Linear Impulse and Momentum
Lecture 27
ME 231: Dynamics
An ice-hockey puck with mass of 0.20 kg has a velocity of 12 m/s before being struck by the stick. After a 0.04 s impact, the puck moves in the new direction shown with a velocity of 18 m/s.

Determine the magnitude of average force $F$ exerted by the stick on the puck during contact.
Outline for Today

• Question of the day
• From $\mathbf{F}=ma$ to impulse and momentum
• Linear impulse and momentum
• Linear impulse-momentum principle
• Conservation of linear momentum
• Answer your questions!
Recall: Possible Solutions to Kinetics Problems

- Direct application of *Newton’s 2nd Law*
  - force-mass-acceleration method
    - Chapters 3 and 7
- Use of *impulse* and *momentum* methods
  - Chapters 5 and 8
- Use of *work* and *energy* principles
  - Chapter 4
From $F=ma$ to impulse and momentum

- **Integrate** equations of motion with respect to **time**
- **Linear impulse** $(F*t)$ on $m$ equals change in **linear momentum** $(G)$ of $m$
- Facilitates the **solution** of problems where **forces** act over **specified time** interval or during extremely **short periods of time** (e.g., **impact**)

\[ \sum F = ma \]

\[ \int_{t_1}^{t_2} \sum F \, dt = \int_{t_1}^{t_2} \frac{d}{dt}(mv) \, dt \]

\[ \int_{t_1}^{t_2} \sum F \, dt = \int_{t_1}^{t_2} \dot{G} \, dt \]
Linear impulse and momentum

- Particle of mass $m$ is located by position vector $\mathbf{r}$
- **Velocity** $\mathbf{v}$ is tangent to its path
- **Resultant** $\Sigma \mathbf{F}$ of all forces on $m$ is in the direction of its acceleration $\mathbf{a}$
- Valid only when mass $m$ is **constant**

$$\Sigma \mathbf{F} = m\dot{\mathbf{v}} = \frac{d}{dt}(m\mathbf{v})$$

$$\Sigma F_x = \dot{G}_x$$
$$\Sigma F_y = \dot{G}_y$$
$$\Sigma F_z = \dot{G}_z$$

$G = mv$
Linear Impulse-Momentum Principle

\[ \sum F = \dot{G} \]

\[ \int_{t_1}^{t_2} \sum F \, dt = \int_{t_1}^{t_2} \dot{G} \, dt \]

\[ G_1 + \int_{t_1}^{t_2} \sum F \, dt = G_2 \]

**Impulse-momentum diagram**

\[ m(v_1)_x + \int_{t_1}^{t_2} \sum F_x \, dt = m(v_2)_x \]

\[ m(v_1)_y + \int_{t_1}^{t_2} \sum F_y \, dt = m(v_2)_y \]

\[ m(v_1)_z + \int_{t_1}^{t_2} \sum F_z \, dt = m(v_2)_z \]

\[ G_2 = m \mathbf{v}_2 \]

- **Integrate** to describe the effect of the **resultant force** \( \sum F \) on **linear momentum** over a finite period of **time**
Conservation of Linear Momentum

\[ \Sigma F = \dot{G} \]

\[ \int_1^2 \Sigma F \, dt = \int_1^2 \dot{G} \, dt \]

\[ G_1 + \int_1^2 \Sigma F \, dt = G_2 \]

\[ \Delta G = 0 \]

or

\[ G_1 = G_2 \]

- If the \textbf{resultant force} \( \Sigma F \) is zero, then \textbf{linear momentum} remains \textbf{constant}, or is said to be \textbf{conserved}

- Linear momentum may be \textit{conserved} in \textit{one coordinate} (e.g., \( x \)), but \textbf{not necessarily} in \textit{others} (e.g., \( y \) or \( z \))
A jet fighter with a mass of 6450 kg requires 10 seconds from rest to reach its takeoff speed of 250 km/h under constant thrust $T = 48$ kN.

Determine the time average of the combined air and ground resistance $R$ during takeoff.
A 100-lb boy runs with a velocity of 15 ft/s and jumps on his 20-lb sled. The sled and boy coast 80 ft on level snow before coming to rest.

Determine the coefficient of kinetic friction between the snow and sled.
Linear Impulse-Momentum: Yet Another Exercise

A 2.4-kg particle moves in the $x$-$y$ plane and has the velocity shown at time $t = 0$. A force $F = 2 + 3t^2/4$ Newton's is applied in the $y$-direction at $t = 0$.

Determine the velocity of the particle 4 seconds after $F$ is applied and specify the angle $\theta$ measured counter clockwise from the $x$-axis to the direction of the velocity.
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For Next Time...

- Continue Homework #9 due on **Thursday (11/1)**
- Read Chapter 5, Section 5.3