HOMEWORK #3

BME 473 ~ Applied Biomechanics

Inverse Kinematics and Inverse Dynamics for Course Project and Final Exam

Not Due

1. We discussed different angle set conventions in class. One common convention is a Body-fixed X-Y-Z rotation sequence. With this convention, the B frame is first rotated about $\mathbf{b}_x$ by angle θ_x, then rotated about $\mathbf{b}_y$ by angle θ_y, and finally rotated about by $\mathbf{b}_z$ angle by θ_z. This rotation sequence results in the following rotation matrix (where $c_x = \cos\theta_x$, $s_x = \sin\theta_x$, etc.).

$$
{}^{A}\mathbf{R}^{B}_{BodyXYZ} = \begin{bmatrix} c_y c_z & -c_y s_z & s_y \\ s_x s_y c_z + c_x s_z & -s_x s_y s_z + c_x c_z & -s_x c_y \\ -c_x s_y c_z + s_x s_z & c_x s_y s_z + s_x c_z & c_x c_y \end{bmatrix}
$$

   Using the examples from class as a primer, and using the atan2 function, please derive the formulas necessary to extract the values of angles θ_x, θ_y, and θ_z from the above rotation matrix. For simplicity, you may assume that no angles equal either 0° or 90°. Please express your answers in terms of elements of the rotation matrix (r_{xx}, r_{yz}, etc.)

2. You are interviewing to work in a motion capture laboratory. During the job interview, you are presented with the 3-D marker convention shown in Figure 1. Since you have taken BME 473, you feel confident in your ability to understand this marker convention. You quickly suspect that, for the shank, markers 3-5 are used to establish a Segment Coordinate Frame (*Shank*) and that markers 1,2,6, and 7 are used to establish the Skeletal Coordinate Frame (*Tibia*). You relay your suspicions to your interviewer, who is quite impressed with your ability.

   a. You are told that markers 3-5 do form the *Shank* Coordinate Frame. Marker 4 is used as the origin. The x-axis is directed from marker 4 towards marker 3. The y-axis is normal to the x-axis and the vector from marker 4 towards marker 5. The z-axis is normal to both the x-axis and the y-axis.

      Using the values for the location the markers in the *Lab* Coordinate Frame shown in Figure 1, please derive the 4x4 transformation matrix that expresses the *Shank* Coordinate Frame (defined using markers 3-5) with respect to the *Lab* Coordinate Frame for the conditions shown in Figure 1.

   b. You are once again correct and are told that markers 1, 2, 6, and 7 do form the *Tibia* Coordinate Frame (at this point, you're feeling good about the interview). You are told the following: "The origin for the *Tibia* Coordinate Frame is the midpoint of markers 1 and 2. The z-axis is directed from the origin towards the midpoint of markers 6 and 7. The y-axis is directed anteriorly and is normal to the z-axis and a temp vector from marker 1 to marker 2. The x-axis normal to both the y-axis and z-axis and is directed medially" (Note: numerous labs use this convention.)

Using the values for the location the markers in the *Lab* Coordinate Frame shown in Figure 1, please derive the transformation matrix that expresses the *Tibia* Coordinate Frame (defined using markers 1, 2, 6, and 7) with respect to the *Lab* Coordinate Frame for the conditions shown in Figure 1.

c. Using your solutions to parts a and b, please derive the transformation matrix that expresses the *Tibia* Coordinate Frame (defined using markers 1,2,6, and 7) with respect to *Shank* Coordinate Frame (defined using markers 3-5).

d. You know that the markers used to establish the *Tibia* Coordinate Frame (defined using markers 1, 2, 6, and 7) are frequently removed from a subject during a motion analysis study. As a final test during your job interview, you are asked to find the knee flexion angle (θ_x from question 1), the knee varus/valgus angle (θ_y), and the internal rotation angle (θ_z) using a Body-fixed X-Y-Z rotation sequence for some future instance in time (assume the *Femur* is the fixed frame "A" and that the *Tibia* is the moving frame "B"). Please provide your answer in degrees. To solve for the angles, please use your answer to part c and the following additional information:

$$^{Lab}T^{Shank} = \begin{bmatrix} -0.1514 & 0.9478 & 0.2805 & 15.7069 \\ -0.9224 & -0.2375 & 0.3045 & -4.7283 \\ 0.3552 & -0.2126 & 0.9103 & 25.0473 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{Lab}T^{Thigh} = \begin{bmatrix} -0.4542 & 0.7576 & -0.4687 & 5.1436 \\ -0.7900 & -0.0994 & 0.6049 & -3.2432 \\ 0.4117 & 0.6451 & 0.6437 & 63.9444 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{Thigh}T^{Femur} = \begin{bmatrix} -0.2978 & -0.1859 & -0.9363 & -18.5119 \\ -0.6382 & 0.7682 & 0.0505 & -7.7569 \\ 0.7099 & 0.6127 & -0.3474 & -14.3123 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

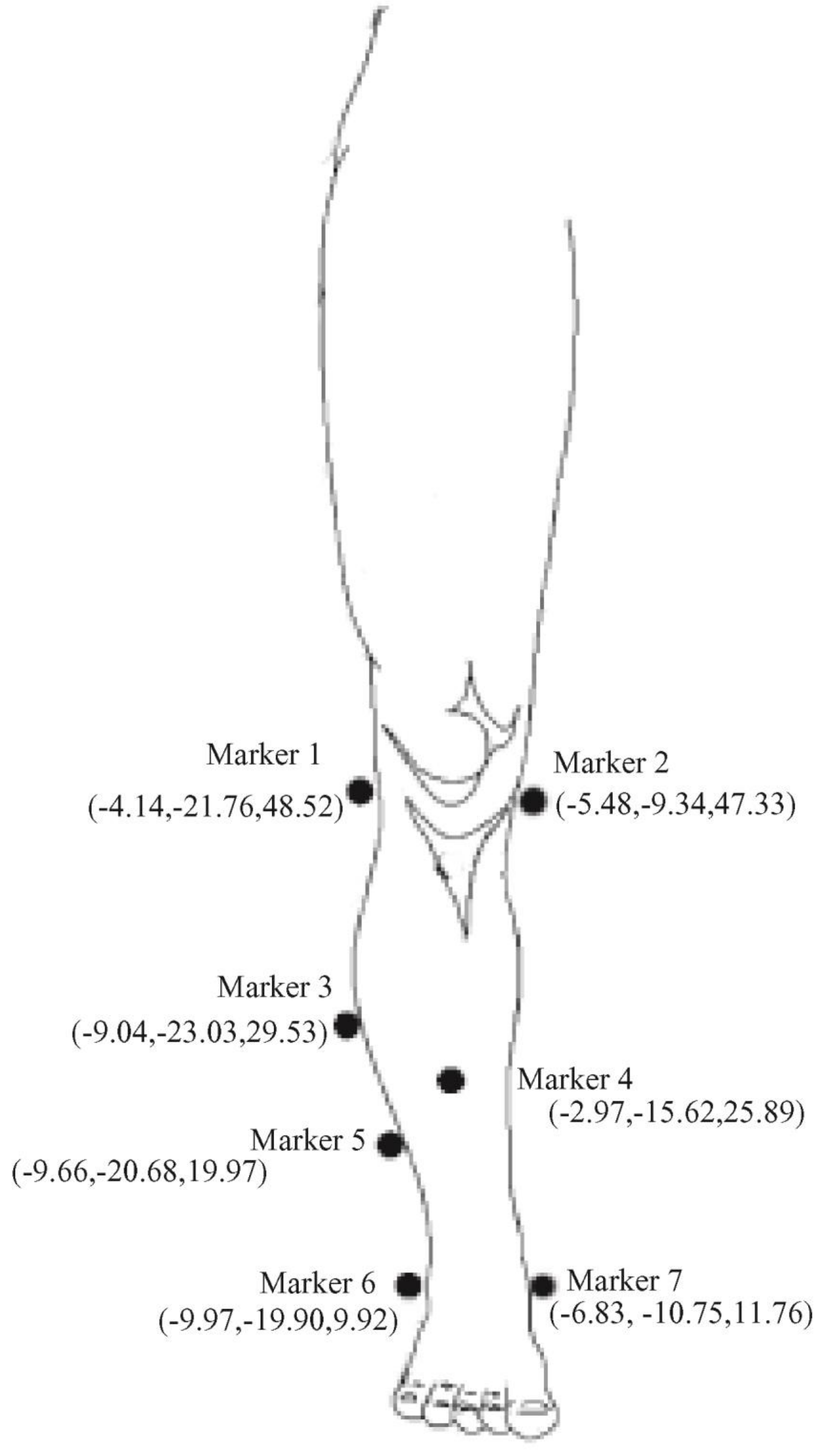

Figure 1: Reflective Makers on a Right Leg

3. A common problem encountered by the elderly is loss of balance, which sometimes results in falling. One biomechanical hypothesis suggests that falling in the elderly results from a lack of sufficient muscle strength. To study the issue of strength, suppose we asked a number of elderly women to rise from a chair in our motion lab. Our aim is to estimate the net joint torques for the major joints of the lower extremity during this task. Once we calculate the joint torques used to stand up from a chair, we can compare them to the maximum joint torques that our subjects can generate (maximum joint torques are a measure of muscle strength). This comparison will show what percentage of our subjects' maximum strength must be used to rise from a chair.

   Suppose we used high-speed video cameras, together with markers positioned judiciously over various body landmarks to estimate the time history of the absolute angular displacements of the shank, thigh, and trunk (i.e., θ1, θ2, θ3 in Figure 2). These data were subsequently filtered and numerically differentiated to obtain the absolute angular velocities and accelerations of each of these body segments over time. Assuming that the sit-to-stand activity can be characterized by predominantly planar (2-D) motion of all the body segments, a simple three-segment, planar linkage is used to model the human skeleton (Figure 2). Further assume that the body segments are connected by frictionless single degree-of-freedom revolute (i.e., hinge) joints.

   In terms of *kinematic data only*:

   a. Please derive complete analytical expressions for the net joint torques exerted at the ankle (T1), knee (T2), and hip (T3) during sit-to-stand motion. Assume that the foot remains flat on the floor.

   b. Please derive complete analytical expressions for the ground reaction forces exerted in the horizontal (fore-aft) and vertical directions during the sit-to-stand motion. Again assume that the foot remains flat on the floor and that it is massless. Note that if you could measure Fgx and Fgy that you could calculate T1 without any kinematic data.

   c. Would you consider this problem to be inverse or direct dynamics? Why?

   d. What is the physical significance of the intersegmental forces (e.g., F1y)? Are the calculated intersegmental forces equal to the true bone-on-bone forces? Why or why not?

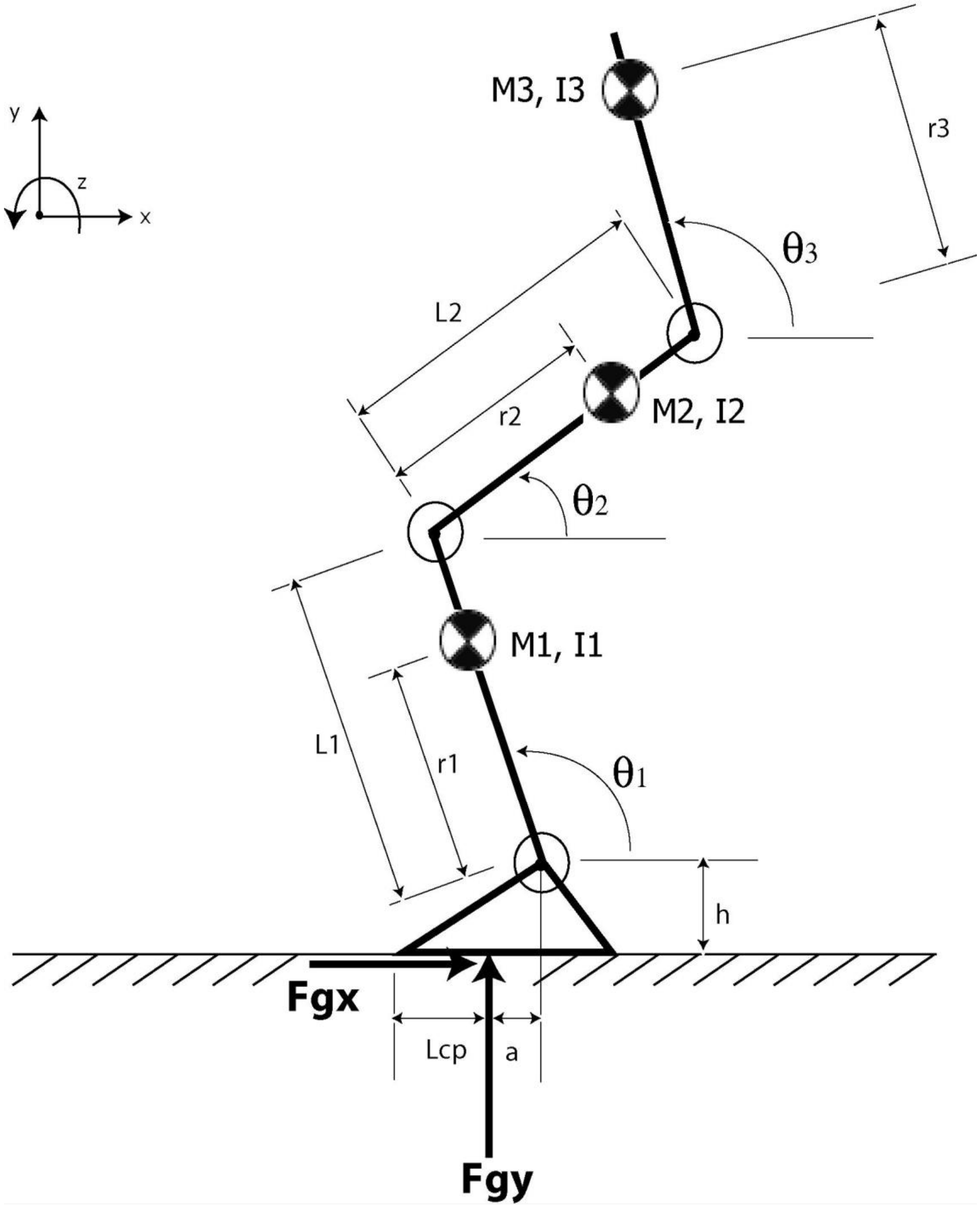

Figure 2: Planar 3 segment model

4. Using our preliminary data from problem 3, we were able to land a small grant and have used the money to purchase a force plate. With our improved experimental set-up we can now measure the horizontal (fore-aft) and vertical ground reaction forces during the sit-to-stand motion. Also, our video system remains operational, and we shall continue to record the time history of the angular displacements of the shank, thigh, and trunk.

   On the basis of the three segment, planar model given in Figure 2, use the *kinematic and force-plate data* now available to:

   a. Derive complete analytical expressions for the net joint torques exerted at the ankle (T1), knee (T2), and hip (T3) during the sit-to-stand motion. Again, assume that the foot remains flat on the floor.

   b. By comparing the terms in your equations for T1, T2, and T3 derived in problems 4a and 3a, discuss the advantages to being able to accurately estimate the ground reaction forces and relevant kinematic parameters when calculating joint torques.