Relative Acceleration
Lecture 13
ME 231: Dynamics
The *acceleration* of the cart is 4 \( \text{ft/s}^2 \) to the right.

Determine the *angular acceleration* of the wheel so that *point A* on the top of the rim has a *horizontal component* of *acceleration* equal to *zero*. 
Outline for Today

- Question of the day
- Relative acceleration due to rotation
- Interpretation of $a_A = a_B + a_{A/B}$
- Solution of relative-acceleration eq.
- Answer your questions!

- What about next week?
Recall: Relative Motion

\[ \mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B} \]

\[ \mathbf{v}_A = \dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \dot{\mathbf{r}}_{A/B} \]

\[ \mathbf{a}_A = \ddot{\mathbf{v}}_A = \ddot{\mathbf{r}}_A = \ddot{\mathbf{r}}_B + \ddot{\mathbf{r}}_{A/B} \]

- Absolute position of \( B \) is defined in an inertial coordinate system \( X-Y \)
- Attach a set of translating (non-rotating) axes \( x-y \) to particle \( B \) and define the position of \( A \)
- Define position of “\( A \ relative to \ B \)” (“\( A/B \)”’) in \( x-y \)
Relative Acceleration Due to Rotation

\[ \mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B} \]

\[ \mathbf{v}_{A/B} = \dot{\mathbf{r}} = \omega \times \mathbf{r} \]

\[ \mathbf{a}_{A/B} = \dot{\mathbf{v}}_{A/B} = \omega \times \dot{\mathbf{r}} + \dot{\omega} \times \mathbf{r} \]

From *translating* (non-rotating) axes \( x-y \) attached to point \( B \), the acceleration is a simply due to *circular motion* about \( B \)
Interpretation of Relative-Acceleration Eq.

**Translational portion**

\[ \mathbf{a}_A = \mathbf{a}_B + (\mathbf{a}_{A/B})_t + (\mathbf{a}_{A/B})_n \]

**Rotational portion**

\[ (\mathbf{a}_{A/B})_t = \mathbf{a} \times \mathbf{r} \]

\[ (\mathbf{a}_{A/B})_n = \mathbf{\omega} \times (\mathbf{\omega} \times \mathbf{r}) \]
Outline for Today

• Question of the day
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• Interpretation of $\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$
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Solution of Relative-Acceleration Eq.: Exercise

A truck has forward \textit{acceleration} \( a = 12 \text{ ft/s}^2 \) rolling without slipping its 24” tires.

Determine the \textit{velocity} of the truck when point \( P \) in the \textit{position} shown will have \textit{zero horizontal component} of \textit{acceleration}.
Solution of Relative-Acceleration Eq.: Exercise

Calculate the *angular acceleration* of the plate, where \( OA \) has a constant *angular velocity* \( \omega_{OA} = 4 \text{ rad/s} \) and \( \theta = 60^\circ \) for both links.
Solution of Relative-Acceleration Eq.: Exercise

Link $OA$ has constant angular velocity $\omega = 4 \text{ rad/s}$. Determine the angular acceleration $\alpha_{AB}$ of link $AB$ when $OA$ is parallel to the horizontal axis through $B$. 
Outline for Today

• Question of the day
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• Interpretation of $a_A = a_B + a_{A/B}$
• Solution of relative-acceleration eq.
• Answer your questions!

• What about next week?
What about next week?

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For Next Time...

- Continue Homework #5 due next Wednesday (9/26)
- Read Chapter 6, Sections 6.3 and 6.4