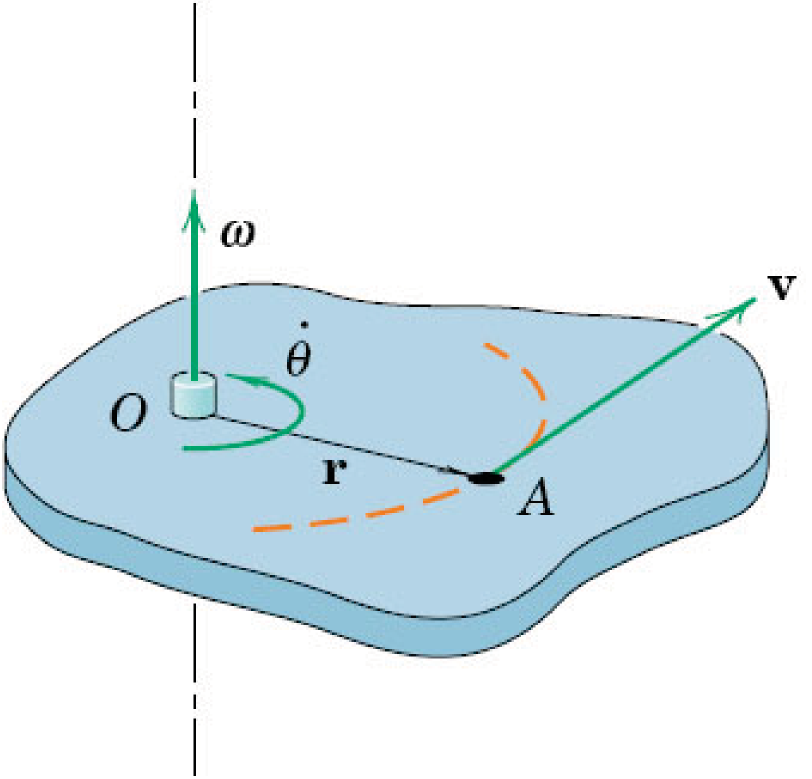
$$a_t = r\alpha$$

# Vector Representation: Angular Velocity

$$\boxed{\mathbf{v} = \dot{\mathbf{r}} = \boldsymbol{\omega} \times \mathbf{r}}$$

$$\boxed{-\mathbf{v} = \mathbf{r} \times \boldsymbol{\omega}}$$

- ***Angular velocity*** ($\omega$) is normal to plane of rotation and sense governed by the right-hand rule
- ***Velocity*** ($\mathbf{v}$) is the cross product of $\omega$ and $\mathbf{r}$
- Cross product gives the ***magnitude*** and ***direction*** for $\mathbf{v}$



# Vector Representation: Angular Acceleration

- ***Acceleration*** ($\mathbf{a}$) is the time derivative of ***velocity*** ($\mathbf{v}$)

$$\mathbf{a} = \dot{\mathbf{v}} = \boldsymbol{\omega} \times \dot{\mathbf{r}} + \dot{\boldsymbol{\omega}} \times \mathbf{r}$$

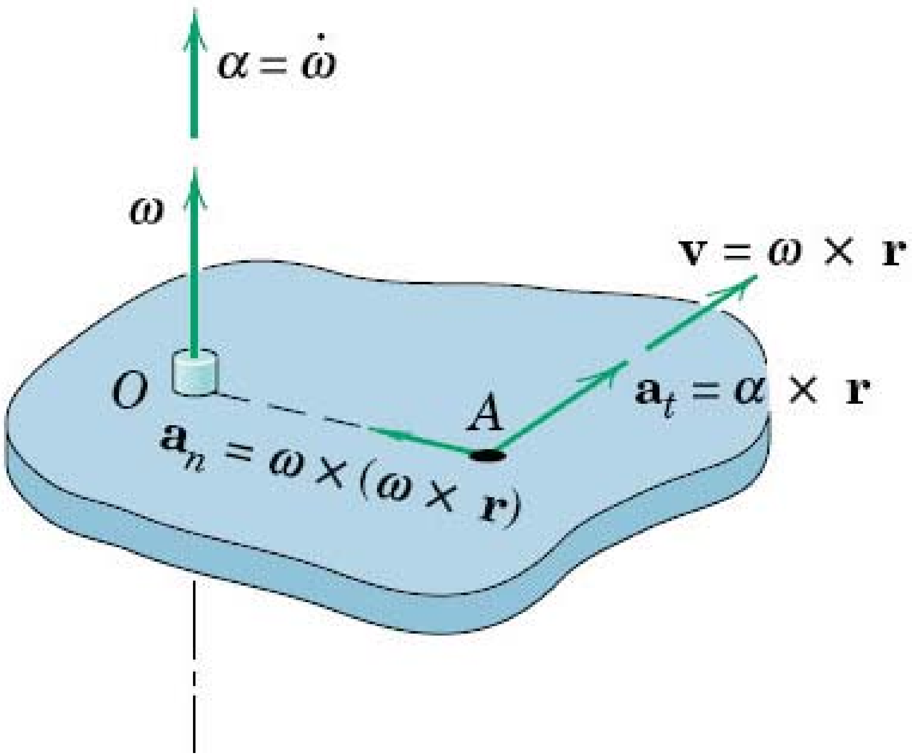
$$\mathbf{a} = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \dot{\boldsymbol{\omega}} \times \mathbf{r}$$

$$\mathbf{a} = \boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\alpha} \times \mathbf{r}$$

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$$

$$\mathbf{a}_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

$$\mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r}$$



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## For Next Time…

- Begin Homework #4 due next week (9/19)
- Read Chapter 6, Sections 6.1 and 6.2